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Physiographical space-based kriging for regional flood frequency estimation at ungauged sites

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[1] A physiographical space-based kriging method is proposed for regional flood frequency estimation. The methodology relies on the construction of a continuous physiographical space using physiographical and meteorological characteristics of gauging stations and the use of multivariate analysis techniques. Two multivariate analysis methods were tested: canonical correlation analysis (CCA) and principal components analysis. Ordinary kriging, a geostatistical technique, was then used to interpolate flow quantiles through the physiographical space. Data from 151 gauging stations across the southern part of the province of Quebec, Canada, were used to illustrate this approach. In order to evaluate the performance of the proposed method, two validation techniques, cross validation and split-sample validation, were applied to estimate flood quantiles corresponding to the 10, 50, and 100 year return periods. Results of the proposed method were compared to those produced by a traditional regional estimation method using the canonical correlation analysis. The proposed method yielded satisfactory results. It allowed, for instance, for estimating the 10 year return period specific flow with a coefficient of determination of up to 0.78. However, this performance decreases with the increase in the quantile return period. Results also showed that the proposed method works better when the physiographical space is defined using canonical correlation analysis. It is shown that kriging in the CCA physiographical space yields results as precise as the traditional estimation method, with a fraction of the effort and the computation time. *INDEX TERMS:* 1860 Hydrology: Runoff and streamflow; 1869 Hydrology: Stochastic processes; 1821 Hydrology: Floods; *KEYWORDS:* regional flood estimation, kriging, canonical correlation analysis, principal components analysis, frequency analysis, streamflow

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1. Introduction

[2] Streamflow data are essential for optimal surface water resource management. Normally, information about this resource is derived from flow gauging stations. However, in many parts of the world, hydrometric networks are characterized by their low density and their inconvenient spatial distribution. Also, even when hydrometric networks are well developed historical hydrological data are not always available at the site of interest. Consequently, regional frequency analysis is commonly used for the estimation of flow characteristics at sites where little or no data are available. Numerous regional estimation procedures and techniques were proposed and applied in various areas over the world [*Dalrymple*, 1960; *Burns*, 1990; *Rosbjerg and Madsen*, 1994; *GREHYS*, 1996a, 1996b; *Ouarda et al.*, 1999; *Pandey and Nguyen*, 1999; *Daviau et al.*, 2000; *Ouarda et al.*, 2001; *Grover et al.*, 2002; *Mic et al.*, 2002]. Moreover, there is a large consensus that regional frequency analysis yields much more reliable flood quantile estimators than does the at-site approach (local frequency analysis) when only short records are available [*Durrans and Tomic*, 1996]. Indeed, spatial information is used to make up for the paucity of temporal information.

[3] In regional flood frequency analysis, the two main steps are the identification of groups of hydrologically homogeneous basins and the application of a regional estimation method within each delineated region to estimate flood characteristics at the site of interest [*GREHYS*, 1996a]. The delineation of homogeneous regions aims to identify sites which show a similar hydrological behavior. In fact, variation in flow characteristics is closely related to the variation in regional physiographical and climatic factors [*Pandey and Nguyen*, 1999]. Physiographic and meteorological characteristics can then be used to produce estimates of flow statistics at ungauged sites or at sites where records are short.

[4] Homogeneous regions can be defined as geographically contiguous regions, geographically noncontiguous regions, or as hydrological neighborhoods. The use of geographically defined homogeneous regions is convenient for practical purposes. However, geographical proximity is not a guarantee of hydrological similarity [*Reed et al.*, 1999; *Ouarda et al.*, 2001]. *GREHYS* [1996b] clearly indicated that the neighborhood approach is superior to the fixed region approach. In the neighborhood approach each target site is assumed to have its own homogeneous region. The

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neighborhood is composed of a set of gauged sites for which the proximity to the target site is represented by a distance within a multidimensional space defined by the hydrological, physiographical and meteorological characteristics of gauged catchments. Two main neighborhood methods were proposed: the region of influence [*Burns*, 1990]; and canonical correlation analysis [*Cavadias*, 1990; *Ribeiro-Corréa et al.*, 1995; *Ouarda et al.*, 2000]. Several estimation methods such as the index flood approach [*Dalrymple*, 1960; *GREHYS*, 1996a, 1996b] and linear, nonlinear or nonparametric regression [*Gingras and Adamowski*, 1992; *Pandey and Nguyen*, 1999; *Grover et al.*, 2002] may be used to estimate flood frequencies in combination with one homogeneous regions delineation method.

[5] The application of the neighborhood concept in its classical way is laborious and time consuming. For instance, neighborhood delimitation methods, such as canonical correlation analysis, need elaborated statistics for the definition of a neighborhood ellipse [*Ribeiro-Corréa et al.*, 1995; *Ouarda et al.*, 2001]. Furthermore, this neighborhood definition is a compromise between the quantity of information (number of stations within the neighborhood) to consider in the regional analysis and the hydrological homogeneity of the group of neighbors [*Ouarda et al.*, 1999].

[6] On the other hand, despite the reserves raised against the use of the geographical proximity in regional flood frequency estimation, several authors assumed that flow characteristics can be regarded as continuous variables in the geographical space and still apply interpolation techniques such as kriging to flow regionalization [*Delhomme*, 1978; *Villeneuve et al.*, 1979; *Gottschalk*, 1993a, 1993b; *Huang and Yang*, 1998; *Daviau et al.*, 2000; *Haberlandt et al.*, 2001; *Eaton et al.*, 2002; *Grover et al.*, 2002]. However, hydrological variables are rather discrete variables in the geographical space. They may change dramatically over adjacent catchments since flood generation mechanisms are specific to each basin. Consequently, the direct application of interpolation methods in the geographical space runs up against a serious problem [*Sauquet*, 2000].

[7] Flood quantiles at a given site represent the hydrological response to the prevailing climate and reflect the signature of the basin physical and geomorphological characteristics. Therefore an appropriate interpolation technique over the physiographical space (a multidimensional space defined by the physiographical and meteorological characteristics of the gauged basins) may have a real potential for the regionalization of hydrological variables. Indeed, while they are discontinues in the geographical space, quantiles can be regarded as continuous variables in the physiographical space. In other terms, one can estimate flow quantiles at an ungauged site, knowing flow quantiles at "neighboring" gauged sites, and by using an appropriate interpolation technique.

[8] In the present study, we present the results of two models for the regional estimation of flood quantiles over the physiographical space. These methods are based on the use of a geostatisical technique, namely ordinary kriging, for the interpolation of flood quantiles over the physiographical space. Two approaches for defining the physiographical space were tested and compared: canonical correlation analysis and principal component analysis. The physiographical space-based kriging results were also compared to those produced by the traditional canonical correlation analysis regional estimation method [*Ouarda et al.*, 2001]. The proposed regional estimation procedure was applied to data from the hydrometric station network of the province of Quebec (Canada) to estimate flood quantiles corresponding to the 10, 50 and 100 year return periods.

2. Proposed Approach

[9] The proposed approach is based on the use of the basins coordinates in the physiographical space rather than the geographical space to interpolate the hydrological variables of interest over the physiographical domain. There are several ways which may be used to construct the physiographical space. In this work, we choose to apply the canonical correlation analysis and the principal component analysis techniques in order to characterize the interpolation space.

[10] The canonical correlation analysis (CCA) method is a statistical multivariate analysis tool which permits to describe the relationship of dependence existing between two sets of random variables [*Muirhead*, 1982]. It allows determining pairs of linear combinations of each set of variables, named the canonical variables, in such way that the correlation between the canonical variables of a pair is maximized and the correlation between the variables of different pairs is null. One thus obtains a set of canonical variables for the two sets of random variables. It is then possible to find the canonical variables of one set knowing the canonical variables of the other set. One can also calculate a distance between two canonical scores. This property is actually used as mentioned earlier to determine hydrologic neighborhoods.

