Non-Stationary Hydrologic Frequency Analysis using B-Spline Quantile Regression

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Abstract

Hydrologic frequency analysis is commonly used by engineers and hydrologists to provide the basic information on planning, design and management of hydraulic and water resources systems under the assumption of stationarity. However, with increasing evidence of climate change, it is possible that the assumption of stationarity, which is prerequisite for traditional frequency analysis and hence, the results of conventional analysis would become questionable. In this study, we consider a framework for frequency analysis of extremes based on B-Spline quantile regression which allows to model data in the presence of non-stationarity and/ or dependence on covariates with linear and non-linear dependence. A Markov Chain Monte Carlo (MCMC) algorithm was used to estimate quantiles and their posterior distributions. A coefficient of determination and Bayesian information criterion (BIC) for quantile regression are used in order to select the best model, i.e. for each quantile, we choose the degree and number of knots of the adequate B-spline quantile regression model. The method is applied to annual maximum and minimum streamflow records in Ontario, Canada. Climate indices are considered to describe the non-stationarity in the variable of interest and to estimate the quantiles in this case. The results show large differences between the non-stationary quantiles and their stationary equivalents for an annual maximum and minimum discharge with high annual non-exceedance probabilities.

Keywords: Quantile regression, B-Splines, Bayesian, Streamflow, AMO, PDO.

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INTRODUCTION

Understanding the temporal variability of hydrological processes and their associated statistics is essential for better water resource management. Frequency analysis of extreme hydrologic data has been widely used for problems related to engineering design, flood risk management, river navigation planning and water quality management. Generally, current methods of hydrological frequency analysis have been most often based on the assumption of stationarity. Indeed, classical frequency analysis is based on the assumption of underlying independent and identically distributed (i.i.d.) random variables. The last assumption is not valid in non- stationary circumstances. In the context of hydrological processes, non-stationarity is often present because of seasonal effects, perhaps due to different climate patterns in different months, or in the form of trends, possibly due to long-term climate changes (e.g., Stocker et al., 2013; Bates et al., 2008). Basically, strict-sense stationarity means that the distribution remains constant over time. From a practical point of view, hydrologists assume second-order stationarity, which implies that the first two moments (mean and variance) do not vary over time. Several tests are used to detect non-stationarity in time series including the KPSS test (Kwiatkowski et al., 1992), the Leybourne-McCabe test (Leybourne and McCabe, 1994) and the Mann Kendall test (Mann, 1945). The last one is the most commonly used one in hydro-climatological studies (e.g., Dry and Wood, 2005; Cunderlik and Burn, 2002; Cunderlik and Ouarda, 2009; Nasri et al., 2013; Fiala et al., 2010; Khaliq et al., 2009). The frequency analysis of a non-stationary series calls for a different understanding than the conventional approach involving stationarity. In fact, in the context of climate change, the distribution parameters and the distribution of hydrological extremes were likely to be modified. As a consequence, the exceedance probability used to estimate the return period also varies over time. Recently, several methods are proposed to take into account, at least partially, non-stationarity in the context of frequency analysis. The most popular approach is the frequency analysis with covariates method.

The idea underlying the covariates approach is to incorporate the covariates into the distribution parameters (e.g., Coles, 2001; Olsen et al., 1999; Vrac and Naveau, 2007; Aissaoui-Fqayeh et al., 2009; Cannon, 2010; Ouarda and Adlouni, 2011; El Adlouni and Ouarda, 2009). Two distributions are generally used in this case: The Generalized Extreme Value (GEV) (e.g., Fisher and Tippett, 1928; Jenkinson, 1955; Hundecha et al., 2008; El Adlouni et al., 2007) and the Generalized Pareto Distribution (GPD) for a Peaks Over a Threshold (POT) approach (Pickands, 1975; Ehsanzadeh et al., 2007). In the case of stationary data, these distributions are based on limit results of extreme value theory (TVE) (e.g., Fisher and Tippett, 1928; Pickands, 1975). The use of GEV and GPD for non-stationary data proposed by (Coles, 2001) are not based on the extreme value theory. In the fact, it is a "natural" extension of the two models (GEV and GPD) where the stationarity hypothesis is not satisfied. Introducing covariates in one of these distributions can be done through any parameter. The effect of a covariate can be modeled by making one or more parameter linearly (e.g., Coles, 2001; Cannon, 2010) or nonlinearly (e.g., Chavez-Demoulin and Davison, 2005; Nasri et al., 2013; Neville et al., 2011) dependent on the covariate. The covariate method is largely developed and has been used in the literature to understand the variation in hydrological time series. Climate indices are commonly used as covariates. This approach works well in the case of linear or quadratic dependence and in the case of one covariate. However, it suffers from several disadvantages in the case of several covariates: (i) the introduction of several covariates in this model increases the number of hyper-parameters, which decreases the model parsimony and potentially increases estimation errors (ii) the interactions of the predictors make the model much more complicated because it requires multivariate function modelling. For this reason, some recent studies (e.g., Nasri et al., 2013, 2015) suggest to use the quantile regression method, which was introduced by (Koenker and Bassett, 1987). Quantile regression provides the conditional quantiles of the response variable for a fixed value of covariates rather than only the conditional mean. This model can be a good alternative to overcome the problems of convergence raised by the covariate method. Some recent studies in the hydroclimatology context have used linear quantile regression to estimate non-stationary extreme events (e.g.,Cannon,2011; Tareghian and Rasmussen, 2013). The linear quantile model assumes that the relationship between the variable of interest and covariates is linear. However, in hydroclimatology, the dependence between covariates and variables of interest can take different structures. For this reason, we should investigate the use of a quantile regression model with a more general form of dependency. The nonparametric quantile model allows the assumption of linearity to be relaxed. This model aims to identify the best function according to the data distribution, rather than imposing a restrictive parametric model.Several nonparametric quantile methods have been proposed in the literature (e.g., Koenker et al., 1994; Hendricks and Koenker, 1992). The most popular is a smoothing regression (or splines regression). Briefly, splines regressions are obtained by joining smoothed polynomial functions separated by a sequence of knots. A larger number of knots lead to a more flexible curve and hence, a better fit. Splines regressions for quantile smoothing have been introduced by (Hendricks and Koenker, 1992). Several variants have been suggested by (Koenker et al., 1994) who proposed to use natural polynomial splines. This nonparametric quantile regression method is also used in hydrology, (see Donner et al. (2012)). Recently (Nasri et al., 2013) proposed to use B-splines to model nonlinear dependencies. Indeed, B-spline functions are linear combinations of non-negative piecewise polynomials (real functions). This type of functions has some advantages: B-splines do not depend on the response variable, or the variable of interested, but depend only on: (1) the support of the covariates, (2) the number and position of knots and (3) the degree of the B-spline function (De Boor, 2001). The objective of the present study is to use B-Spline quantile regression for modelling non-stationary hydrological extreme events (floods and drought) for some rivers located in the province of Ontario (Canada). This province has undergone a number of extreme events over the past two decades. For instance, in 2001: the aggregate level of the Great Lakes plunged to its lowest value in more than 30 years, with lakes Superior and Huron displaying near record lows (Mitchell, Septembre,2002; Ashkar and Ouarda, 1996) and in 2005: heavy rainfall and associated flooding resulted in CAD 500 million $ in insured damages (Sandink, 2013). Hydrologic extreme events are often linked to atmospheric circulation patterns. In fact, several recent studies in North America have modelled the non-stationarity of precipitation and streamflow using climate indices. The most used climate indices in this context are El Nino Southern Oscillation (ENSO) (e.g., Regonda et al., 2005; Cannon, 2010; Nasri et al., 2013), Pacific Decadal Oscillation (PDO) (e.g., Brabets and Walvoord, 2009; Khaliq and Gachon, 2010; Cannon, 2010; Nasri et al., 2013), Atlantic Multi-decadal Oscillation (AMO) (Teegavarapu et al., 1969) and North Atlantic Oscillation (NAO) (Hurrell and Van Loon, 1997). To our knowledge, no studies have previously studied streamflow extremes using nonparametric quantile regression incorporating B-Spline functions. In the next section, the theoretical background of the nonparametric quantile regression model and its estimation are provided. Data are then presented in Section 3 and results are presented in Section 4. Section 5 provides a conclusion.

