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Optimal replacement of water pipes

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[1] Pipe breaks are used as indicators of the structural state of pipe network. The approach used considers times to failure between pipe breaks as random variables. Pipe lifespan is divided into two periods, the first one characterized by time-dependent hazard functions (nonexponential period) and the second one characterized by constant hazard functions (exponential period). Closed-form expressions have been derived for probability density functions of occurrence of breaks for all break orders as well as expressions for the time evolution of the average number of pipe breaks per unit time. An optimal replacement criterion is defined on a pipe-to-pipe basis based on a cost function using conditional probabilities to estimate the expected future costs. Minimization of this cost function leads to a replacement criterion involving hazard functions. When applied to models with constant hazard functions, this criterion identifies a critical pipe break order at which replacement should be made. *INDEX TERMS:* 1848 Hydrology: Networks; 1869 Hydrology: Stochastic processes; 6304 Policy Sciences: Benefit-cost analysis; *KEYWORDS:* water network, urban infrastructure, break model, replacement cost

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1. Introduction

[2] Many studies have been realized and published in recent years that reveal the importance of underground infrastructure deterioration [*American Water Works Association* (*AWWA*), 1994; *Siddiqui and Mirza*, 1996; *Desbiens*, 1997; *Villeneuve et al.*, 1998]. Important investments will be necessary in order to improve the overall structural state of these infrastructures. For example, *Villeneuve et al.* [1998] estimated at around 8 billion Canadian dollars the necessary investments to maintain at its present level the average structural state of water and sewer networks in the province of Quebec over a 20-year period.

[3] The actual situation is the result of the lack of care given to these infrastructures over the years. As obvious it appeared that the structural state of their underground infrastructures was deteriorating, many municipalities now recognize the importance to develop methodologies and strategies in order to assist water managers in their replacement/rehabilitation decision making process [*Sægrov et al.*, 1999; *Malandain et al.*, 1998]. The overall objective is to use the readily available information on the structural state and develop methodologies that optimally use this information to plan interventions on the network. Planning the interventions in order to minimize related costs and maintain a structural state that will ensure an adequate service to the customers is one way to optimally use the information [*O'Day et al.*, 1986; *Le Gauffre*, 1998; *Sægrov et al.*, 1999; *Engelhardt et al.*, 2000].

[4] However, this task is complex since very little information is available to characterize the structural state. For water pipe networks, the usual indicator of the structural state is the number of pipe breaks. Although valuable, this indicator is an indirect one and must be used with caution. For example, in most municipalities, the number of pipe breaks recorded corresponds to cases where water has reached the surface and has been detected. Depending on the type of soil and the proximity of the sewer network, important leaks can remain undetected for long periods of time. Also, operational constraints can have an immediate impact on pipe breaks. For example, a water manager reported that he obtained a short-term diminution of 20% of the annual number of breaks by simply reducing by a few percent the pressure in some parts of the network. What this means is that pipe breaks are the result of the deterioration of the structural state of pipes but that the effective time of occurrence of breaks depends on variables and parameters that are also related to the structural and operational stress applied on pipes [*Makar*, 1999; *Makar et al.*, 2001].

[5] The time of occurrence and the location of pipe breaks are the usual available data. Although simple to obtain and record, only few municipalities possess long recorded pipe breaks. Many municipalities have recently begun to systematically record their histories of pipe breaks. The scarcity of data, combined with the fact that they are usually incomplete (incomplete data, inaccurate locations, approximate date of occurrence) make the modeling exercise difficult and special effort must be made to adapt models to the actual municipal reality [*Mailhot et al.*, 2000; *Pelletier et al.*, 2003].

[6] From a practical point of view, one of the problems water managers are facing is to know when they should replace a pipe. Having observed a given number of pipe breaks on a given street, should they wait or should they proceed and replace the pipes? *Shamir and Howard* [1979] were among the first to define a methodology to estimate the optimal time for pipe replacement. Following Shamir and Howard, many other authors have used similar approaches [e.g., *Walski and Pelliccia*, 1982; *Kleiner et al.*, 1998a,

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1998b; *Kleiner and Rajani*, 1999]. Shamir and Howard coupled a simple model describing the evolution of average pipe breaks as a function of time with a cost function. This function takes into account the expected maintenance cost and the replacement cost. The optimal time of replacement is defined as the optimal point where increasing discounted maintenance costs due to structural deterioration become larger than discounted replacement costs. From an operational perspective, this approach is interesting since it defines, based on a model describing the evolution of pipe breaks and on economic parameters (e.g., replacement costs, maintenance costs, etc.), an optimal replacement time.

[7] Shamir and Howard's approach is however based on a description of an average ageing behaviour of pipes. This approach only considers the evolution over time of the average number of pipe breaks. Although interesting, it is inapplicable on an individual pipe basis since it does not take into account the historical data of each pipe.

[8] Other approaches have been proposed since then. *Gustafson and Clancy* [1999b] used Monte Carlo simulations to define what they called the economic loss of replacement. Considering a given pipe break record, this economic loss corresponds to the difference between the cost of replacing a pipe segment after the *n*th break and the minimal cost knowing the complete pipe break record. The proposed economic decision criterion is to replace water mains at the break order that minimizes this economic loss function averaged over all possible pipe break histories.

[9] This paper addresses the following question: how to formally extend Shamir and Howard's approach for an application on an individual pipe basis that takes into account the historical break record of each pipe? Probability density functions (pfd) of the times between breaks should be considered. This modeling scheme has been considered by *Eisenbeis et al.* [1999], *Mailhot et al.* [2000] and *Gustafson and Clancy* [1999a]. The first part of the paper focuses on the definition of a general framework for the description of the probability of occurrence of pipe breaks using this modeling scheme. Probability density functions (pdfs) are defined to describe the distribution of time between consecutive breaks. A general class of models is considered for which low order breaks are described by nonexponential pdfs while high order breaks are described by exponential pdfs. Closed-form expressions are derived for this class of models. One simple case is also considered from which the expression of the average number of breaks is derived and corresponds to the exponential form used by Shamir and Howard.

[10] The second part of the paper presents the economic analysis. An economical function is developed based on a given pipe break history. Minimization of the cost function leads to the definition of an optimal time to replacement based on the historical record of a pipe segment. The application of these criteria to the class of models with exponential pdfs for high order breaks shows that, in that case, the optimal time to replacement is achieved through the identification of an optimal break order. This is illustrated with an example.

2. Modeling Water Main Breaks: A Literature Review

[11] We briefly present in this section three approaches currently used to describe ageing processes of water pipes (this review is not exhaustive; for a complete review, see, for example, *Elnaboulsi and Alexandre* [1996] and *Pelletier* [2000]). The three approaches are the aggregated models [*Shamir and Howard*, 1979; *Kleiner and Rajani*, 1999], the survival cohort model [*Herz*, 1996a, 1996b], and statistical models using pdf to describe the time between successive breaks [e.g., *Eisenbeis et al.*, 1999; *Brémond*, 1998; *Mailhot et al.*, 2000]. The variables used to describe the evolution of the structural state evolution are somewhat different from one approach to another.

2.1. Aggregated Models

[12] These models describe the evolution of the average break rates as a function of time. Mainly, two expressions have been proposed, a linear and an exponential ones [*Shamir and Howard*, 1979; *Kleiner and Rajani*, 1999], which have the following forms:

$$N_i(t) = N_i(t_0) + A_i(t - t_0)$$
$$N_i(t) = N_i(t_0)\exp[A_i(t - t_0)] \qquad (1)$$

where $N_i(t)$ is the number of breaks per unit length, per unit time (usually a year) at time t for pipe i, $N_i(t_0)$ is the number of breaks per unit length, per unit time at time $t = t_0$ for pipe i and A_i is the breakage rate growth for pipe i. Using data to estimate parameters $N_i(t_0)$ and A_i, it is important to realize that expressions in (1) give the *average* number of breaks per unit length, per unit time. The exponential expression has been reported to give better results than the linear expression [*Shamir and Howard*, 1979; *Walski and Pellicia*, 1982; *Kleiner and Rajani*, 1999]. Pipes can be grouped in classes with the same characteristics (type of material, diameter, installation periods, etc.) and parameter values estimated for each of these classes [*Kleiner and Rajani*, 1999]. This simple approach gives no information about the probability of occurrence of breaks or the pdfs of times between successive breaks.

[13] It is noteworthy to mention that *Constantine et al.* [1996] have proposed a power law to describe the evolution of the average number of breaks as a function of time:

$$N(t) = a\, t^b \qquad (2)$$

Parameters a and b vary for different pipe classes. Application of this expression to parts of the Melbourne (Australia) network led to values of b very close to 2.