[11] Given a set of hydrological variables, X, (streamflow quantiles in our case) and a set of physiographic and climatic variables, Y, characterizing the basins of interest, CCA aims to link the two sets using vectors of canonical variables: V for the physiographic/climatic variables and W for the hydrological variables. V and W are defined as linear combinations of X and Y. The linear combinations coefficients are estimated by maximizing the correlation between the random variables V and W. An interesting property of canonical variables is that the correlation coefficient between each pair of elements of either vector V or W is null. Thus both canonical variable vectors V and W may constitute an orthogonal basis for a physiographical and hydrological space, respectively. Knowing the linear combination coefficients vectors, one can precisely locate each available gauged basin within one or the other space. For complete description of the mathematical background of this approach and the use of CCA in regional flood frequency analysis, the reader is referred to *Ouarda et al.* [2001].

[12] Principal component analysis (PCA) is an exploratory multivariate statistical method for simplifying complex data sets [*Jackson*, 1991; *Basilevsky*, 1994]. Given an original set of variables, for instance, basin physiographic and climatic characteristics, PCA is used to generate a new set of variables, called principal components. Each principal component is a linear combination of the original variables. PCA consists of a transformation (rotation) of the original axis of the multidimensional physiographic/climatic space,

where axis are defined along each physiographic variable, to a new axis system defined along the principal components. The data variance along each axis is then maximized. All principal components are orthogonal (they are uncorrelated) so there is no redundant information. Thus principal components as a whole form an orthogonal basis for the data space.

[13] Since the physiographical and climatic characteristics of ungauged basins are known, one can place these basins in a CCA or a PCA physiographic/climatic space. Assuming the hydrological variable is a continuous variable over the transformed physiographical space, it is therefore possible to estimate its value for the ungauged basin of interest using its position in the transformed physiographical space and the information of the surrounding basins for which the hydrological variable values are already known.

[14] Geostatistics are powerful statistical techniques designed to study spatially autocorrelated variables [*Isaaks and Srivistava*, 1989; *Rossi et al.*, 1992]. They permit estimating the local value of a variable using sparse local measurements. These techniques take into account the spatial structure and distribution of the variables through tools known as structure functions such as variograms, covariograms or correlograms. These structure functions express the covariance between the observed points according to the distance which separates them. They describe the intensity and the pattern of the variable spatial autocorrelation.

[15] Ordinary kriging, the most popular geostatistical technique, produces an unbiased and optimal linear estimation of the unknown values. Thus it provides the best possible estimate using neighborhood information. The estimate is obtained by weighting each neighboring value. With respect to the spatial structure, the closest values receive higher weights because they are more likely to be similar to the unknown value being estimated. The unbiasedness is ensured by the "universal condition" where the sum of the weighting coefficients is equal to 1. The kriging estimation can be expressed as follows:

$$\begin{cases} Z^*(x_0) = \sum_{i=1}^{n} w_i Z(x_i) \\ \sum_{i=1}^{n} w_i = 1 \end{cases}, \quad (1)$$

where Z is the continuous variable of interest, $Z^*(x_0)$ its value being estimated at the unsampled position x_0, $Z(x_i)$ its known values at the n sampled locations x_i and w_i are the corresponding weighting coefficients.

[16] The exact weighting coefficients are calculated by modeling the spatial autocorrelation expressed in the structure function. The experimental (observed) structure function cannot be used directly in the calculation of the weights w_i, since it represents a discrete estimate of the spatial autocorrelation. Consequently, and in order to ensure the positive definiteness of the covariance matrix in the kriging system, a theoretical model selected among a limited set of authorized models is fitted to the observed structure function [*Isaaks and Srivistava*, 1989; *Arnaud and Emery*, 2000]. The choice of the model is the most crucial step and most difficult in the kriging application, since estimation quality depends on it. It must also be based on the real knowledge of the phenomenon rather than on the accuracy of the mathematical adjustment only. The model is used to calculate the coefficients w_i.

[17] It should be noted here that ordinary kriging relies on the basic assumption that the variable being estimated should be second-order stationary over the interpolation domain; that is, the variable values fluctuate around a constant value in space and these fluctuations have the same dispersion over all the whole space [*Arnaud and Emery*, 2000].

3. Case Study

3.1. Data

[18] The proposed approach was applied to southern Quebec's (Canada) hydrometric station network. Data of 151 hydrometric stations managed by the ministry of the environment of Quebec (MENVIQ) services were used. The selected stations were identified according to certain criteria. First, the gauged river should present a natural flow regime or, at least, influenced just on a daily scale. The station should present a historical record period longer than 15 years. The gauged basin area is superior to 200 km^2 but less than 100000 km^2. Finally, the selected stations must be located at the inhabited region of Quebec (between 45°N and 55°N). 90 out of 151 stations are still active. Figure 1 illustrates the location of these hydrometric stations.

[19] An at-site flood frequency analysis was carried out at each station of the database [*Kouider et al.*, 2002]. Data were tested for homogeneity and stationarity, and appropriate statistical distributions were fitted to data in order to estimate local flood quantiles corresponding to several return periods. In this study, we focused on spring flood quantiles corresponding to 10, 50 and 100 year return periods. *Eaton et al.* [2002] indicated that, in order to investigate the underlying physical behavior of drainage systems, scale effects must be eliminated from data. Consequently, we used specific quantiles (flood quantiles standardized by the basin area), noted $q10$, $q50$ and $q100$, in order to account for the scale effect.

[20] Initially, several physiographical and meteorological variables were available for each station. They were extracted from a spatial database gathered and implemented within the ArcView software [*Gignac et al.*, 2002]. Basins boundary, area, slope and land occupation as well as the drainage network were extracted from the MENVIQ hydrological database (BDH) and from the topographic digital maps of Quebec. Meteorological variables were calculated using interpolated historical data observed on the MENVIQ meteorological network across the province of Quebec. For each gauging station, the meteorological variables were estimated by an area weighted average of the variable of interest across the whole catchment.

[21] Only four variables were considered in the present study; two physiographical variables: basin mean slope (PMBV) and the fraction of the basin area covered with lakes (PLAC); and two meteorological variables: annual mean total precipitations (PTMA) and annual mean degree-days over 0°C (DJBZ). Table 1 lists the hydrological, physiographical and meteorological variables used herein as well as their descriptive statistics. The selected physiographical and meteorological variables appear to be the most relevant for the study at hand according to their

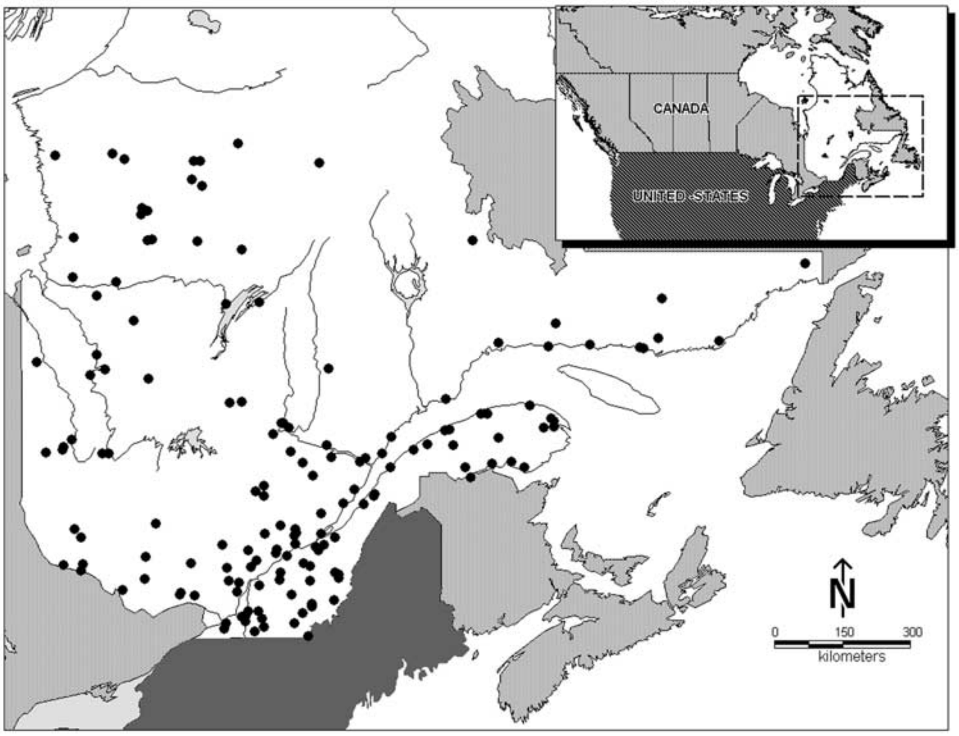

Figure 1. Localization of hydrometric stations across the province of Quebec (Canada).

correlation degree with the specific flood quantiles. Figure 2 shows the hydrological, physiographical and meteorological variable histograms in addition to their correlation coefficients and their interrelations.