1. THEORETICAL BACKGROUND

1.1. Linear Quantile Regression Model

Linear quantile regression is related to linear least-squares regression in that both are used to study the linear relationship between a response variable and one or more independent or explanatory variables. However, whereas the least-squares regression is concerned with modelling the conditional mean of the response variable, quantile regression provides a model of the conditional pth quantile of the response variable, for some value of $p \in]0,1[$. For example, the conditional median corresponds to $p = 0.5$.

For a vector $y = (y_1, \ldots, y_n)$, the sample mean $\hat{y}$, solves the least squares problem:

$$\arg\min_{\mu \in \mathbb{R}} \sum_{i=1}^{n} (y_i - \mu)^2.$$

In many situations, the response Y, the conditional mean of Y depends on some covariates $\mathbf{X} = (X_1, \ldots, X_d)$ (Example: Y can be the maximum annual precipitation or discharge and $\mathbf{X}$ can be a covariate, such as a climate index.). Based on sample (y_i, x_i), where $\mathbf{x}_i = (x_{1i}, \ldots, x_{di})$, for the linear regression model, we suppose that $y_i = \alpha_0 + \mathbf{x}_i^{'}\alpha + \varepsilon$, where $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_d)$, $E(\varepsilon \mid X) = 0$ and $var(\varepsilon \mid X) = c$. An estimate of α_0 and $\boldsymbol{\alpha}$ is obtained by minimizing the following quantity:

$$\sum_{i=1}^{n} \left(y_i - \mathbf{x_i}^{'}\boldsymbol{\alpha} - \alpha_0 \right)^2.$$

For quantile regression, we suppose that $y_i = \alpha_0 + \mathbf{x}_i^{'}\alpha + \varepsilon$., where the p th quantile of $\varepsilon \square \mathbf{x}_i$ is defined as: $Q_p(\varepsilon \square \mathbf{x}_i) \equiv \inf\left\{\varepsilon \mid x_i : F(\varepsilon \mid x_i) \geq p\right\} = 0$. Quantile regression can be derived in a similar manner by specifying the p th conditional quantile as $Q_p(Y \mid X) = \mathbf{X}\alpha(p) + \alpha_0(p)$ and estimating $\boldsymbol{\alpha}(p)$ and $\alpha_0(p)$ as the solution to

$$\arg\min_{\mu \in \mathbb{R}} \sum_{i=1}^{n} \rho_p\left(y_i - x'_i\alpha - \alpha_0\right). \quad (1)$$

where ρ_p (z) is a loss function defined as:

$$\rho_p(z) = \begin{cases} z(p-1)\ if\ z < 0; & z \in \mathbb{R} \\ zp\ \ otherwise. \end{cases} \quad (2)$$

In the case of the linear ordinary regression, the loss function can be written as $\rho(z) = z^2$. Figure 1 gives a simulation example to show the difference between the linear regression and linear quantile regression with their loss function.

1.2. Nonparametric Quantile Regression with B-Spline Functions

Nonparametric regression allows the assumption of linearity to be relaxed (Fox, 2000), and it restricts the analysis to smooth and continuous functions. The aim of nonparametric regression is to identify the best regression function according to the data distribution, rather than estimating the parameters of a specific model.

Let us consider the simplest regression case of one explanatory variable:

$$y = \boldsymbol{f}(x) + \varepsilon. \quad (3)$$

In nonparametric regression, the function $\boldsymbol{f}$ is not specified and it is commonly assumed that the errors are independent and identically distributed with mean equal to zero. Also, we assume that the errors are independent of the covariate. In the framework of nonparametric quantile regression, several methods are proposed in the literature. In this work, the B-spline quantile regression is proposed. B-splines, in this case, will be used to approximate the function $\boldsymbol{f}$. A B-spline is a piecewise polynomial function of degree k and is defined over a domain $x \in [t_0, t_m]$, where m is integer. The points where $x = t_j$ for $j = 1 \ldots m$ are known as knots. A B-Spline of degree k is a linear combination of basis B-Splines, $B_{i,k}(x)$, of degree k and is given by:

$$f(x) = \sum_{i=1}^{m} \beta_i B_{i,k}(x), \qquad x \in [t_0, t_m]. \quad (4)$$

The βi are called control points and the integer m is the number of knots. Expressions for the polynomial pieces, $B_{i,k}(x)$, can be derived by means of a recursive formula following the definition of the initial polynomial:

$$B_{i,0}(x) = \begin{cases} 1 \, if \; t_i \le x < t_{i+1} & \forall \quad i = 0 \ldots (m-1) \\ 0 \;\; otherwise. \end{cases}$$

and

$$B_{i,k}(x) = \frac{x - t_i}{t_{i+k} - t_i} B_{i,k-1}(x) + \frac{t_{i+k+1} - x}{t_{i+k+1} - t_{i+1}} B_{i+1,k-1}(x) \qquad \forall \quad i = 0 \ldots m-k-1.$$

In this case, B-Spline quantile regression will be described as follows:

$$y = \sum_{i=1}^{m} \beta_i B_{i,k}(x) + \varepsilon. \quad (5)$$

1.3. Parameter Estimation

The classical approach to estimate parameters $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_m)$ is to use the simplex methods or the interior point methods described in (Koenker, 2005). In the literature, other methods for estimating the $\boldsymbol{\beta}$ have been developed such as the Bayesian method of (Yu and Moyeed, 2001). In this work, a Bayesian framework is used to estimate $\boldsymbol{\beta}$ parameters for the B-Spline quantile regression model. The Bayesian approach provides the full distribution for estimators of the parameters based on the likelihood function and a prior distribution.

For the prior density of β, we consider a multivariate normal distribution (see Green and Silverman (1994), pp. 51-52, for a discussion about the use of multivariate normal density as prior in this context). Unfortunately, in the multivariate case, we do not have a large choice. In the literature, four multivariate parametric distributions are considered: Student, Normal, Gamma and Lognormal. Multivariate Lognormal and Multivariate Gamma distributions are defined for positive random vectors (R^m_+), while the normal and the student distributions are defined for random vectors with real support (R^m). Therefore, it seems natural to consider normal or student distributions. In the present study, we take the multivariate normal distribution. For more flexible distribution the copula function can provide a good alternative. Our prior probability density distribution for β in this study is therefore defined by means of the multivariate normal density:

$$\pi_{\boldsymbol{\mu},\boldsymbol{\Sigma}}(\boldsymbol{\beta}) = \frac{1}{(2\pi)^{m/2}\,|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(\boldsymbol{\beta}-\mu)'\Sigma^{-1}(\boldsymbol{\beta}-\mu)\right\} \quad (6)$$

where $\boldsymbol{\mu}$ is the mean of $\boldsymbol{\beta}$, Σ is the variance-covariance matrix of $\boldsymbol{\beta}$ and m is the number of parameters.