2.2. Cohort Survival Model

[14] Inspired from demographic models, *Herz* [1996a, 1996b] has proposed the cohort survival model, which assumes that the current pipe lifespan can be considered as a random variable. The cohort survival model is then not a pipe breaks model but a model assessing directly the evolution of pipe life-time. A distribution was also proposed by this author to represent pipe lifespans, since then called the Herz's distribution [*Herz*, 1996a, 1996b]. In his example using Stuttgart's network [*Herz*, 1996b], parameter values were estimated for different types of material. Starting with the 1990 pipe stock, the model was used to compare two infrastructure renewal strategies over a 60-year period. The first strategy was based on pipe replacement while the second considered a combination of pipe replacement and

rehabilitation. Simulations have shown that the second strategy would be less expensive than the one based on pipe replacement alone. Although interesting, it is important to note that Herz's approach does not use pipe breaks as an indicator of the structural state. By considering pipe lifespan as a random variable, this approach assumes that pipe replacements historically occur at the end of the useful lifespan. Therefore it cannot be used to define a criteria that would lead to the selection of pipes that should be replaced [*Eisenbeis et al.*, 1999].

2.3. Statistical Models Using pdfs to Describe the Time Between Pipe Breaks

[15] This approach is based on the observation that the time between successive breaks on a given pipe segment becomes shorter as the break order increases and that the number of breaks already observed is the main variable that affects the probability of occurrence of future breaks [e.g., *Clark et al.*, 1982, 1988; *Andreou et al.*, 1987]. The number of pipe breaks is an indicator of the structural state of a pipe or surrounding pipes and some spatial and time correlations exist among pipe breaks. The time-to-failure between successive breaks is then represented as a statistical variable and different distributions are used for different break orders. For a given pipe, as we will see in section 3.4, the probability of break occurrences is a function of the pipe break record. An economical analysis on an individual pipe basis is thus possible.

[16] This modeling approach was used by many authors. *Eisenbeis* [1994], for example, considered the data of two networks in France. His results clearly show that different distributions must be used to describe the pdfs of the time between pipe breaks of different orders. He showed that the time between the installation and the first break and between the first and second breaks are well described by Weibull distributions while higher break orders are best described by exponential distributions. In a later publication, *Eisenbeis et al.* [1999] reported the application of a similar model to three networks, one in Norway and two in France. A proportional hazard model was used to identify the most significant factors that influence the time to failure and the failure rate. The number of previous failures was identified as the most important and used as a "stratification variable": the data set was split into different subsets according to the number of previous breaks and analyses were performed on these subsets. For other examples of application and validation of this approach, see *Mailhot et al.* [2000], *Pelletier et al.* [2003], *Le Gat and Eisenbeis* [2000], *Le Gauffre* [1998], and *Brémond* [1998].

[17] Among recent studies, it is interesting to mention the one by *Gustafson and Clancy* [1999a]. Using data from the City of Saskatoon (Canada), these authors used a model where the pdf for the time between the installation and the first break is described by a generalized gamma distribution. Exponential distributions were used for subsequent breaks up to the 11th break order. Their results also indicate that the mean time between breaks was almost constant after the occurrence of the 5th break.

[18] Survival analysis is usually used to statistically estimate distribution parameters of pipe break models [*Kalbfleisch and Prentice*, 1980]. This statistical method allows the possibility to account for right-censored data.

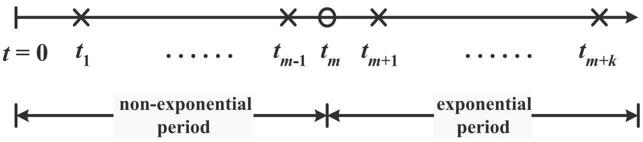

Figure 1. Schematic representation of nonexponential and exponential periods.

Data stratification of time between breaks is done according to the number of breaks and survival analysis is performed on each data subset. *Mailhot et al.* [2000] extended this approach to consider the case usually encountered where the installation time is prior to the period of recorded pipe breaks. In that case, it is impossible to know if the first observed pipe break is really the first one occurring.

[19] While the different case studies reported in the literature seems to demonstrate the validity of this type of approach, very few studies have analysed model parameter uncertainties. *Mailhot et al.* [2000] compared results obtained using different distributions in order to determine which one was more appropriate considering the available data. It was shown, for example, that the use of a Weibull distribution to describe the probability of the time-to-failure between the first and second pipe breaks could not be statistically justified when compared to a model using an exponential distribution.

3. A General Framework for Water Main Breaks Modeling

3.1. Description

[20] The modeling approach adopted, as described hereafter, is the one where the time between breaks of successive orders are random variables described by pdfs. These pdfs correspond to the probability of occurrence of the jth break as a function of the time elapsed since the occurrence of the $(j-1)$th break. According to this approach, one must specify the types of distributions used for each break order and parameter values associated with these distributions.

[21] In the following development, we assume that the ageing process of a pipe can be divided into two distinct periods. The first one, called the nonexponential period, is characterized by the fact that distributions used to describe pdfs of the time between breaks are not of the exponential type. This occurs in the early stages of the pipe life. In contrast, pdfs used in the second period, called the exponential period, to characterize distributions of times between breaks of that period are of the exponential type. A critical break order, the mth break order, delimitates these two periods. Thus the last nonexponential pdf is used to describe times from the $(m-1)$th break to the mth break while the first exponential pdf is used to characterize the mth to the $(m+1)$th break (Figure 1).

[22] This representation is coherent with previous studies [see, e.g., *Eisenbeis*, 1994; *Andreou et al.*, 1987; *Gustafson and Clancy*, 1999a]. No specific distribution is assumed for all pdfs used in the nonexponential period. Weibull distributions, as proposed by *Eisenbeis* [1994] and *Mailhot et al.* [2000], or generalized gamma distributions, as proposed by *Gustafson and Clancy* [1999a] can be used. Time between breaks for orders higher than the mth break are all described by exponential distributions with parameters λ_j where j is

the break order. The pdf for exponential distributions has the following form:

$$f_j(t) = \lambda_j \exp\left[-\lambda_j\left(t - t_{j-1}\right)\right] \quad (3)$$

t_{j-1} is the time of occurrence of the $j-1$ th break. The use of exponential distributions implies that the hazard function is independent of time and is given by λ_j for the jth break order (see *Cox and Oakes* [1994] for a definition of the hazard function). A constant hazard function means that the system has "no memory" as to when happened the last break and it thus follows a Poisson process.

3.2. Pdf of Occurrence of the *j*th Break

[23] We first estimate the pdf describing the probability of occurrence of the jth break at time t, $\phi_j(t)$ for pipe segments of length ℓ. In a general case, for the jth break, the probability density function is given by the following expression:

$$\phi_j(t) = \int_0^t du\, \phi_{j-1}(u) f_j(t-u) \quad (4)$$

In this expression t_j is the time of occurrence of the j th break and f_j is the pdf of the time between the $(j-1)$th and the jth breaks. Expression (4) shows that the function ϕ_j (t) is obtained by the convolution of $\phi_{(j-1)}$ and f_j. Using the convolution theorem [*Doetsch*, 1971; *Schiff*, 1999], we can show that:

$$\varphi_j(s) = \prod_{i=1}^{j} F_j(s) \quad (5)$$

φ_j and F_j are the Laplace transforms of ϕ_j and f_j respectively. This expression is general and applies to whatever pdfs used to describe the times between different breaks order. Suppose now that time $t = t_m$ corresponds to the time of occurrence of the mth break and assuming that $t > t_m$, the Laplace transform of the exponential pdf is simply [*Doetsch*, 1971; *Schiff*, 1999]:

$$F_j(s) = \frac{\lambda_j}{s + \lambda_j} \quad (6)$$

After substitution in Equation (5), the inverse Laplace transform of the resulting expression is easily computed and the final expression for ϕ_j (t) is:

$$\phi_j(t|t_m) = \sum_{i=m+1}^{j} \eta_{i,j}^m \exp[-\lambda_i(t - t_m)] \quad (7)$$

with:

$$\eta_{i,j}^m = \frac{\prod_{k=m+1}^{j} \lambda_k}{\prod_{\substack{i'=m+1 \\ i' \neq j}}^{j} (\lambda_{i'} - \lambda_i)} \quad (8)$$

Equations (7) and (8) give the pdf for time of occurrence of the jth break as a function of the parameters of exponential distributions associated with the different break orders considering that the mth break has occurred at time t_m. The pdf for times of occurrence of the jth break with $j > m$ from $t = 0$, the installation time of the pipe, is given by:

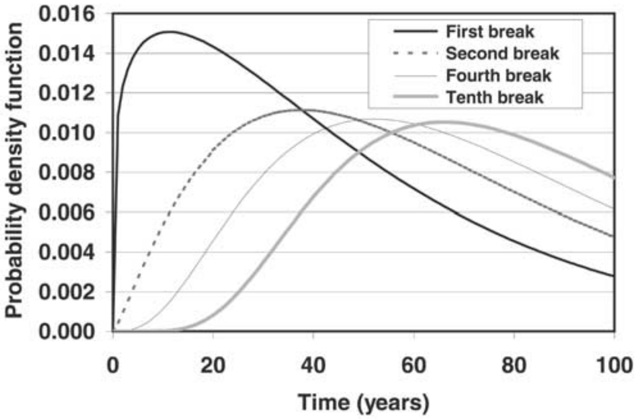

Figure 2. Probability density function of the time of occurrence for the first break, second break (shaded dashed curve), fourth break (thin dashed curve) and tenth break (thick curve) in the case of a model with a Weibull distribution for the first break and exponential distributions for subsequent breaks. Hazard functions are assumed linear with break orders for the exponential period (equation 13). Values of κ, p, λ_0 and α are 0.02, 1.2, 0.05 and 0.08, respectively.

$$\phi_j(t) = \int_0^t dt_m \phi_m(t_m) \phi_j(t|t_m) \quad (9)$$

This is obtained by integrating the product of the pdfs of having the jth break at time t, assuming that the mth break has occurred at $t = t_m$, $\phi_j(t|t_m)$, and the pdf that the mth break has occurred at $t = t_m$, ϕ_m (t_m), over all possible values of time t_m. Using expression (7) for $\phi_j(t|t_m)$, we then have for $j > m$:

$$\phi_j(t) = \sum_{i=m+1}^{j} \eta_{i,j}^m I_i(t) \quad (10)$$

where:

$$I_i(t) = \int_0^t dt_m \phi(t_m) \exp[-\lambda_i(t - t_m)] \quad (11)$$

In general, there will be no closed form for $\phi_m(t_m)$ when $m > 1$. This function will be the result of a multiple integral similar to the one appearing in equation (4). Integration is performed over all possible values of times of occurrence from the first to the mth order. Considering the usual distributions used to describe time of occurrence of the first or the second breaks, there will be no analytical solution to integrals on the right hand side of equation (11). However, for $m = 1$ or $m = 2$, numerical integration can be easily be performed. Figure 2 gives examples of pdfs for $m = 1$ and with a Weibull distribution for times of occurrence of the first break. The Weibull pdf is given by [*Kalbfleisch and Prentice*, 1980]:

$$\phi_1(t) = f_w(t) = \kappa\, p\, (\kappa\, t)^{(p-1)} \exp[-(\kappa\, t)^p] \tag{12}$$

In this example, parameters λ_j describing hazard functions for the exponential period are linearly related to break orders:

$$\lambda_j = \lambda_0 + \alpha\,(j - m - 1) \qquad j > m \tag{13}$$

The parameter α defines the slope of the linear relationship between the break order and λ values. This model with a Weibull distribution for the first break order and exponential distributions with hazard functions linearly related to the break order (equation 13) will be referred, from now on, as the Weibull-exp. model.

3.3. Average Number of Pipe Breaks as a Function of Time

[24] The average number of pipe breaks per unit time, per unit length, for a pipe segment of length ℓ at time t is given by:

$$N(t) = \frac{1}{l} \sum_{k=1}^{\infty} \phi_k(t) \tag{14}$$

In the general case, using equation (10) for the pdfs of the times between successive pipe breaks during the exponential period, we have:

$$N(t) = \frac{1}{l}\left\{ \sum_{j=1}^{m} \phi_j(t) + \sum_{j=m+1}^{\infty} \sum_{i=m+1}^{j} \eta_{i,j}^m\, I_i(t) \right\} \tag{15}$$

If we consider the average number of breaks during the finite interval $[t, t']$, we integrate the last equation and obtain:

$$N(t, t') = \frac{1}{l}\left\{ \sum_{j=1}^{m} \int_t^{t'} d\tilde{t}\, \phi_j(\tilde{t}) + \sum_{j=m+1}^{\infty} \sum_{i=m+1}^{j} \eta_{i,j}^m \int_t^{t'} d\tilde{t}\, I_i(\tilde{t}) \right\} \tag{16}$$

We assume, in the following development, that the pipe segment is in the exponential period, meaning that the mth break has already occurred at time t_m. The function describing the evolution of the average number of breaks over time takes a simple form, in the case where the parameters associated with the different exponential distributions are related to break orders, as prescribed by equation (13). To derive this relationship, we first define $\psi_j(t|t_m)$, the probability that $(j - m)$ breaks have occurred during the interval $[t_m, t]$. Since the hazard function is independent of time during the exponential period, it means that the probability of occurrence of a $(j + 1)$th break during the interval $[t, t + dt]$ is given by $\psi_j(t)\, \lambda_{j+1} dt$. The "balance equation" for $\psi_j(t)$ leads to the following equation:

$$\frac{d\psi_j(t|t_m)}{dt} + \lambda_{j+1}\, \psi_j(t|t_m) = \lambda_j\, \psi_{j-1}(t|t_m) \tag{17}$$

with:

$$\frac{d\psi_m(t|t_m)}{dt} + \lambda_{m+1}\, \psi_m(t|t_m) = 0 \tag{18}$$

According to the definition of ϕ_j, we have for $j \geq m$:

$$\phi_j(t|t_m) = \lambda_j\, \psi_{j-1}(t|t_m) \tag{19}$$

Multiplying equation (18) by λ_{j+1}/ℓ and taking the sum over j from $(m + 1)$ to ∞ on both sides, we find:

$$\frac{dN(t|t_m)}{dt} = \frac{1}{l} \sum_{i=m+1}^{\infty} (\lambda_{i+1} - \lambda_i)\, \phi_i(t) \tag{20}$$

where

$$N(t|t_m) = \frac{1}{l} \sum_{i=m+1}^{\infty} \phi_i(t|t_m) \tag{21}$$

$N(t|t_m)$ is the average number of pipe breaks per unit time, per unit length to occur at time t considering that the mth break has occurred at $t = t_m$. Now, using the expression (13) to describe the parameter dependency on the break order during the exponential period, we find, after substitution in equation (20) and using initial condition $N(t_m|t_m) = \lambda_{m+1}/l$:

$$N(t|t_m) = \frac{\lambda_{m+1}}{l} \exp[\alpha\,(t - t_m)] \tag{22}$$

This is an interesting result since it shows that an exponential increase in the average number of breaks as a function of time can be obtained, if exponential distributions are used to describe the times between successive breaks at different orders, and if the hazard function is related linearly with the break order. In that case, the argument of the exponential corresponds to the slope of the relationship between the hazard function and the break order (parameter α).

[25] The average number of breaks per unit time, per unit length, at time t considering the contribution from both the nonexponential and exponential periods is given by:

$$N(t) = \frac{1}{l}\left\{ \sum_{i=1}^{m} \phi_i(t) + \int_0^t dt_m \phi_m(t_m) N(t|t_m) \right\} \tag{23}$$

where $\phi_i(t)$ is the pdf describing the occurrence of the ith break. The sum on the right hand side accounts for the contribution of the 1st to the mth break, while the integral corresponds to the contribution from break orders $j > m$. If we use equation (22) for $N(t|t_m)$, we find, for large values of t while neglecting the contribution from the nonexponential period:

$$N(t) \approx \tilde{\lambda}_{m+1} \exp(\alpha\, t) \tag{24}$$

with

$$\tilde{\lambda}_{m+1} = \frac{\lambda_{m+1}}{l} \int_0^{\infty} dt_m \phi_m(t_m) \;\; \exp(-\alpha\, t_m) \tag{25}$$

Equation (25) shows that the average number of breaks at time t will be well described, for large values of t, by an

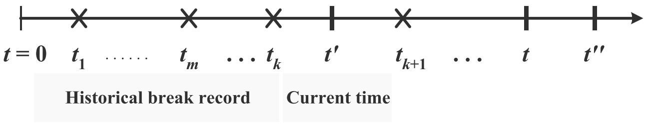

Figure 3. Schematic representation of the time axis with the different characteristic times used for the derivation of conditional probabilities.

exponential function, as long as the hazard function linearly increases with the break order in the exponential period. The average number of breaks per year can be obtained by the integration of expression (24) and we have:

$$\tilde{N}(t) = \frac{\tilde{\lambda}_{m+1}}{\alpha} \exp(\alpha\, t)[\exp(\alpha) - 1] \approx \tilde{\lambda}_{m+1} \exp(\alpha\, t) \qquad (26)$$

$\tilde{N}(t)$ is the average number of breaks per unit length that will occur during year t and the approximation used to derive the right hand term is valid for $\alpha \ll 1$. Equation (26) has been used by many authors to describe the evolution of the average annual number of pipe breaks as a function of time [e.g., *Shamir and Howard*, 1979; *Walski and Pellicia*, 1982; *Kleiner and Rajani*, 1999]. The last development is interesting since it shows that, for the exponential period, a linear relationship between hazard functions and break orders will result in an exponential dependence over time of the average number of breaks per year.