[22] According to Figure 2 and Table 1, the selected physiographical and meteorological variables as well as the hydrological variables show an evident asymmetry. However, since CCA requires variable normality, the variables should be transformed prior to analysis. Thus a logarithmic transformation was applied to the specific flood quantiles, PMBV, PTMA and DJBZ. As for PLAC, a root transformation appeared to be more adequate. Even if the PCA does not require variable normality, the transformed variables were nevertheless used to calculate the principal components. Indeed, the first components explain more total variance when the variables are transformed than if they are not. As well, in order to account for scaling effects, all variables were standardized prior to CCA and PCA analysis.

3.2. Results

[23] Figure 3 presents the spatial distribution of the specific quantiles of the gauging stations through the CCA and PCA physiographical spaces. For the CCA, the space axes are made up of the two first canonical physiographical variables (V1 and V2). As for the PCA, the space axes consist of the two first principal components (CP1 and CP2). It should be noted that CP1 and CP2 explain 82.2% of the total variance (58.4% and 23.8%, respectively). Thus the specific quantiles exhibit an evident spatial pattern in both physiographical spaces. Nevertheless, the spatial patterns are different from one space to another. While they are more scattered over the PCA space, the gauging stations are grouped around the origin in the case of the CCA space. However, in the latter case, some gauging stations are located apart from the rest of the group: in the lower left and upper right corners. This may result in a higher risk of extrapolation within the CCA physiographical space. In both spaces, specific quantiles values vary inversely to the first axis of the space: CP1 for the PCA and V1 for the CCA. However, this variation is less marked along the second axes (CP2 and V2).

[24] In order to identify and measure the spatial structure within the hydrological variables at hand, the isotropic experimental variograms were calculated over both spaces (Figure 4). All variables demonstrate an obvious spatial

Table 1. Descriptive Statistics of Hydrological, Physiographical, and Meteorological Variables

Variable	Unit	Notation	Mean	Median	Max	Min	SD	Skewness	Kurtosis
100 year specific flood	$m^3/s.km^2$	$q100$	0.31	0.26	0.94	0.03	0.20	0.76	2.99
50 year specific flood	$m^3/s.km^2$	$q50$	0.28	0.24	0.77	0.03	0.18	0.64	2.65
10 year specific flood	$m^3/s.km^2$	$q10$	0.22	0.21	0.53	0.03	0.13	0.41	2.17
Basin mean slope	%	PMBV	2.43	2.14	6.81	0.96	0.99	1.17	5.10
Percentage of the basin occupied by lakes	%	PLAC	7.72	5.00	47.00	0.00	7.99	1.94	8.23
Annual mean total precipitation	mm	PTMA	988	996	1534	646	154	0.72	5.31
Annual mean degree-days over 0°C	degree-day	DJBZ	16,346	14,390	29,631	8589	5385	0.90	2.77

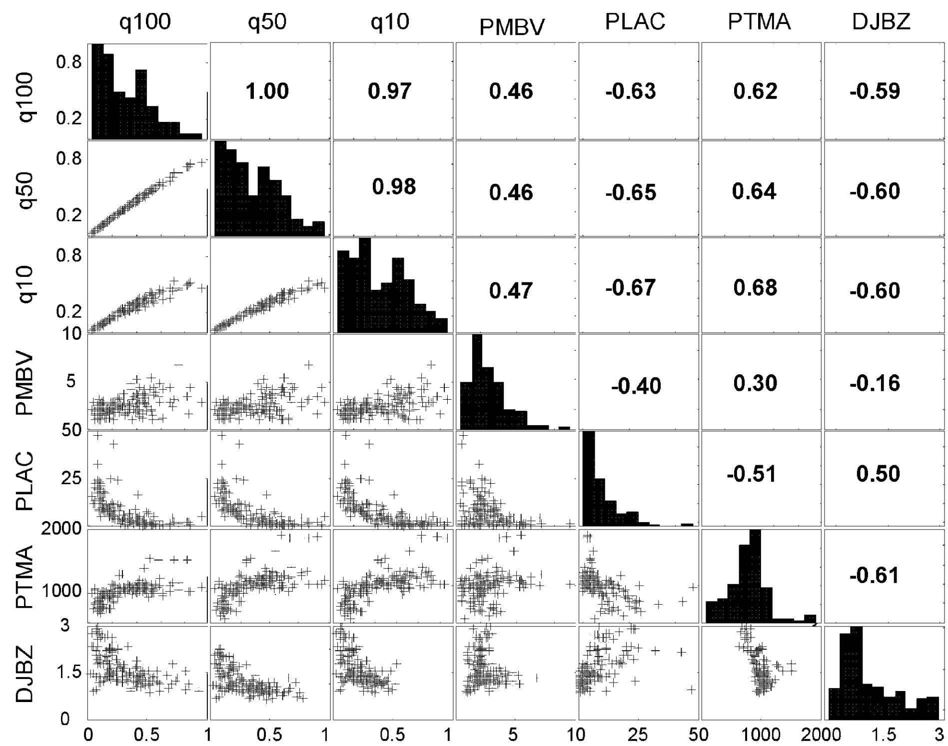

Figure 2. Variable histograms, correlation coefficients, and interrelations.

structure (spatial autocorrelation). Indeed, the experimental variograms show a small variance at short distance. For CCA, the variance then increases rapidly according to the separation distance before reaching a plateau, which indicates the disappearance of the spatial correlation. This variance level, commonly called the sill, is reached at a distance, known as the range, about 5, in the case of the CCA space. As for the PCA space, the experimental variograms show no sill, at least, at the scale of the observed sample. We assumed therefore that the hydrological variables reach their sills, in the PCA physiographical space, at a range located beyond the extent of the observed sample. This means that their spatial autocorrelation is prolonged beyond the observed sample range. In fact, one should not forget that this space was designed using observed combinations of basins physiographical and meteorological characteristics, which do not cover inevitably all the possible combinations of physiographical and meteorological variables. Consequently, the sample of gauging stations does not cover all the extent of the PCA physiographical space. However, the absence of sill in the PCA space variograms could be also explained by the fact that the specific quantiles would not stationary in this space. This means that there might be a gradual trend in variable values; that is, the variable values doest not fluctuate around a constant value in space but they vary according to space coordinates.

[25] Results also indicate that, for both spaces, $q100$ shows sills higher than $q50$ and $q10$. This is due to the fact that the $q100$ values are in turn higher than $q50$ which are higher than $q10$. Their respective sills are consequently ordered.

[26] The Gaussian model was fitted to all experimental variograms (Figure 4). Table 2 presents the characteristics of the theoretical models. The models were selected according to the pattern of the spatial structure shown by the experimental variograms. The Gaussian model is a transition model, which reaches its sill asymptotically and exhibits a parabolic behavior near the origin [*Isaaks and Srivistava*, 1989; *Arnaud and Emery*, 2000]. The latter property represents the distinguishing feature of the Gaussian model. It is used to model continuous phenomena like the one at hand. In fact, all experimental variograms show a slow progression of the variance at a short separation distance, indicating that the variables of interest are quite continuous over both spaces. This spatial continuity is the most pronounced for $q10$ and the least pronounced for $q100$ in both cases of the PCA space and the CCA space.