The final step in our Bayesian approach is to define the likelihood of (x_i, y_i). The proposed approach is in accordance with (Yu and Moyeed, 2001) and (Thompson et al., 2010). In these papers, the authors show that the minimization of the loss function is exactly equivalent to the maximization of a likelihood function formed by combining independently distributed asymmetric Laplace densities. Let us recall the properties of the asymmetric Laplace distribution. A random U is said to follow the asymmetric Laplace distribution if its probability density is given by:

$$L_p(u) = p(p-1)\exp\{-\rho_p(u)\}; \quad -\infty < u < +\infty \quad and \quad p \in]0,1[$$

Substituting u by $y - \sum_{i=1}^{m} \beta_i B_{ik}(x)$. The resulting likelihood takes the form:

$$L(\mathbf{y}|\boldsymbol{\beta}) = p^n(1-p)^n \exp\left\{-\sum_{j=1} \rho_p\left(y_j - \sum_{i=1} B_{ik}(x_j)\boldsymbol{\beta}_i\right)\right\} \quad (7)$$

where ρ_p is the standard quantile regression loss function defined in (2). For more information concerning the Laplace distribution in this case, please see Appendix A.

Combining $\pi_{\mu,\Sigma}(\boldsymbol{\beta})$ and $L\,(\mathbf{y}|\boldsymbol{\beta})$, we can write the posterior density function of $\boldsymbol{\beta}$ as:

$$\pi\,(\boldsymbol{\beta}|\mathbf{y}) \propto L\,(\mathbf{y}|\boldsymbol{\beta})\,\pi\,(\boldsymbol{\beta})\,. \tag{8}$$

We now simulate realizations of $\boldsymbol{\beta}$ for the posterior density using a Monte Carlo Markov Chain (MCMC) approach implemented through the Metropolis-Hastings (M-H) algorithm. Our inference is based on these posterior realizations. In particular, we use the posterior mean of $\boldsymbol{\beta}$ to produce our estimated quantile regression. Our algorithm can be summarized as follows:

M-H algorithm

Initialize $\left(\beta^{0} \sim \Phi\left(\beta\right)\right)$ (Φ is called the proposal distribution)

for iteration $i = 1, 2, \ldots.$ **do**

  Propose: $\beta^{*} \sim \Phi\left(\beta^{i}|\beta^{i-1}\right)$ (β^{*} is called a candidate)

  Acceptance probability $\quad \alpha\left(\beta^{*}|\beta^{i-1}\right) = \min\left\{1, \frac{Q(\beta^{i-1}|\beta^{*})\pi(\beta^{*})}{Q(\beta^{*}|\beta^{i-1})\pi(\beta^{i-1})}\right\}$

  $u \sim$ Uniform $(u; 0, 1)$

  if $u < \alpha$

    Accept the proposal $\beta^{i} \leftarrow \beta^{*}$

  else

    Reject the proposal $\beta^{i} \leftarrow \beta^{i-1}$

  end if

end for

The first step is to initialize the sample value for parameter vector $\boldsymbol{\beta}$ (this value is often sampled from the parameter's prior distribution). The main loop of the M-H algorithm consists of three components: (1) Generate a proposal sample $\boldsymbol{\beta^*}$ from the proposal distribution $\Phi\,(\boldsymbol{\beta^i}|\boldsymbol{\beta^{i-1}})$; (2) Compute the acceptance probability via the acceptance function based upon the proposal distribution and the full joint density $\pi(.)$; (3) Accept the candidate sample with probability α, the acceptance probability, or reject it with probability $1 - \alpha$.

Proposal Distribution: The M-H algorithm starts with simulating a ``candidate`` sample β from the proposal distribution Φ (.). Note that samples from the proposal distribution are not accepted automatically as posterior samples. These candidate samples are accepted probabilistically based on the acceptance probability α. In the literature, the proposal distribution is often the same as the prior distribution (e.g., Gelman et al., 1995; Gilks et al., 1996). In our study, we choose the multivariate normal distribution as a proposal distribution function.

1.4 Criteria to choose the best model

The B-spline functions depend on two parameters: number of knots (*m*) and degree (*k*). When (*m, k*) = (1, 1), the B-spline quantile regression model is exactly the linear quantile regression model. And when (*m, k*) = (1, 2), the B-Spline quantile regression model is exactly the quadratic quantile regression model. To select the "Best" model, two performance methods are used: (i) the "coefficient of determination" based on the quantile and (ii) the Bayesian information criterion (BIC) for quantile regression.

1.4.1 Coefficient of determination for quantiles

The coefficient of determination for quantiles was proposed in the first time by (Donner et al., 2012) and developed by (Noh et al., 2012). This coefficient aims to quantify the goodness of fit measures in the framework of quantile regression. This coefficient aims to compare the mean of residual error between two models. (Noh et al., 2012) defined this performance metric as follows:

$$R(p) = 1 - \frac{\sum_{i=1}^{n} \rho_p \left(y_i - \hat{f}(x_i) \right)}{\sum_{i=1}^{n} \rho_p \left(y_i - \hat{f}_c(x_i) \right)} \quad (9)$$

where, for fixed $0 < p < 1$. $\boldsymbol{f}$ is the nonparametric conditional *pth* quantile of *Y* given *X* and $\widehat{f}$ is the conditional *pth* quantile of *Y* given *X* for the comparative parametric model (e.g. linear or quadratic). *R* (*p*) can be positive, negative or null. When it takes positive values, it means that the nonparametric model is better than the comparative model and when it takes negative values, it means that the comparative model is better. Otherwise the two models are equivalent.

1.4.2 Bayesian information criterion for quantiles

The BIC is a criterion for model selection among a finite set of models; the model with the lowest BIC is preferred. It is based, in part, on the likelihood function. When fitting models, it is possible to increase the likelihood by adding parameters, but leads to overfitting estimation. To overcome this problem, the BIC introduces a penalty term depending on the number of parameters in the model. The BIC was developed, in the first time, by (Schwarz, 1978). The BIC criterion can be written as follows:

$$BIC = -2\log\left(\hat{L}(\mathbf{y} \mid \boldsymbol{\beta})\right) + p\log(n) \quad (10)$$

where $L(\boldsymbol{y}|\boldsymbol{\beta})$ is the estimator of the likelihood function and it is the same function given in 7, *p* is the number parameters and *n* denotes sample size. In quantile regression, several references are studied for this criterion (Nishii, 1984; Wu and Zen, 1999; Zhang et al., 2010). For a review of literature on the usage of BIC, see (Lee et al., 2014) where they give the BIC form of the linear and nonlinear quantile regression. In hydroclimatology works, we can see the application of this criterion in (Donner et al., 2012) as well as (Koenker and Schorfheide, 1994).

2. Data

This study is based on two applications. These applications respectively focus on (a) annual maximum and (b) minimum streamflow records in Ontario, using climate indices to model non-stationarity. For each application, we selected 5 stations. The data from each station were checked, in particular to look for obvious shifts, outlier values and missing data. The stations that were selected have more than 30 years of complete daily data with identified non-stationarity and they are correlated with at least one climate index.

For this study, we tested the dependence between the variables of interest and the following covariates: Pacific Decadal Oscillation (PDO), Atlantic Multi-decadal Oscillation (AMO), North Atlantic Oscillation (NAO) and El Nino Southern Oscillation (ENSO) indices (see Appendix B for definitions). Ultimately, only the covariates that have significant dependence with the variables of interest were kept, which are AMO and PDO indices. AMO and PDO were found to have, respectively, significant dependence with maximum and minimum streamflow time series. The dependence in this case is estimated by using the Kendall rank correlation coefficient (Kendall, 1948). Streamflow data come from the HYDAT database of Environment Canada ftp://arccf10.tor.ec.gc.ca/wsc/software/HYDAT/. Climate indices data come from NOAA (National oceanic and atmospheric administration Earth System Research Laboratory) website http://www.esrl.noaa.gov. Figure 2 illustrates the geographic location of all stations selected for each application. Table 1 gives a summary of the description of the selected stations with long precipitation records, the results of Kendall's tau rank correlation coefficients and the stationary quantile estimation for 2 and 10 return periods, for applications (a) and (b). Figures 3 and 4 show the variations of maximum and minimum annual streamflows at each station. Figures 5 and 6 show, respectively, the variation of maximum and minimum streamflows against AMO and PDO oscillation for each station.