[26] If we consider the case $m = 1$, we have

$$N(t, t') = \frac{1}{l} \int_t^{t'} dt'' f_1(t'') + \frac{\lambda_2}{l} \int_t^{t'} dt'' I_1(t'') \exp(\alpha\, t'') \qquad (27)$$

If we use the Weibull pdf to describe the times of occurrence of first breaks, the numerical integration of the second integral on the right hand side can be easily performed.

3.4. Conditional Probability of Break Occurrences

[27] We now consider the situation where, at time t', k breaks have already been observed on a pipe segment at times $\{t_1, t_2, \ldots, t_k\}$ (Figure 3). What is then the probability to observe a $(k + n)$th break during the interval $[t, t + dt]$ $(t > t')$? We define $\phi_{k+n}(t|k; t')$ as the pdf of occurrence of the $(k + n)$th break during the interval $[t, t + dt]$ considering that k breaks have already occurred at times $\{t_1, t_2, \ldots, t_k\}$ during the period $[0, t']$. For the case $n = 1$, this is given by the following conditional probability:

$$\phi_{k+1}(t|k; t') = \frac{f_{k+1}(t - t_k)}{F_{k+1}(t' - t_k)} \qquad (28)$$

$F_{k+1}(t' - t_k)$ is the survivor function [*Cox and Oakes*, 1994]. For the general case with $n > 1$, we have:

$$\phi_{k+n}(t|k; t') = \frac{\int_t^{t''} dt_{k+1} f_{k+1}(t_{k+1} - t_k) \phi_{k+n}(t|t_{k+1})}{F_{k+1}(t' - t_k)} \qquad (29)$$

If $k \geq m$, we can use equation (10) for $\phi_{k+n}(t'|t_{k+1})$ and after integration, we have for $n > 1$:

$$\phi_{k+n}(t|k; t') = \lambda_{k+1} \sum_{i=k+2}^{k+n} \frac{\eta_{i,k+n}^{k+1}}{(\lambda_i - \lambda_{k+1})} \cdot \{\exp[-\lambda_{k+1}(t - t')] - \exp[-\lambda_i(t - t')]\} \qquad (30)$$

For $n = 1$, the corresponding equation is

$$\phi_{k+1}(t|k; t') = \lambda_{k+1} \exp[-\lambda_{k+1}(t - t')] \qquad (31)$$

These equations can be integrated to obtain the probability of occurrence of the $(k + n)$th break during the interval $[t, t'']$ conditional on the appearance of k breaks during the interval $[0, t']$, $\phi_{k+n}(t, t''|k, t')$ and we find for $n = 1$:

$$\phi_{k+1}(t, t''|k; t') = \exp[-\lambda_{k+1}(t - t')] - \exp[-\lambda_{k+1}(t'' - t')] \qquad (32)$$

and for $n > 1$:

$$\phi_{k+n}(t, t''|k; t') = \lambda_{k+1} \sum_{i=k+2}^{k+n} \frac{\eta_{i,k+n}^{k+1}}{(\lambda_i - \lambda_{k+1})} [\{\exp[-\lambda_{k+1}(t - t')] - \exp[-\lambda_{k+1}(t'' - t')]\} - \frac{\lambda_{k+1}}{\lambda_i} \cdot \{\exp[-\lambda_i(t - t')] - \exp[-\lambda_i(t'' - t')]\}] \qquad (33)$$

Figure 4 gives some example of these pdfs for different n values conditional on the occurrence of a break during the period $[0, t']$ $(k = 1)$.

3.5. Average Number of Breaks Conditional on a Given Historical Record

[28] The average number of breaks per unit time and unit length at time t considering that k breaks have occurred during the period $[0, t']$ $(t' < t)$ at times $\{t_1, \ldots t_k\}$ for a pipe segment of length ℓ, $N(t|k, t')$, is given by:

$$N(t|k, t') = \frac{1}{l} \sum_{n=1}^{\infty} \phi_{k+n}(t|k, t') \qquad (34)$$

Again the equation for the average number of breaks takes a simple form when we consider a linear dependence of λ's as

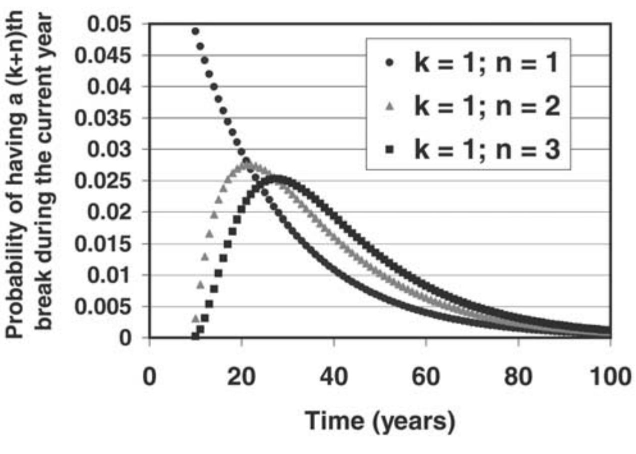

Figure 4. Examples of conditional probability of occurrence of breaks using the Weibull-exp. model. Time t' has been set at 10 years and $t'' = t + 1$.

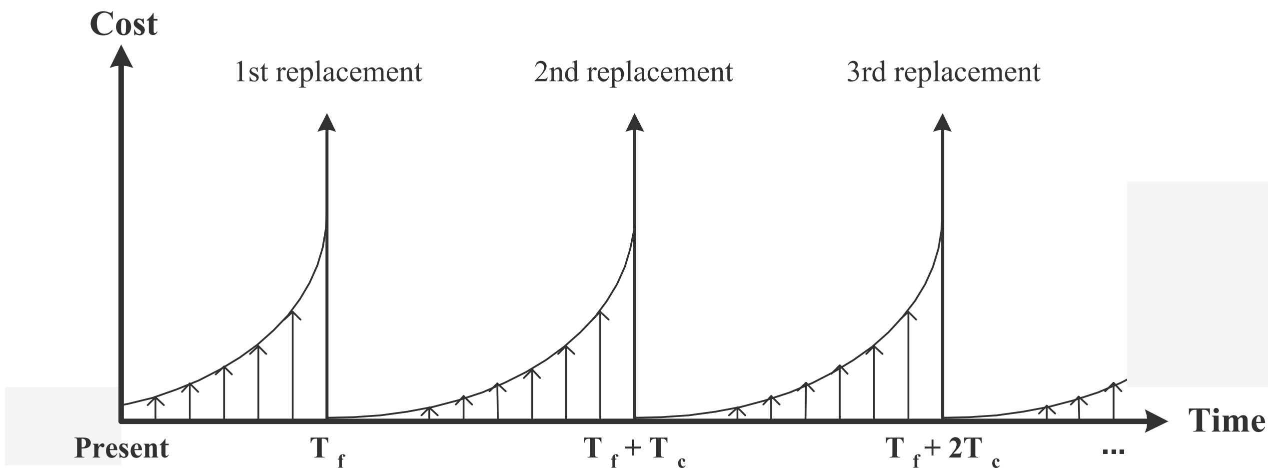

Figure 5. Stream of costs for a given pipe.

a function of break orders (equation 13). To derive this expression, valid for $j > k \geq m$ and $t > t'$, we use a set of equations similar to (20) and (21). The solution is then:

$$N(t\,|\,k;t') = \frac{\lambda_{k+1}}{l} \exp[\alpha\,(t-t')] \quad \forall\, t > t' \tag{35}$$

$N(t|k;t')\,dt$ is the average number of pipe breaks during the interval $[t, t + dt]$ conditional on the occurrence of k breaks before time t' ($t' > t_m$). Equation (35) shows that the conditional probability of occurrence of breaks is still described by an exponential relationship with a coefficient equal to the hazard function associated with the $(k + 1)$th break.

4. Economical Analysis of Pipe Replacement Timing

4.1. Definition of the Minimum Cost Replacement Timing (MCRT)

[29] *Shamir and Howard* [1979] proposed a methodology to estimate the optimal replacement time of pipes. They defined the Minimum Cost Replacement Timing (MCRT) for which the estimation is based on an economical analysis where maintenance and replacement costs as well as discount rates are considered. We adopted, in the following development, the terminology used by *Kleiner et al.* [1998a].