[27] In addition, a nugget effect was added to the Gaussian model. This parameter is used to model the discontinuity at the origin observed in the experimental variograms (Figure 4). The discontinuity is due to several factors related mainly to local estimation, sampling and/or localization errors. Indeed, the local quantiles used herein are estimated with a level of uncertainty, which increases with the return period. This appears clearly in the values of the estimated nugget effect (Table 2). Thus in both spaces, $q10$ presents a nugget effect value that is lower than that of $q50$ which in turn is lower than that of $q100$. One should note here that, as expected, the ranges of the theoretical models in the PCA space are larger than the observed sample extent (Table 2).

[28] The spatial autocorrelation models thus adjusted can be used for the estimation of specific quantiles using the kriging technique, at any point of the CCA and PCA physiographical space. It should be noted here that the estimation is carried out with an optimal neighborhood structure. Ideally, this is represented by a searching circle

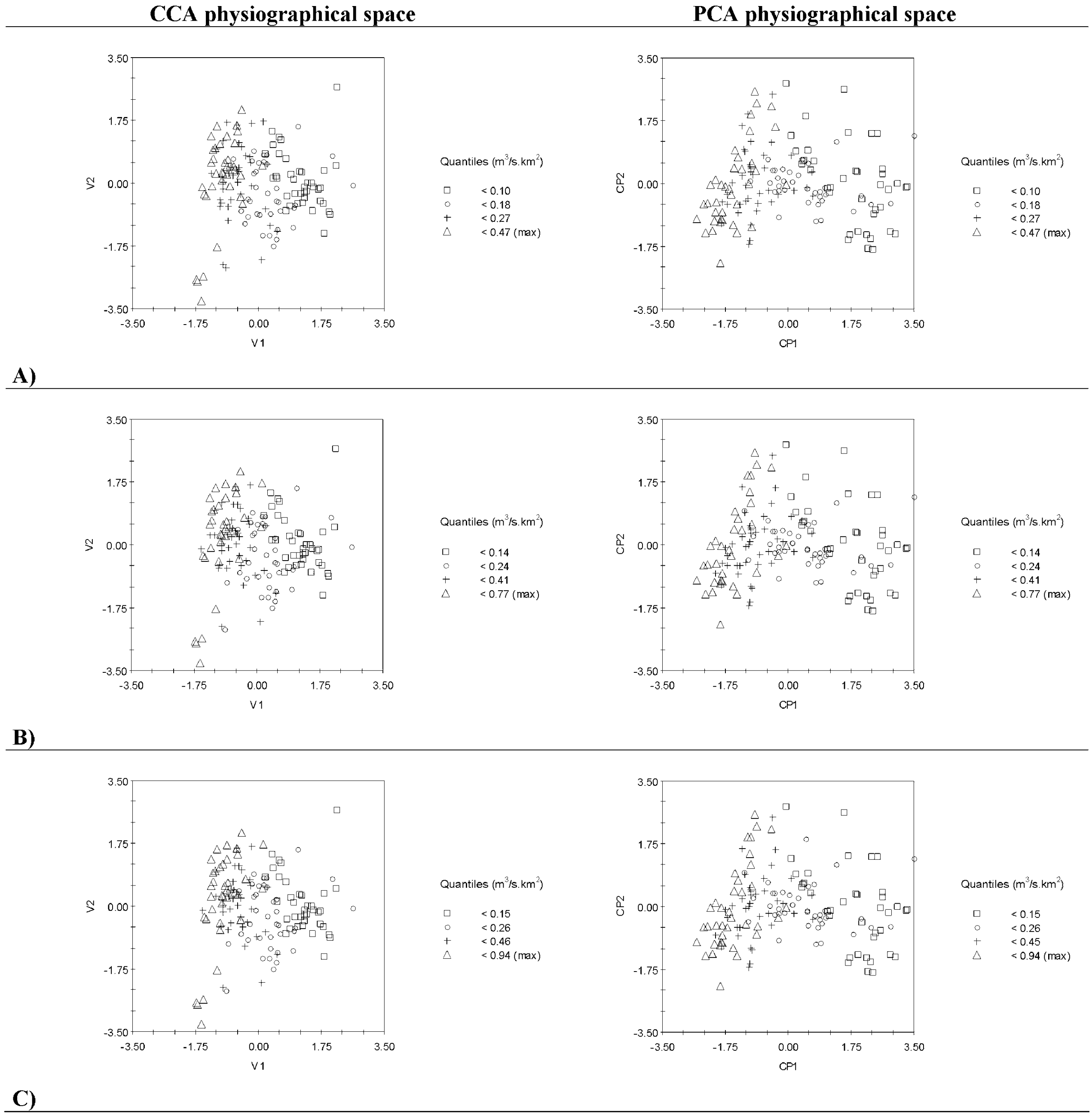

Figure 3. Flood quantiles across the CCA and PCA physiographical space: (a) $q10$, (b) $q50$, and (c) $q100$.

or ellipse (in the case of anisotropic spatial autocorrelation) that is divided into several sectors (at least four sectors) with an optimal number of observations by sector. The subdivision of the neighborhood structure aims to minimize the clustering effect as well as to limit the occurrence of extrapolation in the regions where there are few observations. However, the geostatistical software [*Gamma Design Software*, 2000], used in the present study does not allow the design of such structure. Consequently, we used a neighborhood structure with one sector and a searching radius of the same size as the extent of the observed sample. The use of such searching radius is a reasonable choice, since, as demonstrated by the variograms where ranges exceed the sample spatial coverage, the hydrological variables are very continuous over both spaces. Within this neighborhood structure, a maximum number of 64 observations were used in the estimation.

[29] In order to evaluate the performances of the kriging technique over both spaces and compare their results to the traditional approach, we conducted two types of validation: a cross validation (jackknife) and a split-sample validation. In the first validation technique, the value of a given station is temporarily removed from the sample. The value for this observation is then estimated using the remaining stations. This operation is repeated for the whole station set. Then, the estimated values are compared