3. RESULTS

For model development, the following functions are first fitted:

- Maximum annual streamflow$_i = f_{1i}(AMO) + \varepsilon$; $i = 1, \ldots, 5$

- Minimum annual streamflow$_i = f_{2i}(PDO) + \varepsilon$; $i = 1, .., 5$

f_1 and f_2 are estimated by the B-spline approach. To select the degree k and the number of knots m, we calculate the determination coefficient and BIC criterion for several values of the couple (k, m). For each application in each station, we choose the couple (k, m) that maximizes (resp. minimizes) the determination coefficient (resp. the BIC criterion). The following paragraph shows the results of the determination coefficient and the best model of each station. The next one describes quantile estimation.

For application (a), these criteria are calculated for quantiles 0.5 and 0.9, which correspond to return periods of 2 years and 10 years and for application (b), these coefficients are calculated for quantiles 0.1 and 0.5, which correspond to return periods of 10 years and 2 years. Tables 2 and 3 show, the determination coefficient, for different values of the couple (m, k), for application (a) and application (b) respectively. Tables 4 and 5 show, the BIC criterion, for different value of (m, k), for application (a) and application (b) respectively. From Table 2, it can be noticed that the best model chosen using the determination coefficient is the model with 3 knots and 3 degrees for all stations. From Tables 3, we can notice that the best model is model with 2 knots and 2 degrees for all stations except for station 02HC029, where the model with 3 knots and 3 degrees is selected. Different results are provided using BIC criterion. In fact, for all the studied cases and for both application, we can conclude that the models selected using the BIC criterion have less degrees and number of knots than those chosen using the coefficient of determination. For both applications, the values of the degrees and knots are less or equal to (2,3). This result was expected, as the BIC criterion penalizes for the number of parameters to estimate and favors parsimony. In contrast, the coefficient of determination only considers estimation errors. For this reason, quantiles estimation will be done by using the models selected by the BIC criterion.

Figures 7 and 8 show the estimated 2 and 10-year return period maximum and minimum streamflow quantiles, respectively as function of the covariates AMO and PDO. It can be seen that generally, quantiles take different values as the covariate values change. Non-stationary quantiles can take much larger values than stationary quantiles. For example, the results for station 02AC001 show that the stationary median is equal to 48 (m^3/s). However, the non-stationary median can reach 140 (m^3/s). This difference between stationary and nonstationary quantiles becomes very large for high quantile levels. In fact, for station 02AC001, the 0.9 stationary quantile is equal to 96.9 (m^3/s), but the 0.9 nonstationary quantile can reach 250 (m^3/s). For stationary quantile, please see Table 1. We can see similar results for all stations in both applications which confirm the importance of considering the nonstationary quantiles for better water resource management practices.

4. DISCUSSION AND CONCLUSION

The two last decades have witnessed the development of a large number of statistical modelling approaches for extreme value variables in the presence of non-stationarity or dependence on covariates. In this study, we present the B-spline quantile model, a nonlinear and nonparametric approach which model nonlinear conditional quantiles or quantile with covariates and offers great flexibility and smoothing for quantile estimation. Estimating the parameters of the proposed model is carried out using the Bayesian approach. It combines observed and prior information, estimates the entire posterior distribution of the parameters and quantiles and allows giving better or similar estimation results than the frequentist approach (similar results, in the case of non-informative prior and better if we have a prior information concerning the parameters).

Despite the advantages of the nonparametric model, this kind of model is often criticized in the literature for the possibility of over-parameterization, leading to a parsimony problem. Some studies suggested to use classical models such as a linear or quadratic quantile models to avoid this problem. In this study, we propose to adapt a performance criterion for quantiles that allows the comparison of B-Spline approach with classical models, by using a coefficient of determination for quantiles and BIC criterion for quantiles. However, only the BIC criterion allows to select the best performing model with fewer parameters.

In this work, two case studies are proposed to show the model performance. The first one is focused on maxi- mum annual streamflow quantiles for five stations in Ontario using the AMO oscillation index, and the second one estimates the quantiles for minimum annual streamflow at five other stations in Ontario, using the PDO oscillation index. Our results show that:

1. Different covariates can influence different metrics of one variable of interest in the same study area. In this case, we have the PDO index that influences the minimum annual streamflow while it does not affect the annual maximum streamflow and conversely for the AMO index. Looking at the time series of PDO and AMO oscillations, we noticed that the relationship between the AMO and PDO are negative between the period of 1942-1965 and between 1968-1998 (Rowan and Daniel, 2005). That can explain the influence of AMO for maximum annual discharge and PDO for minimum annual flow values.
2. Moreover, it can be noticed that, although the shape of the relationship between AMO and floods is similar for all five studied stations, it is not the case for low flows and PDO. Two of the five stations, located further north show a negative relationship between low flows and PDO, while for the three stations located in southern Ontario, this relationship is positive. Looking at the daily datasets of flows in these stations, we noticed that minimum discharge values in the northern stations are most often observed late in the winter, generally between March and April. However, the minimum flow values at the other stations are often observed during the summer or autumn period, especially between July and November. This can explain the difference in signs (+ or -) of Kendall's tau.

3. The quantile regression model was used in a framework that includes nonparametric smoothing B-spline functions. These functions can capture linear and nonlinear dependence between covariates (e.g climate indices) and the variables of interest (annual minimum and maximum streamflows). For both case studies, conditional quantiles are calculated for two return periods T = 2 years and 10 years. Quantiles for higher return periods (e.g. T = 50, T

= 100 years) could be calculated if we disposed of longer data sets. The proposed model allows to modulate conditional quantile estimation as a function of low frequency atmospheric patterns and in some cases, this can lead to quantile estimations that are much higher than those obtained in a stationary framework. For nonstationary quantiles, several values are possible for a given return period, depending on the value of the covariate. Also, we see that nonlinear quantile values are superior to linear quantile values and to stationary quantile values for all stations.

4. The proposed model shows several advantages and some drawbacks. Indeed, according to the results described above, we can easily conclude that it is a relatively complex model for describing the linear and nonlinear quantiles in the presence of covariates. However, it is a flexible model that allows to reach more extreme values than classical models like linear and quadratic quantile regression models. However, this model also has some disadvantages. Indeed, the optimal number of knots and degrees of smoothing for the B-spline functions are always based on the calculation of a specific criterion (coefficient of determination and BIC criterion in this case), which can take much computing time and some programming skill.

5. In this work, a Bayesian framework is used to estimate $\boldsymbol{\beta}$ parameters for the B-Spline quantile regression model. The Bayesian approach provides the full distribution for estimators of the parameters. Posterior distributions are calculated for each β, in each station, and each probability level. In the paper, we excluded the posterior distribution to reduce the size of paper. We give an example of the MCMC results for one station 04JF001 and for one quantile level = 0.5. Please see figure 9.