[30] The life cycle of a pipe is represented as periodic replacements between which pipes get older until the next replacement. The period between two replacements is called a cycle. Figure 5 presents the corresponding stream of costs. Four different times are considered in the following development: the pipe installation time, t_0, the present time, t_p, the time at which costs are discounted, t_a, the time when the first replacement occurs T_f. We also define the time interval between subsequent replacements T_c. The steady-state approach is used when replacements occur periodically at times T_f, $T_f + T_c$, ..., $T_f + n\,T_c$, ... [*Kleiner et al.*, 1998a]. Pipe characteristics and the evolution of pipe break number are assumed identical for all cycles. Maintenance costs increase between replacements due to the increase in the average number of breaks. Taking $t_0 = 0$ as the installation time and the beginning of the cycle, we have, for the total cost per unit length for one cycle, discounted at the beginning of the cycle, $C_{tot}(T_c)$, which is given by:

$$C_{tot}(T_c) = C_r e^{-rT_c} + C_b \int_0^{T_c} dt\, N(t)\, e^{-rt} \tag{36}$$

C_r and C_b are respectively the pipe replacement cost per unit length and the cost of a single repair. The continuous discount rate is r and $N(t)$ is as before the average number of breaks per unit time, per unit length, at time t. Pipe indices have been omitted for simplicity. The total cost, discounted at the beginning of the cycle ($t = 0$) for an infinite series of pipe replacements, $C_{\inf}(T_c)$, is given by:

$$C_{\inf}(T_c) = \sum_{m=0}^{\infty} C_{tot}(T_c)\, e^{-mrT_c} \tag{37}$$

The value of T_c that minimizes the total cost $C_{\inf}$ is defined as T^{**}. It corresponds to the optimal time period between successive replacements and therefore between the installation time and the first replacement. For $r \neq 0$ and $T_c \neq 0$, T^{**} can be found, in the general case, by solving the following equation:

$$N(T^{**}) \left(1 - e^{-rT^{**}}\right) - r\,\chi(T^{**}) = r\frac{C_r}{C_b} \tag{38}$$

where $N(T^{**})$ is the average number of breaks at time T^{**} and $\chi(T^{**})$ is given by:

$$\chi(T^{**}) = \int_0^{T^{**}} dt\ N(t)\, e^{-rt} \tag{39}$$

If an exponential form (equation 1) is used to describe the evolution of the average number of pipe breaks over time, substitution in equation (38) leads to an expression similar to equation (5) of *Kleiner et al.* [1998a]. Figure 6 shows a graph of T^{**} as a function of the ratio C_r/C_b for different values of r in the case of the Weibull-exp. model described

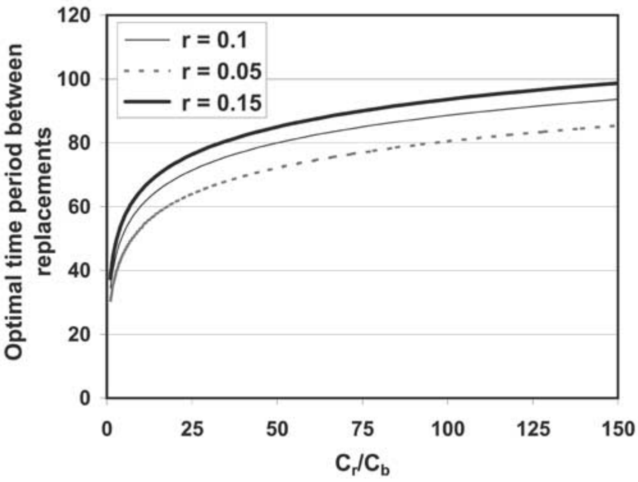

Figure 6. Optimal period between pipe replacements as a function of C_r/C_b for different values of the discount rate r.

previously. The average number of breaks will vary for different types of pipes depending on material, diameter, installation period and therefore different variables should be specified for different homogeneous sets of pipes. We have omitted this dependency for simplicity.

[31] For the general case where we want to minimize the total cost from the present time to infinity, discounted to time t_a, the total cost C_{tot} is given by the following equation:

$$C_{tot}(T_f) = C_r\, e^{-r(T_f - t_a)} + C_b \int_{t_p}^{T_f} dt\ N(t) e^{-r(t-t_a)} + C_{\inf}(T^{**})\, e^{-r(T_f - t_a)} \quad (40)$$

This equation is minimized according to T_f and the corresponding value is defined as T^*. A look at this last equation shows that T^* does not depend on t_p since the dependence of C_{tot} on this variable is included in a term that is independent of T_f. Also, it does not depend on t_a. This nondependence of T^* on t_a and t_p is valid whatever the functional form of N(t). If we consider an exponential form for $N(t)$ (equation 1), the value of T^*, is given by

$$T^* = t_0 + \frac{1}{A_i} \ln\left[r \frac{C_r + C_{\inf}(T^{**})}{C_b N_i(t_0)} \right] \quad (41)$$

The value T^* is called the Minimum Cost Replacement Timing (MCRT) and corresponds to the time when projected maintenance cost will exceed replacement cost. Equation (41) confirms that T^* does not depend on t_a and t_p. The nondependence of T^* on t_p is due to the fact that no specific information on the structural state of the pipe segment at time t_p is taken into account in this type of analysis. This approach is, in some sense, a generic approach based on an average ageing behaviour. Since T^{**} is the time period between replacements, we have

$$T^{**} = T^* - t_0 \quad (42)$$

All the valuable information is already included in T**. The time remaining until the first replacement, T_r, is simply given by:

$$T_r = T^* - t_p \quad (43)$$

Walski and Pellicia [1982] have extended this approach to include two correction factors. One correction factor, the "previous break factor", accounts for the effect of previous breaks on the predicted break rate and the "pipe-size factor" accounts for the impact of the pipe size on the breakage rate. The model used is thus similar to the one of Shamir and Howard except for these multiplicative factors.

4.2. Optimal Replacement Timing Taking Into Account the Actual Pipe Break Record

[32] We first consider $C_1(T_f; \{t_i\}_k)$, the total cost from $t = 0$ (installation time) to the replacement time, T_f, (one cycle of the stream of costs) for a pipe segment of length ℓ which had k failures at time $t_1, t_2, \ldots, t_k$ during that period and for given values of r, C_b and C_r. This cost is given by:

$$C_1(T_f; \{t_i\}_k) = C_r(l) e^{-rT_f} + C_b \sum_{i=1}^{k} e^{-r\,t_i} \quad (44)$$

This is the actual cost associated with the pipe break record for one cycle. C_r is actually a function of pipe segment length ℓ. The probability of occurrence of the sequence of pipe breaks is estimated through the model describing probability distributions of time between successive breaks. Using the Weibull-exp. model, we can estimate the maintenance cost pdf. Figure 7 presents this pdf for: r = 0.05 year^{-1}, $C_r(\ell)/C_b$ = 50 and T_f = 72 years. The minimal maintenance cost occurs when no pipe break is reported during the period from installation to replacement. For reasons of clarity, the peak at C/C_b = 0 has been omitted. Only some cost ranges are allowed for a given number of breaks and, depending on the values of r and T_f, it is possible to have discontinuous distributions (see appendix A). Only one of these "forbidden" ranges of cost values is seen on Figure 7, namely the one defined by $]0, e^{-r\,T_f}[$.

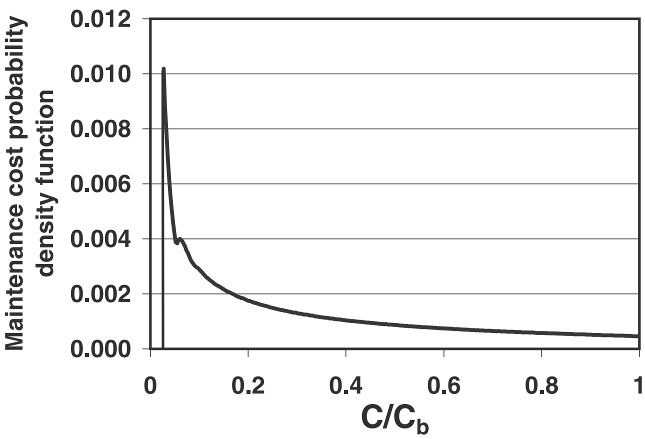

Figure 7. Maintenance cost distribution using the Weibull-exp. model with hazard functions linearly related to the break order for the exponential period. Values of λ_0, p, r, and T_f are 0.04 breaks/year, 0.1 year $^{-1}$, 0.05 year $^{-1}$, and 49.1 years, respectively.