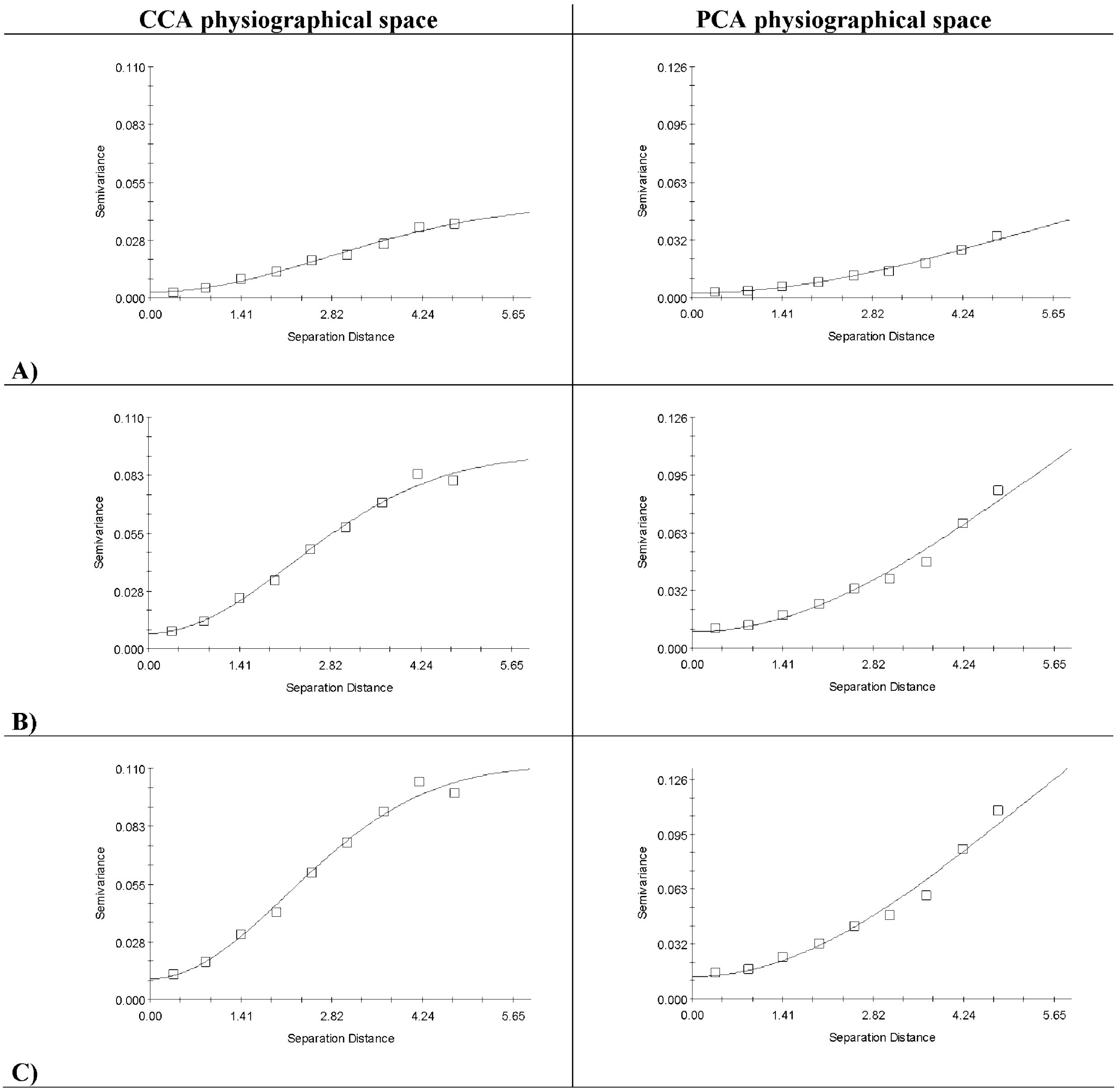

Figure 4. Isotropic theoretical (solid line) and experimental (squares) variograms of the flood quantiles through the CCA and PCA physiographical space: (a) $q10$, (b) $q50$, and (c) $q100$.

with the true one. As for the split-sample validation, 30 gauging stations were randomly selected and removed from the whole observed sample to serve as a validation group. The remaining 121 gauging stations were used as calibration group. In the case of the proposed approach, the physiographical spaces were designed using the data of the remaining 121 gauging stations. The spatial structure of the hydrological variables of the latter group of stations was used to calibrate the kriging system. The physiographical and meteorological characteristics of the validation group were employed therefore to calculate their coordinates within both spaces. Through the spatial location of each station and the neighborhood information using the spatial structure of the hydrological variables identified and quantified by the variogram model, their hydrological variables were estimated and then compared to the local estimates.

[30] To assess the performance of the estimation methods, we considered a certain number of evaluation indices. In addition to the coefficient of determination (R^2), these include the *Nash* criterion (NASH), the root mean square error (RMSE), the relative root mean square error (RMSEr),

Table 2. Characteristics of the Theoretical Models Fitted to the Variables' Experimental Isotropic Variograms

Physiographical Space	Variable	Model	Nugget	Sill	Range	Fitting r^2
CCA	$q10$	Gaussian	0.003	0.045	6.72	0.990
	$q50$	Gaussian	0.007	0.092	5.49	0.992
	$q100$	Gaussian	0.010	0.112	5.23	0.991
PCA	$q10$	Gaussian	0.002	0.086	12.70	0.984
	$q50$	Gaussian	0.009	0.228	13.15	0.980
	$q100$	Gaussian	0.013	0.262	12.56	0.977

Table 3. Cross-Validation Results

	Variables	Kriging in the CCA Physiographical Space	Kriging in the PCA Physiographical Space	Traditional CCA Regional Estimation Method
R^2	$q10$	0.78	0.76	0.78
	$q50$	0.73	0.70	0.72
	$q100$	0.70	0.67	0.69
NASH	$q10$	0.78	0.76	0.78
	$q50$	0.72	0.69	0.72
	$q100$	0.70	0.66	0.68
RMSE, $m^3/s.km^2$	$q10$	0.050	0.053	0.059
	$q50$	0.093	0.098	0.094
	$q100$	0.110	0.116	0.112
RMSEr, %	$q10$	51	66	43
	$q50$	64	82	49
	$q100$	70	86	51
BIAS, $m^3/s.km^2$	$q10$	−0.004	−0.006	0.001
	$q50$	−0.007	−0.008	0.005
	$q100$	−0.008	−0.010	0.007
BIASr, %	$q10$	−16	−20	−9
	$q50$	−21	−25	−11
	$q100$	−23	−27	−11

the mean bias (BIAS) and the relative mean bias (BIASr). They are defined as follows:

$$\mathrm{NASH} = 1 - \frac{\sum_{i=1}^{n} (z_i - \hat{z}_i)^2}{\sum_{i=1}^{n} (z_i - \bar{z})^2}, \tag{2}$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (z_i - \hat{z}_i)^2}, \tag{3}$$

$$\mathrm{RMSE}r = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \left(\frac{z_i - \hat{z}_i}{z_i}\right)^2}, \tag{4}$$

$$\mathrm{BIAS} = \frac{1}{n}\sum_{i=1}^{n} (z_i - \hat{z}_i), \tag{5}$$

$$\mathrm{BIAS}r = \frac{1}{n}\sum_{i=1}^{n} \left(\frac{z_i - \hat{z}_i}{z_i}\right), \tag{6}$$

where z_i, $\hat{z}_i$ and $\bar{z}$ are, respectively, the local and regional estimates at station i and the local mean value of the hydrological variable of interest; n is the sample size.

[31] According to the jackknife method (Table 3), the proposed estimation method produces satisfactory results for both PCA and CCA, especially, in the case of $q10$ for which the NASH criterion is close to 0.8. In fact, the NASH criterion compares the estimation method performance to the use of the observed mean value as an estimate. If it is negative, the estimate is worse than using the mean value. For an estimation method reproducing perfectly the observed data, the NASH criterion is equal to 1. In general, one can expect from a satisfactory estimation that the NASH criterion value is close to 0.8. The results of the new method are comparable to those obtained using the traditional CCA method. CCA-space kriging is slightly better than the traditional method, while PCA kriging is less powerful. It must be recalled that *GREHYS* [1996b] has shown that traditional CCA produced the best performances in comparison with other methods for the delineation of homogeneous regions.

[32] On the basis of the RMSE values, estimates produced using kriging in the CCA space are as precise as those produced by the traditional CCA regional estimation method and more precise than kriging in the PCA space estimates. However, the relative RMSE shows that estimates made by the traditional method are on average less erroneous than kriging in CCA space and kriging in PCA space.

[33] On the other hand, the kriging method seems to overestimate quantile values, since the calculated mean bias is negative, while the traditional approach seems to underestimate the quantiles. The estimation bias is slightly less significant in the case of this last method.

[34] The precision of the quantiles estimated values decreases with the increase of the return period. This represents an expected result, since the precision of the locally estimated quantiles decreases as the return period increases. The local estimation error is propagated through the estimation method. Its impact on the spatial structure of the variables, in the case of the interpolation approach, was discussed earlier. As for the traditional CCA regional estimation technique, less precise locally estimated quantiles lead to a weaker correlation structure between hydrological and physiographical variables.