6. Two criteria are used to choose the best model with less parsimony. The first criterion is the determination coefficient of quantiles and the second one is the BIC criterion. Both criteria are applied for several combination of number of knots and degree of smoothing. These criteria gave different results. In the fact, the best models selected by the BIC criterion contained fewer coefficients than the best model selected by the determination coefficient

for quantiles. The results of the BIC show the advantage of using non- linear functions in this context. Indeed, error estimation decreases with the use of degree greater than 2. This indicates the usefulness of the proposed model.

7. All the BIC results were confirmed by the MCMC confidence interval. In fact, we can notice from Figure 9 that if all the β are different than 0, then the MCMC confidence interval does not contain 0. And we can notice from the results of BIC criterion that, the best selected model for 04JF001 station are the model with degree equal to 2 and the number of knots equal to 1. The same results are shown for all the other stations.

8. In this study, we propose another way to estimate conditional quantile (or nonstationnary quantile). This method is based on the estimation of coefficient of regression model which is easier than the classical model based on the GEV (GPD) with covariates. The results of this work give the return streamflow quantile for each value of covariate (here, we have a range of values of AMO and PDO). In the future and knowing the values of AMO and PDO, we can easily predict the return streamflow quantile. However, this type of model allows the description of the impact of covariates on the variable of interest and cannot be used for predictions outside the interval values of covariate. Indeed, outside this interval, we can never guess how the variable of interest varies depending on the covariate.

In this study, we used a single covariate to explain the temporal variations of the maximum and minimum flows. This unique covariate partially explained these variations. The introduction of more covariates may allow for better quantile estimation. Hence, future efforts may deal with the introduction of additional covariates. In the context of climate change studies, additional covariates could include GCM/RCM outputs or NCEP / NCAR reanalysis predictors to better explain the temporal variation of the flow in relation to climate.

Tables

Table 1: Description of the selected stations with length of discharge records for application (a) and (b). The five first stations are station chosen for application (a) and the five last stations are the stations chosen for application (b). $Q_{T=2}$ and $Q_{T=10}$ correspond to the stationary quantiles, respectively, for 2 and 10 years return period estimated by using the inverse of cumulative distribution function.

Station	Length of data	Latitude	Longitude	Kendall's tau	$Q_{T=2}$	$Q_{T=10}$
02AC001	1971-2010	48.821	-88.534	AMO (-0.25)	48.9	96.6
02HB012	1965-2010	43.301	-79.869	AMO (-0.24)	11.5	18.1
02HD012	1975-2010	43.991	-78.3282	AMO (-0.26)	28.4	52.5
02LA007	1969-2010	45.249	-75.7906	AMO (-0.27)	79.2	133.0
04LM001	1972-2010	50.585	-82.091	AMO (-0.29)	1880.0	2796.0
02FB007	1959-2010	44.522	-80.930	PDO (0.4)	0.4	0.6
02HC009	1959-2010	43.790	-79.584	PDO (0.39)	0.1	0.2
02HC029	1964-1996	43.757	-79.345	PDO (0.42)	0.4	0.5
04FA001	1970-2010	51.823	-89.602	PDO (-0.35)	15.3	20.8
04JF001	1980-2010	50.658	-86.532	PDO (-0.24)	13.3	16.5

Table 2: Coefficient of Determination for B-spline quantile regression model vs linear quantile model (*l*) and quadratic quantile model (*q*) for application (a).

(Degree, Knots)	02AC001		02HB012		02HD012		02LA007		04LM001	
	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$
$(1,1)^l$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(1,1)^q$	0.04	-0.41	0.28	0.05	0.10	-0.13	-0.22	0.03	-0.37	-0.17
$(1,2)^l$	-0.09	0.17	-0.63	-0.37	0.61	-0.56	0.09	-0.02	0.12	0.07
$(1,2)^q$	-0.09	0.28	-0.63	-0.37	0.61	-0.56	0.26	-0.02	0.34	0.07
$(1,3)^l$	-0.09	-0.06	-0.63	-0.13	0.60	0.02	0.12	-0.02	0.04	0.28
$(1,3)^q$	-0.09	0.27	-0.63	-0.13	0.60	0.02	0.26	-0.02	0.38	0.28
$(2,1)^l$	-0.04	0.29	-0.39	-0.06	-0.11	0.12	0.18	-0.03	0.27	0.15
$(2,1)^q$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(2,2)^l$	0.20	0.16	-0.46	-0.01	-0.05	-0.31	0.69	0.01	0.38	0.20
$(2,2)^q$	0.20	0.42	-0.46	-0.01	-0.05	-0.31	0.20	0.01	-0.22	0.20
$(2,3)^l$	0.26	0.11	0.10	-0.25	0.60	0.09	0.32	0.06	0.37	0.35
$(2,3)^q$	0.26	0.43	0.10	0.13	0.60	0.19	0.33	0.06	0.40	0.35
$(3,1)^l$	0.09	0.04	-0.62	-0.15	0.59	0.01	0.10	-0.07	0.04	0.28
$(3,1)^q$	0.09	0.33	-0.62	-0.15	0.59	0.01	0.23	-0.07	0.38	0.28
$(3,2)^l$	0.29	0.12	-0.50	0.13	0.46	0.01	0.02	0.12	0.22	0.28
$(3,2)^q$	0.29	0.50	-0.50	0.13	0.46	0.01	0.20	0.12	0.43	0.28
$(3,3)^l$	0.34	0.31	0.17	0.13	0.95	0.20	0.48	0.20	0.38	0.45
$(3,3)^q$	0.34	0.54	0.17	0.13	0.95	0.20	0.50	0.20	0.49	0.45
$(3,4)^l$	0.15	0.13	0.02	0.03	0.60	0.17	0.30	0.02	1.29	0.29
$(3,4)^q$	0.14	0.39	0.00	0.04	0.33	0.13	0.28	0.04	0.27	0.22

Table 3: Coefficient of Determination for B-spline quantile regression model vs linear quantile model (*l*) and quadratic quantile model (*q*) for application (b).

(Degree, Knots)	02FB007		02HC009		02HC029		04FA001		04JF001	
	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$
$(1,1)^l$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$(1,1)^q$	0.17	0.00	-0.11	-0.07	0.29	-0.47	0.15	0.16	-0.32	0.08
$(1,2)^l$	-0.74	0.06	0.01	0.11	-0.05	-0.08	0.87	-0.11	0.33	0.12
$(1,2)^q$	-0.45	0.06	-0.09	0.05	0.25	-0.59	0.89	0.06	0.11	0.19
$(1,3)^l$	-1.92	0.06	-0.15	0.13	0.63	0.12	0.03	1.12	0.98	1.12
$(1,3)^q$	-1.43	0.06	-0.27	0.07	0.74	-0.30	0.18	1.10	0.97	1.11
$(2,1)^l$	-0.20	0.00	0.10	0.06	-0.41	0.32	-0.18	-0.18	0.24	-0.09
$(2,2)^l$	0.30	0.08	0.27	0.14	0.60	0.12	0.69	0.29	0.79	0.16
$(2,2)^q$	0.42	0.08	0.19	0.08	0.71	0.30	0.74	0.40	0.72	0.23
$(2,3)^l$	0.69	0.14	-0.16	0.01	0.19	0.10	0.62	0.11	0.39	0.11
$(2,3)^q$	0.75	0.14	-0.28	-0.05	0.43	-0.32	0.68	0.25	0.19	0.18
$(3,1)^l$	-0.61	-0.01	0.07	0.07	-0.36	0.32	0.54	-0.26	-0.51	0.02
$(3,1)^q$	-0.34	-0.01	-0.03	0.01	0.04	-0.01	0.61	-0.06	-1.00	0.10
$(3,2)^l$	-1.45	0.17	0.14	0.12	0.54	0.11	0.34	0.11	-0.93	0.11
$(3,3)^l$	0.48	0.20	-0.32	0.11	0.18	0.08	0.32	0.09	0.24	0.08
$(3,3)^q$	0.56	0.20	-0.46	0.05	0.42	-0.35	0.42	0.23	0.00	0.15