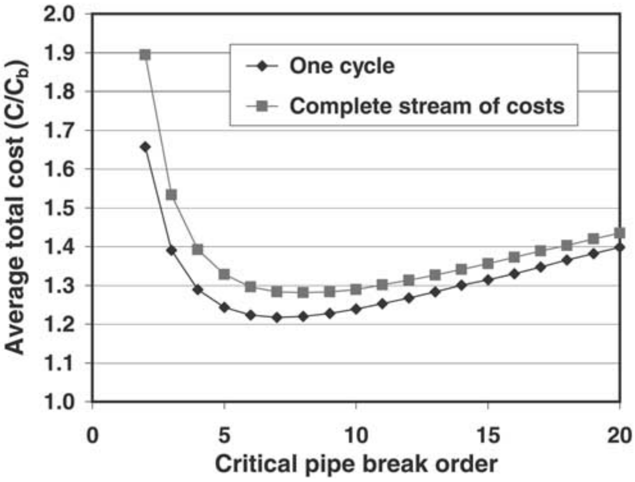

Figure 8. Average total cost for one cycle (diamonds) and for the complete stream of costs (squares) as a function of the pipe break order at which replacement is made. Parameter values are identical to those of Figure 10.

[33] When the total cost (equation 44) is averaged over all the possible break histories that can occur during the period $[0, T_f]$, an average cost is estimated that corresponds to equation (36). This average cost is showed in Figure 8. The estimation of a MCRT, as defined in the previous section, is therefore based on the minimization of this average cost.

[34] Going back to equation (44) and considering a pipe segment of length ℓ which had k breaks from installation to time t', the total cost at time t' can be estimated considering the average number of breaks at time $t > t'$, $N(t|k;t')$.Without loss of generality, we assume in the following that $t_a = t_0 = 0$. If we average total costs over all possible break sequences from time t' to infinity, we have:

$$C_{tot}\left(T_f;l\right) = C_r(l)e^{-rT_f} + C_b \sum_{i=1}^{k} e^{-r\,t_i} + C_b \int_{t'}^{T_f} dt \;\; N(t|k\;;t';l)\; e^{-rt} + C_{\inf} \;\; e^{-rT_f} \tag{45}$$

[35] The dependence of parameters on pipe segment length has been explicitly written. The different terms on the right hand side are related to: (1) the cost of the first replacement at time T_f, (2) the costs of maintenance for k breaks that occurred before time t', (3) the total cost of maintenance from the present time to the first replacement, estimated from the average number of breaks expected for $t > t'$ and (4) the total cost from the time of the first replacement to infinity, C_{inf}, discounted at time $t = 0$. C_{inf} is obtained by averaging $C_{tot}(t_f;l)$ over all possible sequences of breaks from $t = 0$ to infinity. It will depend on the criterion used to define the replacement time for each cycle. Thus we must find the value of T_f that minimizes equation (45) and deriving this equation with respect to T_f, we find, using the Leibnitz rule for derivation:

$$N\left(T_f|k\;;t';l\right) = \frac{r\,[C_r(l) + C_{\inf}]}{C_b} \tag{46}$$

For a given historical pipe break record, k pipe breaks at times $\{t_i\}$, and a functional form of the average number of pipe breaks conditional on this historical record, the last expression defines the time of the first replacement, T_f. Replacement will effectively occur when the estimated replacement time estimated from (46) is equal to the present time ($T_f = t'$). The term of the left hand side of equation (46), when $T_f = t'$, corresponds to the probability of occurrence of a $(k + 1)$th break at t', conditional on the nonoccurrence of this break order during the preceding period $[t_k, t']$. This is simply the hazard function associated with the $(k + 1)$th break. We thus have:

$$h_{k+1}\left(T_f\right) = \frac{r\,[C_r(l) + C_{\inf}]}{C_b} \tag{47}$$

where $h_{k+1}(t_f)$ is the hazard function associated with the $(k +$ 1) th break order. The last equation defines the criterion for determining the optimal time of replacement based on a historical record for a given pipe. However, the value of C_{inf} depends on this criterion. The exact value of the right hand side of equation (47) must be found recursively and optimal values of $C_{tot}(t_f\,;l)$ and C_{inf} are obtained when the value of $C_{\inf}$ used to define the criterion, equation (47), is equal to the value of C_{inf} estimated when this last criterion is used and the total cost is averaged over all possible pipe break sequences from the time of installation to infinity. An example of application is given in the following section for the class of models with an exponential period.

4.3. Application to the Model With an Exponential Period

[36] We suppose in the following that replacement occurs when a pipe is in the exponential period of its ageing process. In that case, hazard functions are constant with time and depend only on break orders. Assuming that:

$$\lambda_j \leq \lambda_{j+1} \quad \forall\, j > m \tag{48}$$

This condition means that the replacement will occur when the k'th break order occurs for which the associated hazard function satisfies the equation:

$$\lambda_{k'} < \frac{r\,[C_r(l) + C_{\inf}]}{C_b} < \lambda_{k'+1} \tag{49}$$

Using equation (13) for $\lambda_{k'}$, we then have for k':

$$k' = \mathrm{int}\left\{\frac{1}{\alpha}\left[\frac{r\,[C_r(l) + C_{\inf}]}{C_b} - \lambda_0\right]\right\} + \; m \tag{50}$$

The expression "int$\{x\}$" means that we consider only the integer part of the real value of x. This expression defines the break order at which the replacement should take place. For a given pipe, replacement time, T_f, corresponds to the time when the k'th break occurs. The corresponding total cost for that pipe at the replacement time is:

$$C_{tot}\left(T_f\right) = C_r(l) \;\; e^{-rT_f} + C_b \sum_{i=1}^{k'} e^{-r\,t_i} \tag{51}$$

The value of k' is given by (50). With this approach, optimal replacement is defined by an optimal break order at which

Table 1. Proposed Ranges of Values for Parameters Used to Estimate Optimal Replacement Time

Parameters	Proposed Ranges
Annual growth rate of breaks - A (year $^{-1}$)	[0.01, 0.19]
Discount rate r (year $^{-1}$)	[0.04, 0.15]
Number of breaks per km in year t_0, $N(t_0)$ (km $^{-1}$ year $^{-1}$)[a]	[0.01, 0.1]
Cost of repairing a break C_b (US \$)	[500, 4850]
Cost of replacing 1 km of pipe C_r (US \$)	[32 800, 492 100]

[a]We assume that t_0 is the time of installation.

replacement should take place. Replacement time will then be different for different pipes depending on their historical break records. The distribution of the occurrence time of the k'th break is given by equation (10).

[37] Figure 8 presents the estimated average total cost for one cycle and the value for the total stream of costs for the Weibull-exp. model with a value of $C_r(l)/C_b$ of 10 and r = 0.05 year^{-1} as a function of the critical break order at which the replacement is done. As can be observed on this graph, the minimal cost value for one cycle is obtained for $k' = 7$, while the minimal cost for the complete stream of costs is obtained at $k' = 8$. The values of C_{inf} and k' obtained satisfy equation (50). However the total cost variation around the minimal value is small and not very sensitive to the critical break order.

[38] Estimation of some optimal replacement break orders was done using some parameter values reported in literature [e.g., see *Shamir and Howard*, 1979; *Kleiner and Rajani*, 1999; *Villeneuve et al.*, 1998; *Chevalier*, 1996] and Table 1 suggests ranges for these parameter values. Exponential functions defined by parameters $N(0)$ and A were used to describe the evolution over time of the average number of pipe breaks. The exponential form was assimilated to a model with exponential pdfs for time-to-failure and hazard functions linearly related to break orders. According to the analysis presented in section 3.3, values for A and $N(0)$ correspond to the values of α and λ_0 in equation (13). Table 2 gives some examples of the optimal break orders, the corresponding average period between replacements, the average numbers of breaks at replacement year estimated for different sets of parameter values. Figure 9 presents the evolution of the optimal break order as a function of the ratio C_r/C_b for four sets of values of (A/r, $N(0)/r$) taken among the possible values defined in Table 1. As can be seen, the relation is nearly linear and the slope of the relation is close to r/A as suggested by equation (50). This means that, for a given C_r/C_b, $N(0)$ and r, the number of

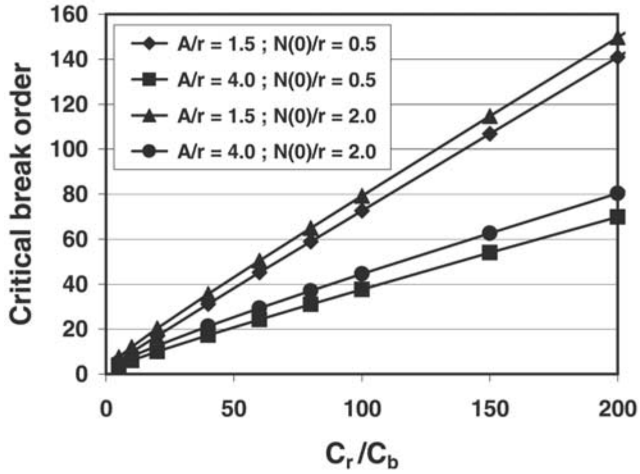

Figure 9. Critical break order as a function of C_r/C_b for four sets of values of (A/r, $N(0)/r$).

breaks over a given period of time will increase as A increases thus favouring an earlier replacement. Also, as intuitively expected, the critical break order will increase as the replacement cost increases compared to the repair cost. Finally, it is observed that an increase in the ratio $N(0)/r$ leads to an increase in the number of pipe breaks and repair costs and consequently, this means a larger value of the critical break order as shown on Figure 9.