[35] The kriging technique produces better results within the CCA physiographical space than the PCA space. Thus it seems that the CCA technique is more capable to characterize the physiographical space in relation to the estimation of the specific quantiles. In fact, the axes of the CCA space are calculated while maximizing the correlation between the basins physiographical characteristics and the hydrological variables. On the other hand, the PCA technique is limited to maximizing the variance along the space axes. This would cause the spatial autocorrelation of the hydrological variables to be better defined within the CCA physiograph-

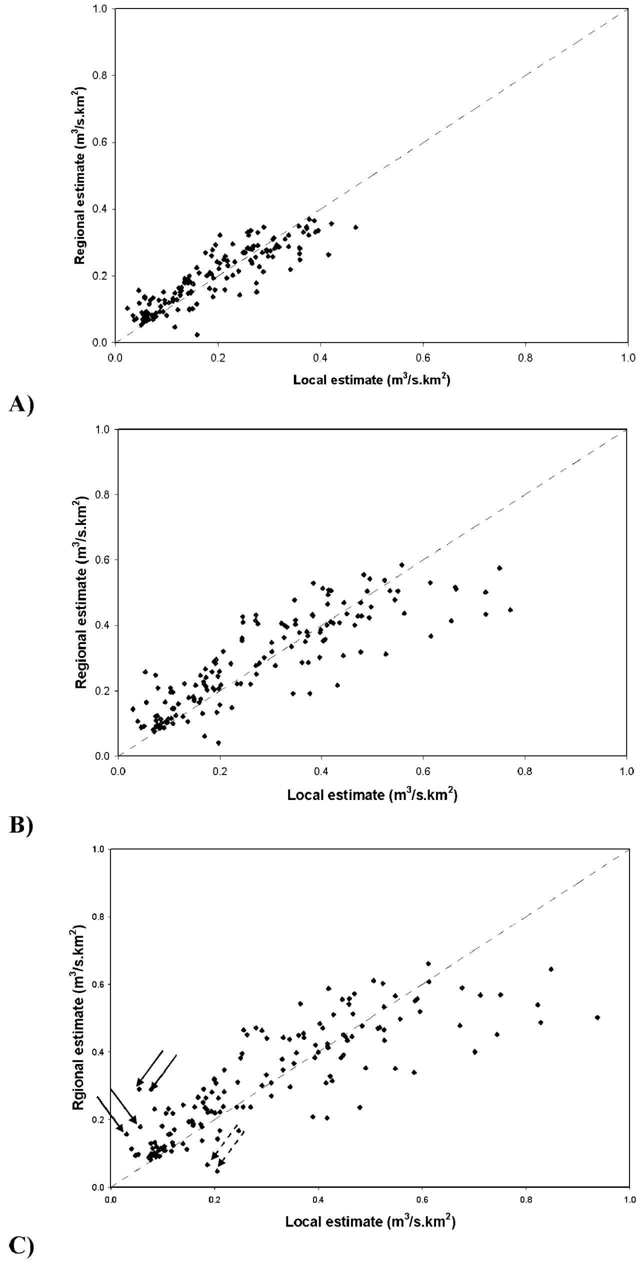

Figure 5. Jackknife estimates using kriging in the CCA physiographical space compared to locally estimated quantiles: (a) $q10$, (b) $q50$, and (c) $q100$. Problematic stations with area problems (solid arrows) and with percentage of lakes problems (dotted arrows) are shown.

ical space than the PCA one. It seems, with the PCA approach, that the observed sample allows characterization of the totality of the physiographical space. Moreover, the range of the models of space autocorrelation is prolonged beyond the observed sample. This may be attributable to the available sample nature, which would not allow to cover a broad range of combinations of physiographical variables and to seize the structure of the spatial autocorrelation of the hydrological variables within the PCA physiographical space. Also, the weaker performances of kriging in PCA physiographical space might be ascribable to the uncomfortableness with the stationary assumption.

[36] Figures 5 and 6 show kriged quantile values estimated with the jackknife technique for the 10, 50, and 100 year return periods. Figure 7 illustrates the results of the traditional CCA-based regional estimation method. These figures emphasize the increase in the estimation variance and bias with the return period. Furthermore, for all return periods, the higher specific quantile values are rather underestimated. These high specific quantile values were found to be associated with relatively small basins, less than 500 km^2. We have already excluded from the study the gauging stations draining very small basins (less than 200 km^2) because of the very low number of such basins. However, it seems that the 200 km^2 threshold is not adequate and the area limit under which the basin can be considered as small should be redefined. Indeed, only

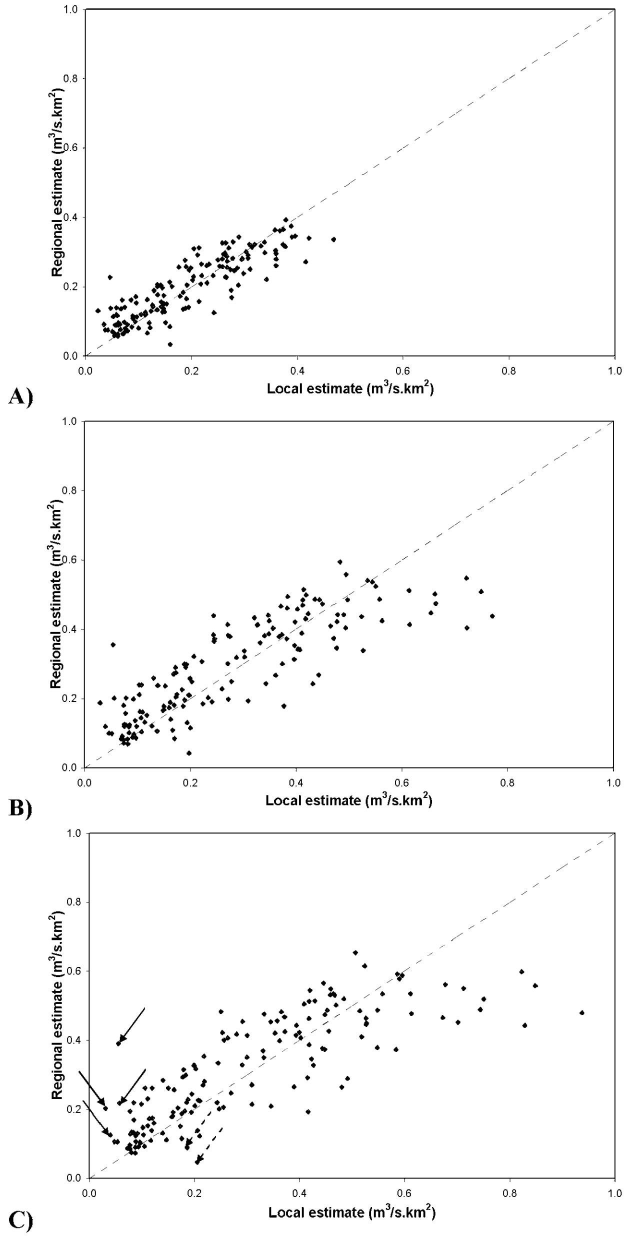

Figure 6. Same as Figure 5, but for the PCA physiographical space.

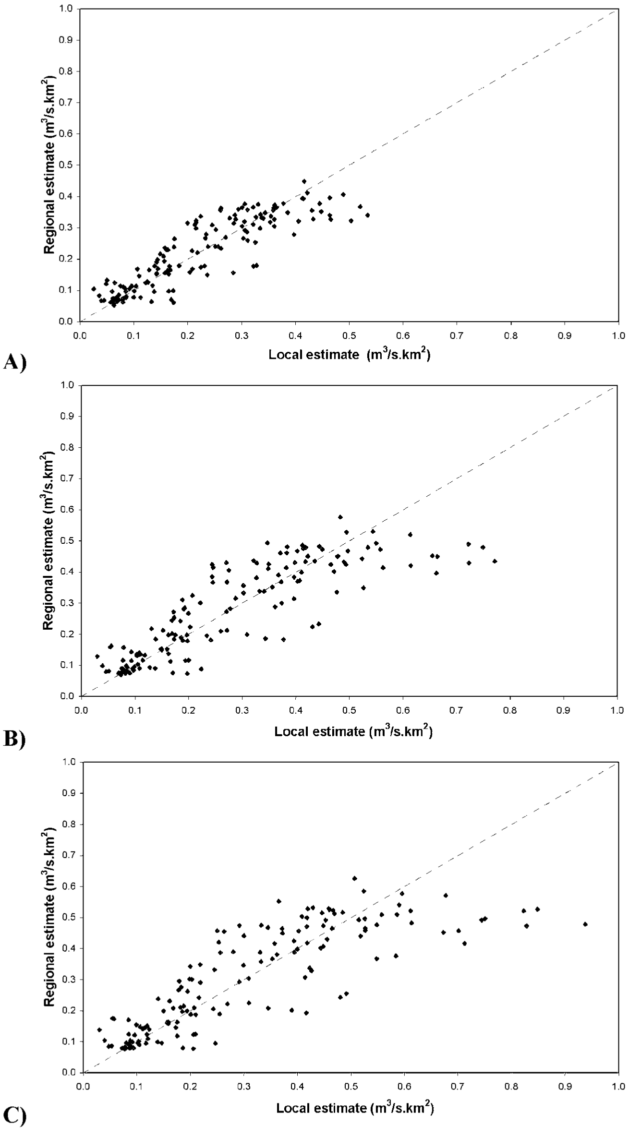

Figure 7. Same as Figure 5, but using the traditional CCA regional estimation method.