Table 4: BIC results for different couple of degree and knots in the B-Spline quantile model for the application (a). The results in the table are $*100$

(Degree, Knots)	02AC001		02HB012		02HD012		02LA007		04LM001	
	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$
(1,1)	252.60	223.40	789.60	346.90	195.36	158.74	273.82	164.18	554.46	488.73
(1,2)	253.60	223.50	793.10	358.10	207.53	157.83	279.10	171.40	559.12	483.90
(1,3)	253.70	223.90	914.20	356.30	209.74	157.91	277.40	169.10	557.48	479.52
(2,1)	252.20	223.20	755.60	353.30	197.70	156.29	273.90	167.06	550.63	473.10
(2,2)	240.60	210.20	853.00	277.20	204.69	162.77	280.41	175.13	550.91	476.80
(2,3)	251.60	222.11	733.20	275.80	194.96	148.89	272.20	163.45	555.00	479.84
(3,1)	252.60	223.11	876.50	310.70	205.72	161.44	280.82	173.58	558.98	483.31
(3,2)	258.56	226.85	917.10	317.70	209.62	163.72	285.01	177.77	562.83	487.26
(3,3)	262.68	226.75	957.00	356.10	211.14	161.65	284.19	182.12	565.67	481.60

Table 5: BIC results for different couple of degree and knots in the B-Spline quantile model for the application (b). The results in the table are $*100$

(Degree, Knots)	02FB007		02HC009		02HC029		04FA001		04JF001	
	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$
(1,1)	-280.53	-386.01	-359.32	-464.85	-326.43	-397.23	-241.20	-541.80	-184.10	-478.50
(1,2)	-268.86	-366.45	-359.06	-452.84	-328.02	-405.39	-502.65	-632.83	-446.78	-555.52
(1,3)	-271.28	-373.30	-349.47	-443.26	-313.80	-383.12	-263.94	-549.12	-278.00	-543.01
(2,1)	-277.18	-377.78	-370.17	-466.85	-323.51	-394.97	-518.20	-652.40	-460.60	-572.70
(2,2)	-291.25	-396.22	-360.28	-456.97	-338.17	-417.93	-272.10	-566.10	-286.60	-559.80
(2,3)	-279.67	-384.85	-347.09	-462.71	-314.13	-387.95	-227.40	-579.10	-337.10	-547.20
(3,1)	-277.18	-374.36	-347.10	-462.91	-310.05	-382.84	-424.10	-516.40	-395.00	-520.90
(3,2)	-269.70	-374.62	-358.66	-462.33	-311.31	-378.48	-423.70	-621.70	-366.60	-489.30
(3,3)	-269.87	-378.03	-354.92	-466.34	-306.19	-374.87	-449.30	-615.30	-365.70	-449.40

Figures

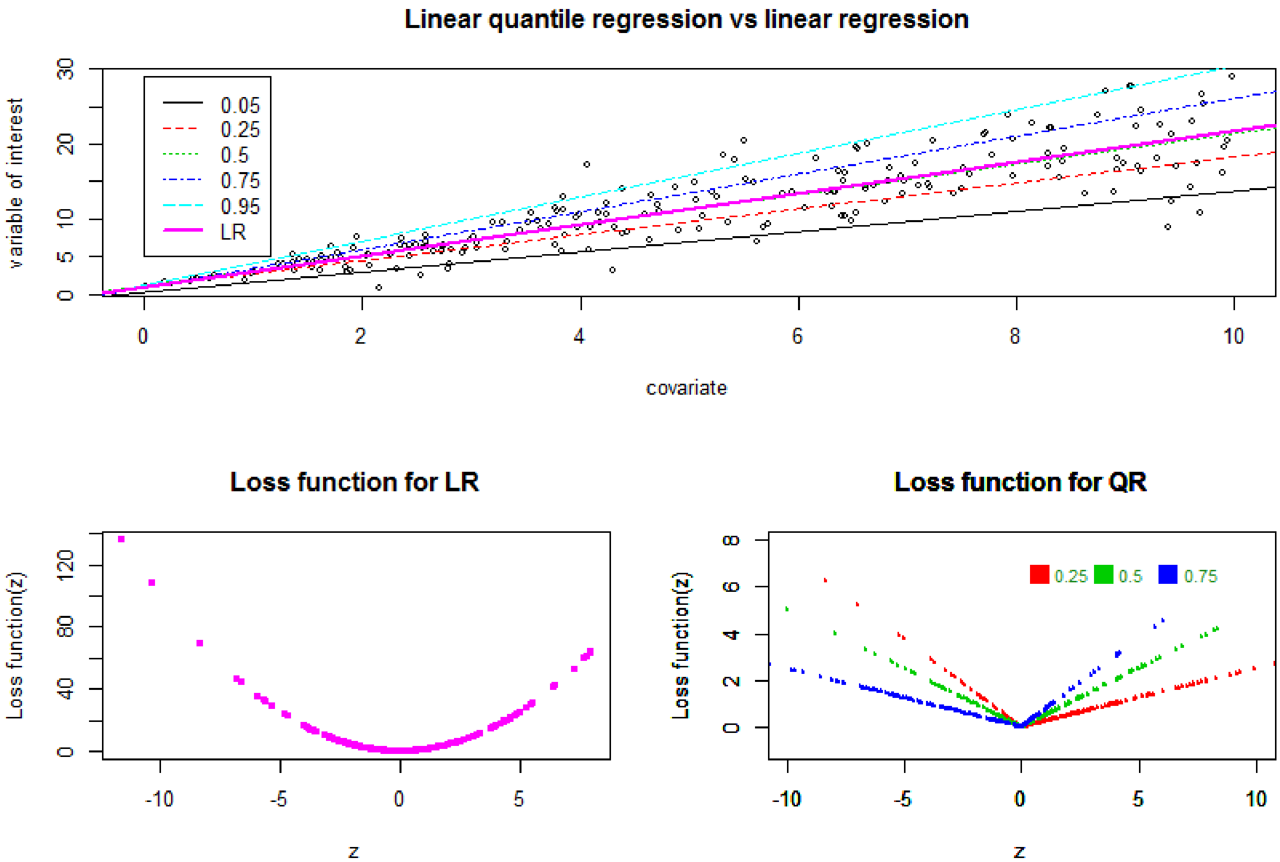

Figure 1: Example of linear regression and linear quantile regression with their Loss function.