[39] The comparison of the total costs estimated using the proposed approach and the Shamir and Howard's approach shows that a substantial reduction in cost is obtained when a replacement criterion based on break order is used. The average time period between replacements is larger than the MCRT obtained with the Shamir and Howard's approach. Figure 10 presents the total cost distribution obtained when the following set of parameter values is used: $A = 0.12$ year^{-1}, $N(0) = 0.07$ break/year, $r = 0.06$ year $^{-1}$ and $C_r(\ell)/C_b = 40$. Simulations were performed using exponential pdfs with hazard functions linearly related to break order. The total cost distribution was estimated by generating sequences of pipe breaks. Pipe replacements were made according to the replacement criterion presented previously and the total cost of each realisation was estimated. Many replacement cycles were considered. The optimal time period between pipe replacements is 40.9 years, the optimal break order is 27 and the average number of breaks during the replacement year is 4 breaks/year (see Table 2). The solid vertical line corresponds to the distribution mean. The average cost obtained using Shamir and Howard's approach

Table 2. Examples of Values of the Optimal Replacement Break Order for Different Parameter Values

Parameter Values				Optimal Replacement Break Orders	Average Period Between Replacements, years	Average breakage Rate at the Replacement Year, breaks/year	Percentage of Reduction of the Total Cost When Compared to Shamir and Howard's Approach
A, year $^{-1}$	N (0), breaks/year	r, year $^{-1}$	$C_r(\ell)/C_b$				
0.08	0.01	0.12	120	180	167	14.9	2.6
0.09	0.04	0.1	100	112	77.5	10.6	9.7
0.1	0.04	0.06	80	53	65.3	6	24.8
0.12	0.07	0.06	40	27	40.9	4	21.6
0.19	0.1	0.04	20	10	21.8	2.6	27.5

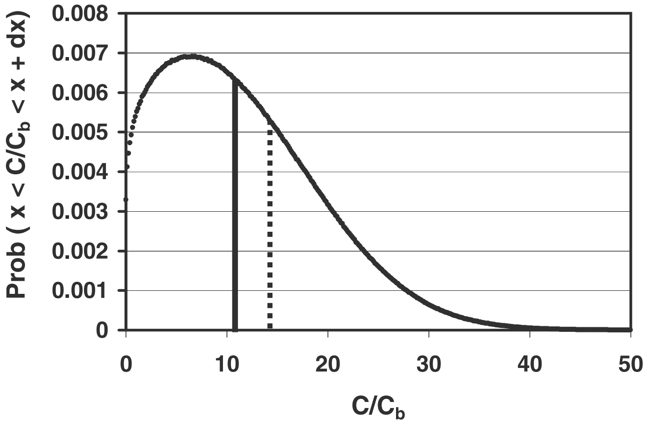

Figure 10. Probability that the total cost value is in the interval $[x, x + dx]$ with $dx = (C_r/C_b + k')/500$. A model with exponential pdfs is used with λ values linearly related to break orders ($\lambda_0 = 0.07$ and $\alpha = 0.12$). Values of C_r/C_b and r are 40 and 0.05 year $^{-1}$, respectively, and the corresponding k' value is 27. The thin line corresponds to the distribution average, while the dotted line corresponds to the optimal total cost obtained from Shamir and Howard's approach.

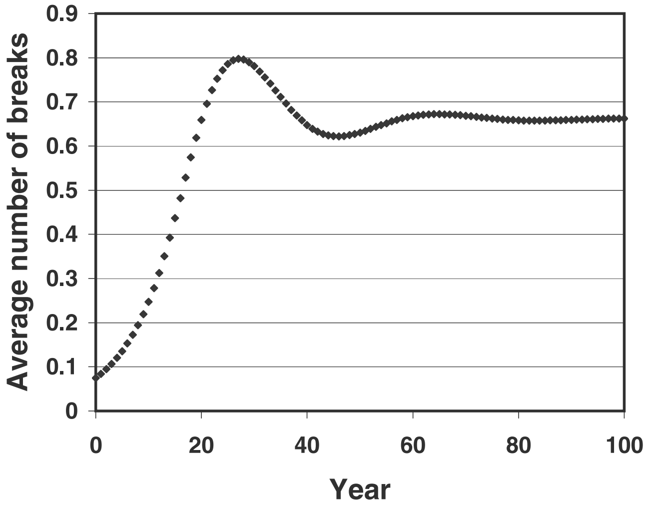

Figure 12. Average number of breaks per year as a function of time. Parameter values are identical to the ones used for Figure 10.

is also presented (dash vertical line). Figure 11 presents the probability that the critical pipe break order appears during the given year. Finally, Figure 12 shows the average number of pipe breaks per year as a function of time. As can be seen, an exponential increase is observed during the early years followed by a period where the slope decreases and a maximum is reached at around the 27th year. In fact, as time goes by, replacements are more likely to occur and consequently, the increase in the average number of pipe breaks stabilizes.

5. Summary and Conclusion

[40] The structural state of underground infrastructures is difficult to assess. One useful indicator of the structural state of water mains is their historical pipe break record. Even if it is an indirect indicator, useful information can be extracted from these data since pipe breaks are usually correlated in space and time. The occurrence of pipe breaks can thus be interpreted as a sign of faster deterioration of the structural state.

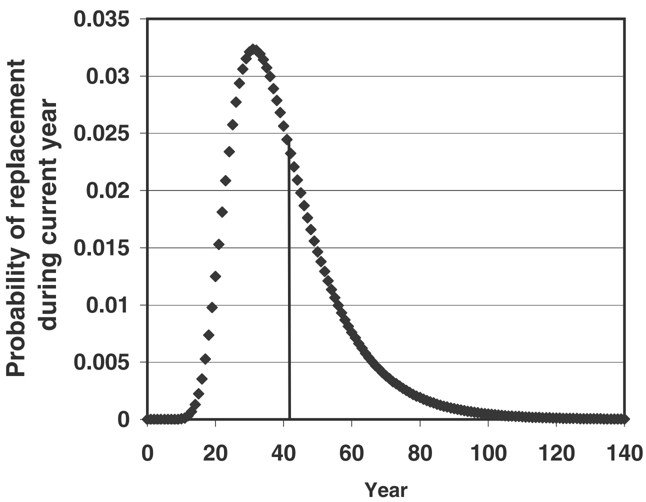

Figure 11. Probability that the first replacement occurs during a given year. Parameter values are identical to the ones used for Figure 10. The thin line corresponds to the average value of the first replacement year.

[41] A framework has previously been proposed to statistically model break occurrences from break record data. According to this, time-to-failure between successive pipe breaks are statistical variables and are represented by pdfs. More specifically, previous studies have showed that time-to-failure between the installation and the first pipe break and between the first and the second pipe breaks can be described by pdf with nonconstant hazard functions. Time-to-failure between higher break orders, on the other hand, are well described by constant hazard functions. The evolution of the structural state of pipes is therefore characterized by two periods. The first one, called the nonexponential period, where times to failure are described by pdfs different from exponential distributions and a second one, corresponding to high order breaks where pdfs are exponential.

[42] The use of exponential distributions makes possible the derivation of a number of analytical expressions. Pdfs for the occurrence of the jth break as a function of time were derived as well as the expression for the time evolution of the average number of breaks. Expressions for cases where low order breaks are described by nonexponential distributions were also derived and can be estimated using numerical integration methods. Conditional pdf and the average number of pipe breaks as a function of time were also derived based on a given historical pipe break record.