9 stations drain basins of area 500 km^2 or less. Another possible reason of the underestimation of small basin quantiles lies in the fact that the surfaces of these basins are themselves underestimated. In fact, the basin areas recorded in the BDH database used in this study were estimated from watershed boundaries. The latter were obtained by sketching manually catchment limits from course-scale printed maps. During this operation the generalization of the watershed shape is inevitable. This generally leads to an undervaluation of the drainage surfaces whose relative importance is more significant in the case of small watersheds.

[37] Moreover, while all estimation methods underestimate specific streamflows superior to 0.55 m^3/km^2, the traditional CCA approach appears to produce more precise values for lower specific streamflows (<0.15 m^3/km^2). Whereas, for the same range of specific streamflow, the interpolation method in both spaces yields values with more constant estimation error. This could explain the lower relative RMSE values observed in the case of the traditional method.

[38] On the other hand, we identified a certain number of problematic gauging stations which are responsible for a significant part of the high observed relative mean square error in both spaces. These suspected stations are indicated by arrows on Figures 5c and 6c. We found that for four stations (identification numbers: 030401, 030402, 041903 and 042607), the catchment areas were underestimated, which caused very high relative errors. The two other stations (080104 and 081101) were found to have, according to DBH, basins with an overevaluated percentage of area covered by lakes when compared to independent digital maps. The exclusion of these six stations improved significantly the overall results. For instance, the RMSEr calculated for $q100$ estimated within the CCA space drops from 70% to 41% and the relative mean bias from −23% to −16%. This means that the method developed in the present study is rather sensitive to the quality of physiographical and meteorological data. Unfortunately, the database used in this study presents certain anomalies and other stations are undoubtedly problematic.

[39] Furthermore, the observed divergence between the locally estimated values and those estimated by the kriging in the physiographical space can partly be attributed to extrapolation. This one occurs on the edge of the observed domain or in isolated regions far from the remainder of the sample (Figure 3). In this case, the sample points are not evenly distributed around the point of interest where an estimate is needed. Thus instead of interpolating between the observed points, information is extrapolated beyond the observed domain which often leads to estimates of lesser quality. A means to limit this problem would be to use a neighborhood structure subdivided into several sectors with an optimal number of observations in each sector as well as a limit on the number of empty adjacent sectors. However, the software used in the present study did not allow the definition of such a structure.

[40] The split-sample validation results over both spaces are presented in Table 4, Figures 8, Figure 9, and Figure 10. The results are similar to those obtained by the jackknife resampling. However, kriging in the CCA yields results as precise as the traditional CCA-based regional estimation method. Here again, the interpolation over the CCA physiographical space produces better results than that in the PCA space. The NASH criterion varies between 0.67 and 0.76 for kriging in the CCA space and between 0.66 and 0.73 in the PCA space. For all estimation methods, the split-sample validation generates RMSE values comparable to those of the jackknife. However, the relative root mean square error values are about half those of the jackknife and the same for the bias. This is due to the fact that most problematic stations were not selected among the validation group. This results also in lower estimation bias. The split-sample validation confirms the tendency of the proposed method to yield flood estimates for which precision decreases with the increase in the return period.

[41] On the other hand, during the split-sample validation, 30 gauging stations were randomly excluded from the calibration of the estimation method. The method yields however results as good as those obtained by a calibration sample of 151 gauging stations. It seems then that the

Table 4. Split-Sample Validation Results

	Variables	Kriging in the CCA Physiographical Space	Kriging in the PCA Physiographical Space	Traditional CCA Regional Estimation Method
R^2	$q10$	0.78	0.74	0.77
	$q50$	0.72	0.69	0.71
	$q100$	0.69	0.66	0.68
NASH	$q10$	0.76	0.73	0.76
	$q50$	0.70	0.69	0.70
	$q100$	0.67	0.66	0.67
RMSE, $m^3/s.km^2$	$q10$	0.055	0.058	0.055
	$q50$	0.086	0.089	0.087
	$q100$	0.103	0.104	0.103
RMSE*r*, %	$q10$	22	31	21
	$q50$	26	35	25
	$q100$	28	37	27
BIAS, $m^3/s.km^2$	$q10$	−0.003	−0.001	0.009
	$q50$	−0.006	0.002	0.014
	$q100$	−0.009	0.002	0.017
BIAS*r*, %	$q10$	−5	−7	1
	$q50$	−6	−8	1
	$q100$	−8	−10	0

proposed estimation method is not very sensitive to the removal of a relatively small part of the sample especially in the case of the CCA.

4. Conclusions and Future Work

[42] In this study, we aimed to develop and apply a new regional flood flow estimation method. This method is a physiographical space-based estimation technique. It consists in the interpolation of flow quantiles over the physiographical/meteorological space rather than the usually employed geographical space. Hence for any ungauged site, one has just to estimate its coordinates in the physiographical space from its basin physiographical and meteorological characteristics, which are generally easily available, and then estimate its flood quantiles by interpolation of local quantile estimates within its physiographical neighborhood. For this purpose, two mutltivariate analysis methods were used to define the physiographical/meteorological space: canonical correlation analysis and the principal component analysis.

[43] It was demonstrated that hydrological variables can be treated as a continuous variable over the physiographical space. Thus geostatistical techniques can be were employed to estimate flood flow quantiles. Variograms and kriging were applied, respectively, to capture the spatial autocorrelation structure of the variables and interpolate them over the physiographical space. Cross-validation and split-sample-validation techniques were used to assess the performances of the proposed estimation method. The validation reveals that physiographical space-based kriging is effective to estimate flood flow quantiles and yields satisfactory results especially in the case of high-frequency quantiles. Results indicate also that the estimation method was more successful within the CCA physiographical space than in the case of PCA physiographical space. However, some anomalies in the physiographical data used in this study were detected in certain gauging stations and led to high estimation errors. This points out the importance of the data quality for the success of the estimation method.

[44] The no sill variograms in the case of PCA space as well as the less conclusive results of kriging in this space

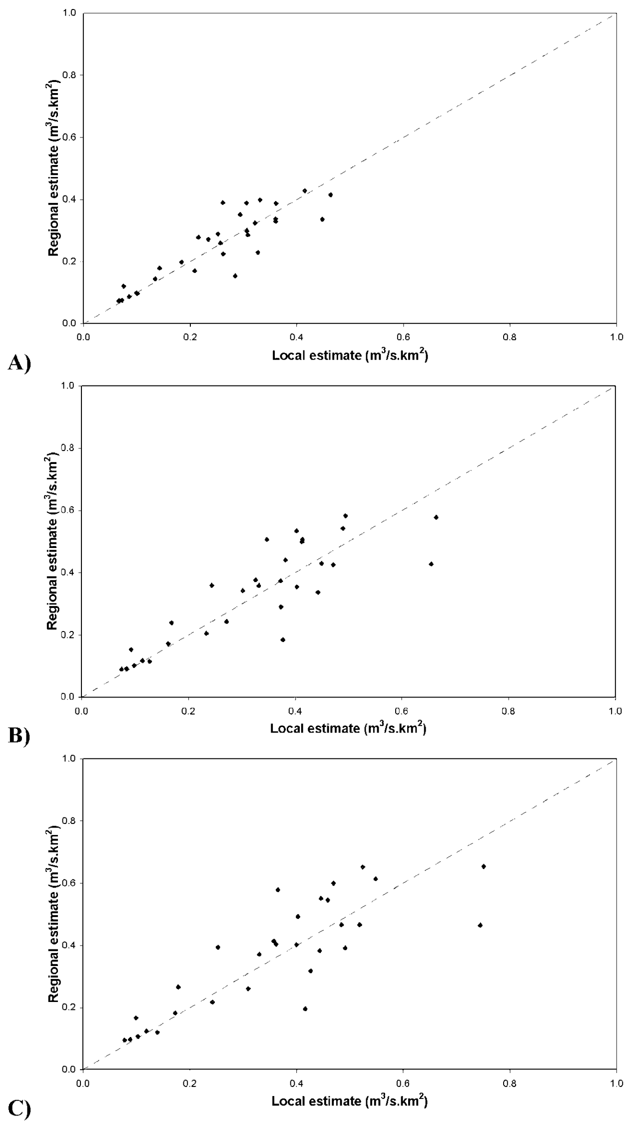

Figure 8. Split sample validation estimates using kriging in the CCA physiographical space compared to locally estimated quantiles: (a) $q10$, (b) $q50$, and (c) $q100$.