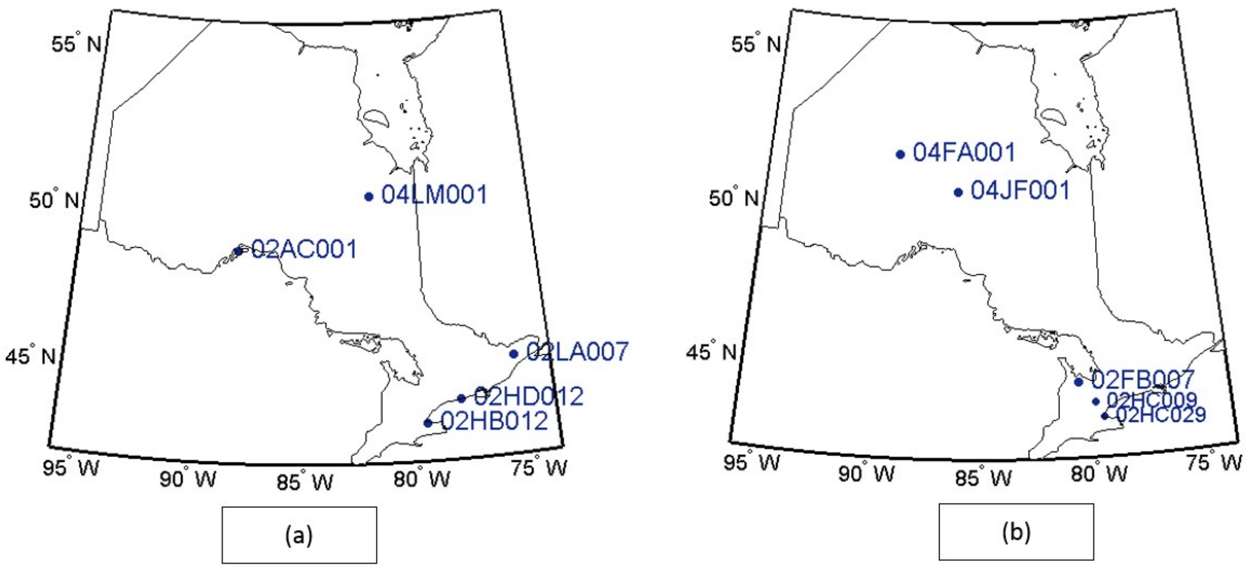

Figure 2: Geographic location of all stations for application (a) and application (b)

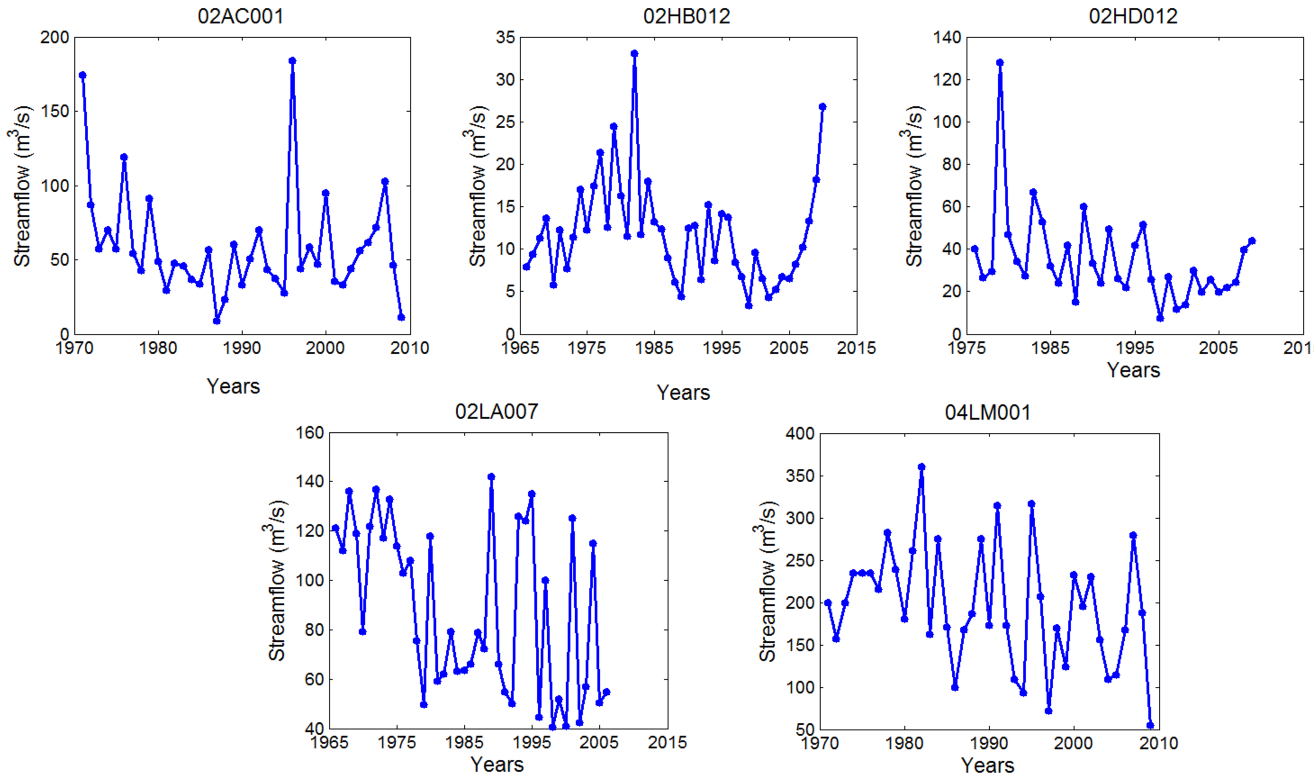

Figure 3: Variation of maximum annual streamflows for each station- Application (a)

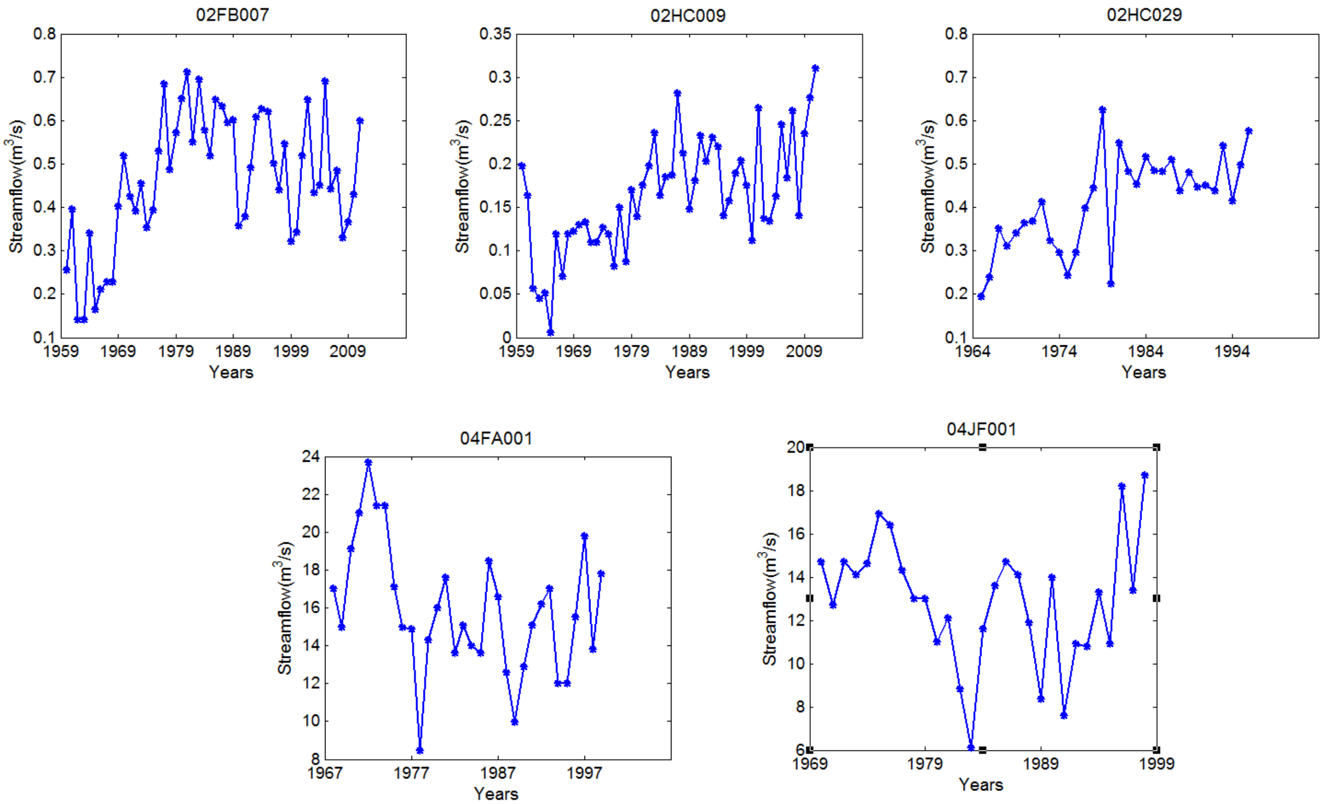

Figure 4: Variation of minimum annual streamflows for each station- Application (b)