[43] The special case where hazard functions for the exponential period are linearly related to break orders was further analysed. It was showed that the resulting function describing the time evolution of the average number of pipe breaks per unit time is exponential. Many authors have used this type of function to describe the time evolution of the average number of pipe breaks. Assuming that the pdfs are exponential and that the nonexponential period is limited to the first and second break orders, this analysis suggests that

hazard functions are somehow linearly related to break orders at the beginning of the exponential period. As suggested by *Gustafson and Clancy* [1999a], it seems likely that hazard functions will be constant after a given number of breaks, leading to a constant average number of breaks per unit time.

[44] One important goal in the development of a model assessing the time evolution of probability of occurrence of pipe breaks is to assist network managers when he has to decide whether or not he should replace a pipe segment. Although many factors have to be taken into consideration, it is crucial to see if the knowledge extracted from the adopted modeling scheme can be fruitfully use to take a decision. From an operational point of view, given a pipe break record on a pipe segment, the model relates future probability of pipe break occurrence to the historical record. Using a framework similar to the one defined by Shamir and Howard, a general expression was derived based on the expected occurrence of pipe breaks given an observed pipe break record. Minimization of the total cost over the complete stream of costs leads to a criterion defining the optimal replacement time and involving the hazard functions associated to different break orders.

[45] Application of this criterion to the case where replacements occur during the exponential period leads to a replacement criterion based on break orders. Indeed, a minimal total cost is obtained when replacement is done after the occurrence of a critical break order defined by equation (50). Using parameter values from the literature, it was showed that minimal total costs obtained with this criterion were smaller than the ones obtained using the Shamir and Howard's approach.

[46] The examples of section 4.3 have showed that an optimal pipe replacement time will generally exist, even if seems unrealistic in some cases, when the average number of breaks per unit time follows an exponential form. This is so because the number of pipe breaks increases indefinitely and will, at some time, exceed replacement costs. However, following *Gustafson and Clancy* [1999a], it may be more reasonable to think that hazard functions will reach a constant value after a number of breaks have occurred. In some situations, no critical pipe break order can be identified. From a strict economical point of view, this would mean that it is better not to replace the pipe segment but repair pipe breaks as they occur even if this means digging the same hole year after year ! This is an unrealistic solution and it demonstrates that, even if an economical analysis is important, other constraints must be integrated in the decision making process along with criteria related to network reliability. These aspects need to be examined in future works.

[47] Among the topics that would also need to be further investigated, the impact of pipe segment length on the identification of an optimal replacement time is an important one. In the analysis presented herein, a pipe segment of length ℓ was considered. Statistical modeling of pipe breaks was performed and the model parameters calibrated for pipe segments of length ℓ. Replacement costs were assumed to be length-dependent and the replacement criterion dependent on pipe segment length through the ratio C_r/C_b. However, it would be important to better describe and integrate into the model spatial correlations between breaks.

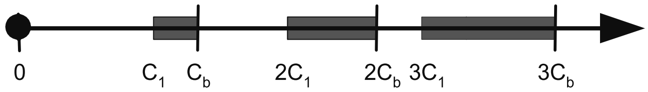

Figure A1. Example of a configuration of possible maintenance cost values. In this example $C_1 = 3/4\ C_b$.

[48] The issue of uncertainties related to this type of model is also of primary importance from an operational point of view. One fundamental question remains: can we expect to define an adequate "optimal replacement strategy" considering the scarce available data and the use of a simplified model with large uncertainties on the parameter values? Very few studies if none have addressed these issues. It is clear that further studies are necessary to precise the accuracy of the pipe break models, to identify sensitive parameters and also to account for the variability of some economical parameters (for example the continuous discount rate).

Appendix A

[49] The range of possible maintenance costs if one break occurs during the interval $[0, T_f]$ is:

$$C_b e^{-rT_f} < C < C_b \tag{A1}$$

The maximal and minimal values correspond respectively to the situations when pipe break occurs at $t = 0$ and at T_f. We define:

$$C_1 = C_b e^{-rT_f} \tag{A2}$$

More generally, if k pipe breaks occur during the period $[0, T_f]$, the range of possible maintenance costs is given by:

$$k\ C_1 < C < k\ C_b \tag{A3}$$

Figure A1 shows a possible configuration of maintenance cost ranges. This configuration is obtained for $C_1 = 0.75\ C_b$. A maintenance cost of zero is possible and corresponds to the case where no break occurs during the period $[0, T_f]$. As can be seen from Figure A1, it is possible to have ranges of unallowed maintenance cost values. Since $C_1 > 0$ (unless T_f goes to infinity), maintenance cost values included in $]\,0, C_1\,[$ are impossible. Furthermore, if $2\ C_1 > C_b$, then the values within the interval $]\,C_b, 2\ C_1\,[$ are not allowed. If we set $2\ C_1 = C_b$ and find T_1, the value of T_f that satisfies this last equation, we find:

$$T_2 = -\frac{1}{r}\ln\left(\frac{1}{2}\right) \tag{A4}$$

The inequality $2C_1 > C_b$ is satisfied when $T_{\rm f} < T_2$. More generally, a forbidden range of maintenance cost values $]\,(k-1)\ C_b, k\ C_1\,[$ will be possible, if we have $T_{\rm f} < T_k$ with:

$$T_k = -\frac{1}{r}\ln\left(\frac{k-1}{k}\right) \tag{A5}$$

As can be seen from Figure A1, ranges of forbidden values get smaller as we increase k so that a critical k can be defined for which no more forbidden ranges of maintenance cost values are possible. Since for $r = 0.05$ year^{-1} (see Table 2), we have $T_2 = 13.9$ years, it is unlikely that forbidden ranges other than the one between 0 and C_1 will occur.

Notation

A_i breakage rate growth for pipe i.

C cost.

C_b cost of a single break repair.

$C_{\inf}(T_c)$ total cost, discounted at the beginning of the cycle (t = 0) for an infinite series of pipe replacements.

$C_r(l)$ cost of pipe replacement for a pipe segment of length ℓ.

$C_{tot}(T_c)$ total cost per unit length, discounted at the beginning of the cycle.

$C_1(T_f;\{t_i\}_k)$ total cost from the installation time to replacement, T_f, for a pipe that had k failures at time $t_1, t_2, \ldots, t_k$ during that period for a given r, C_b and C_r.

$f_j(t)$ probability density function of time between the $(j-1)$th and the jth break.

$f_W(t)$ Weibull probability density function.

$F_j(s)$ Laplace transform of function $f_j(t)$.

$\bar{F}_j(t)$ survivor function associated with the jth break.

$h_j(t)$ hazard function associated with the jth break order.

ℓ pipe segment length.

m number of breaks during the nonexponential period or break order of the last break of the nonexponential period.

$N(t)$ average number of breaks per unit length per unit time.

$N(t, t')$ average number of breaks per unit length during period $[t, t']$.

$N(t|t_m)$ average number of breaks per unit length per unit time considering that the mth break has occurred at time t_m.

$N(t|k;\ t')$ average number of breaks per unit length, per unit time, considering that k breaks have occurred during the period $[0, t']$.

$\tilde{N}(t)$ average number of breaks per unit length occurring during year t.

p "scale" parameter of the Weibull distribution.

r continuous discount rate.

s dummy variable used for the Laplace transform.

t time.

t' ending time of the recorded pipe break period.

t_0 installation time of a pipe.

t_a time at which the costs are discounted.

t_j time of occurrence of the jth recorded break.

t_m time of the mth break defining the end of the nonexponential period.

t_p present time.

T_c time period between successive replacements.

T_f time of the first pipe replacement.

T^* Minimum Cost Replacement Time (MCRT).

T^{**} time that minimizes the total cost from the installation time to infinity.

α slope of the linear relationship between hazard values and break orders.

κ "shape" parameter of the Weibull distribution.

λ_j parameter of the exponential distribution used to describe time between the $(j-1)$th and the jth break.

$\phi_j(t)$ probability density function of occurrence of the jth break.

$\phi_j(t|t_m)$ probability density function of occurrence of the jth break if the mth break has occurred at time t_m.

$\phi_{k+n}(t|k;\ t')$ probability density function of occurrence of the $(k + n)$th break if k breaks have occurred during the period $[0, t']$ $(t > t')$.

$\phi_{k+n}(t, t''|k;\ t')$ probability of occurrence of the $(k + n)$th break during the period $[t, t'']$ if k breaks have occurred during the period $[0, t']$ $(t > t')$.

$\varphi_j(s)$ Laplace transform of function $\phi_j(t)$.

$\psi_j(t|t_m)$ probability that $(j - m)$ breaks occurred during the period $[t_m, t]$ if the mth break has occurred at time t_m.

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