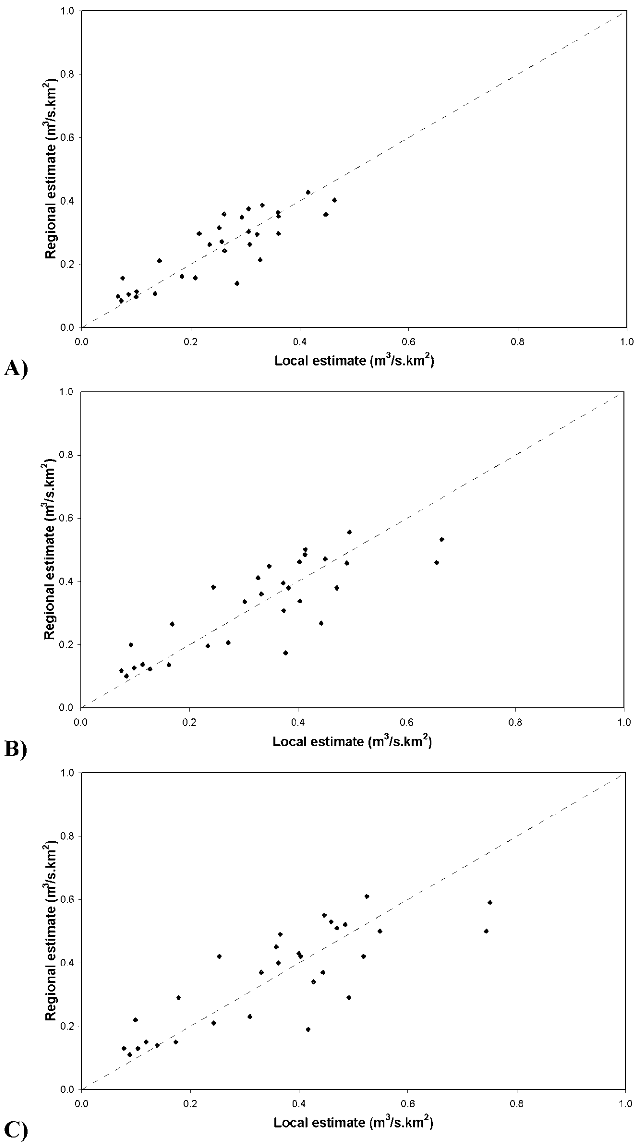

Figure 9. Same as Figure 8, but for the PCA physiographical space.

suggest the absence of stationarity of the specific quantiles data and the existence of a possible spatial trend within the data. If this is the case, ordinary kriging would not be the appropriate estimation technique. One should instead quantify the trend using spatial regression (regression between the specific quantiles and the correspondent coordinates in PCA physiographical space) and apply afterward the ordinary kriging on the regression residuals. The specific quantile estimation at an unsampled location will be then the sum of the quantile value estimated by spatial regression and the kriged residual at the same location. It is as well possible to use an adaptation of ordinary kriging known as universal kriging that can deal with the presence of a trend [*Isaaks and Srivistava*, 1989].

[45] It was demonstrated in this study that the proposed estimation method is as powerful as the most powerful regional estimation techniques. Indeed, when compared to the traditional CCA-based regional estimation approach, the physiographical space-based kriging technique yields equivalent performances and this by using a nonoptimal neighborhood structure. Future developments in relation to the estimation method proposed in the present paper could allow its application in a more effective way. First, one may calculate the directional variograms in order to account for possible anisotropy in the variable spatial autocorrelation. One should also employ a more efficient neighborhood structure during the kriging interpolation, which was impossible with the geostatistics software used in this study. This structure should be defined in relation with the spatial distribution of the observed sample as well as with respect to the spatial autocorrelation structure (anisotropy and range). In addition, it may be interesting to test other space definition techniques such as Multidimensional Scaling [*Kruskal and Wish*, 1978], Factor Analysis [*Lindeman et al.*, 1980; *Stevens*, 1986] or Correspondence Analysis [*Benzecri*, 1973; *Greenacre*, 1984] and compare their results to those presented herein. Additionally, further effort could be devoted to the study of the effect of the choice of

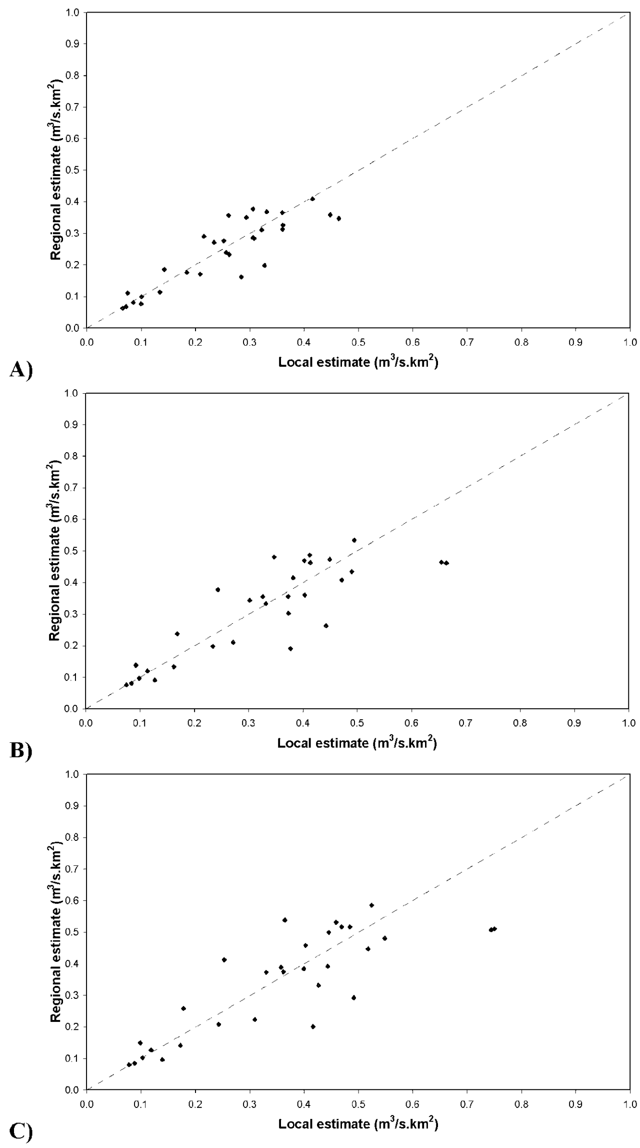

Figure 10. Same as Figure 8, but using the traditional CCA regional estimation method.

physiographical and meteorological variables on the space definition as well as on the estimation quality.

[46] Kriging produces an unbiased optimal estimation, commonly known as BLUE (for Best Linear Unbiased Estimation). However, it requires a significant modeling effort and a good knowledge of the phenomenon of interest. It is also possible to apply to the physiographical space common interpolation techniques such as the inverse distance for their facility of application and speed. However, kriging provides an estimate of the error at any point of the interpolated space, which is not possible with other classical interpolation techniques. This information on the error is very useful. It illustrates the reliability of the regional estimation and could be used to assess the spatial distribution quality of the sampling points.

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