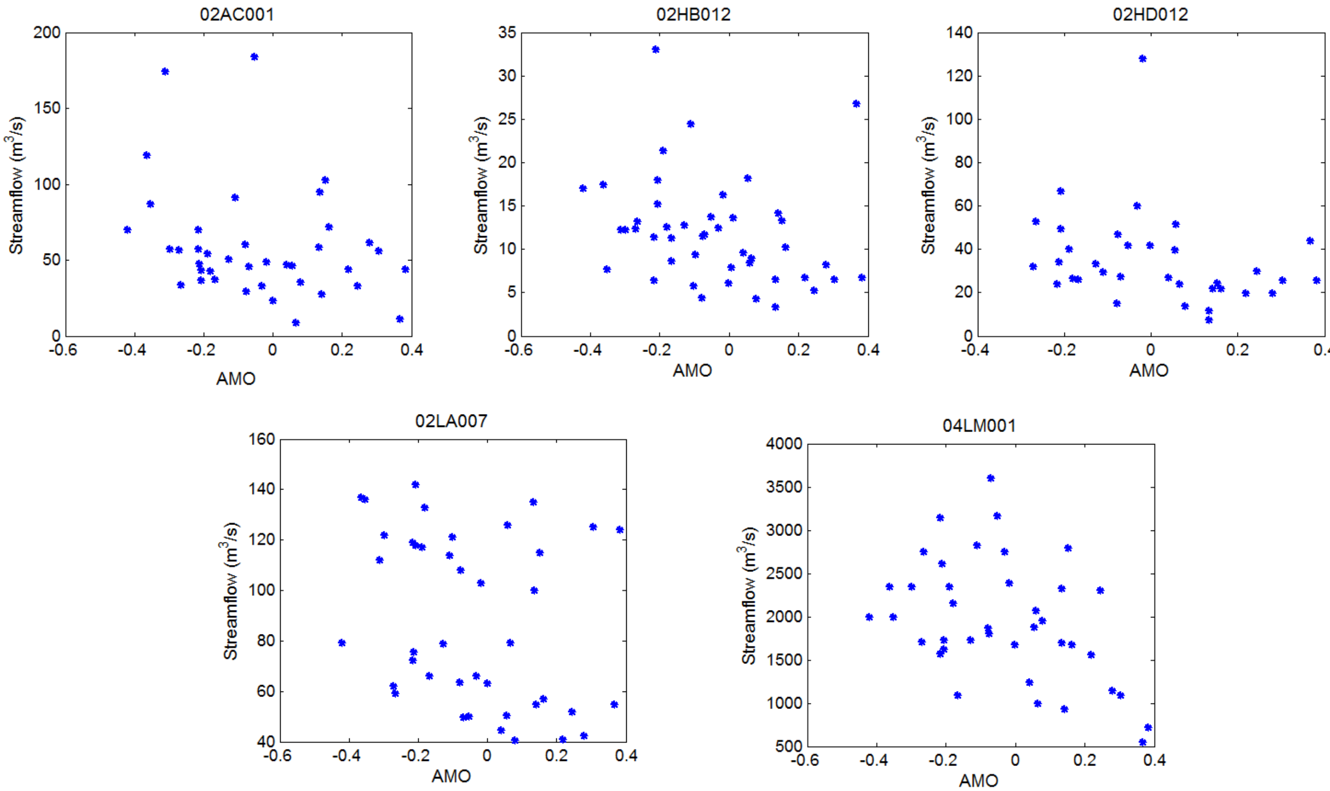

Figure 5: Annual maximum streamflows vs AMO oscillation

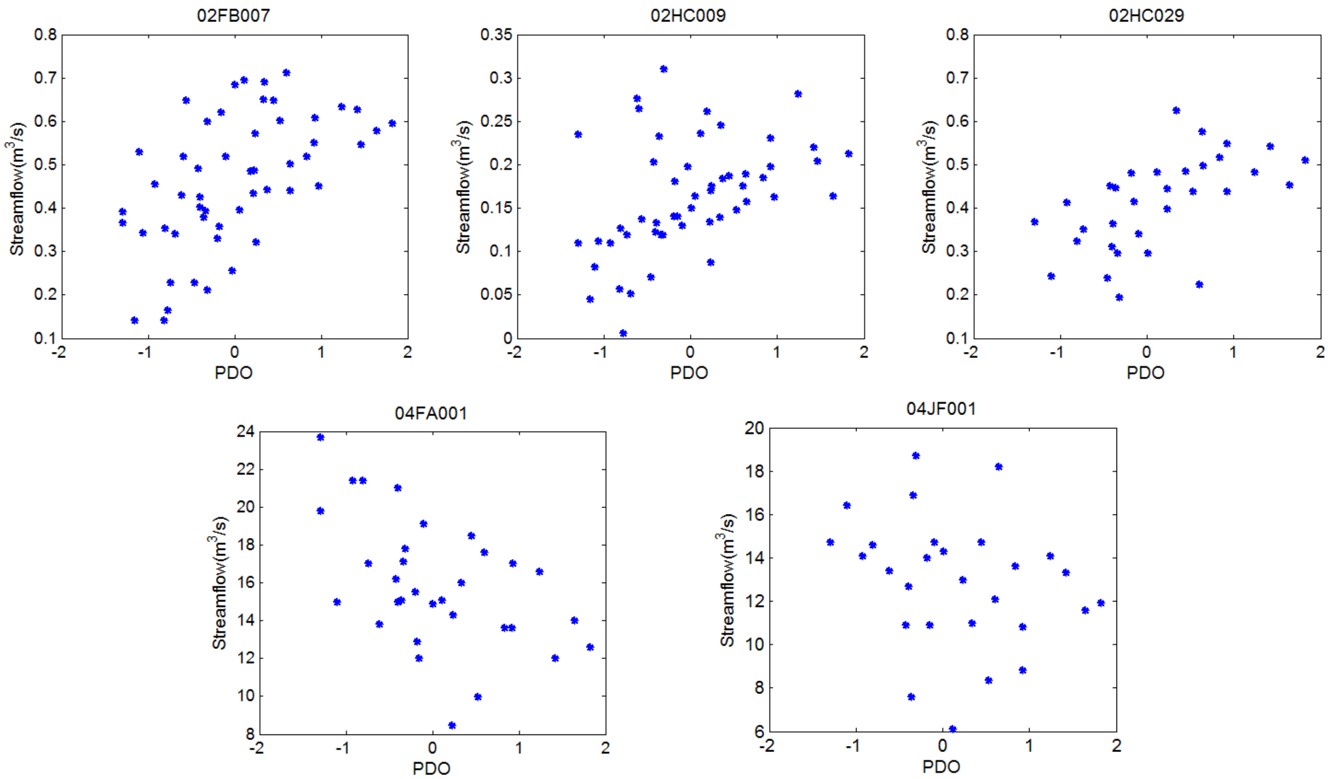

Figure 6: Annual minimum streamflows vs PDO oscillation

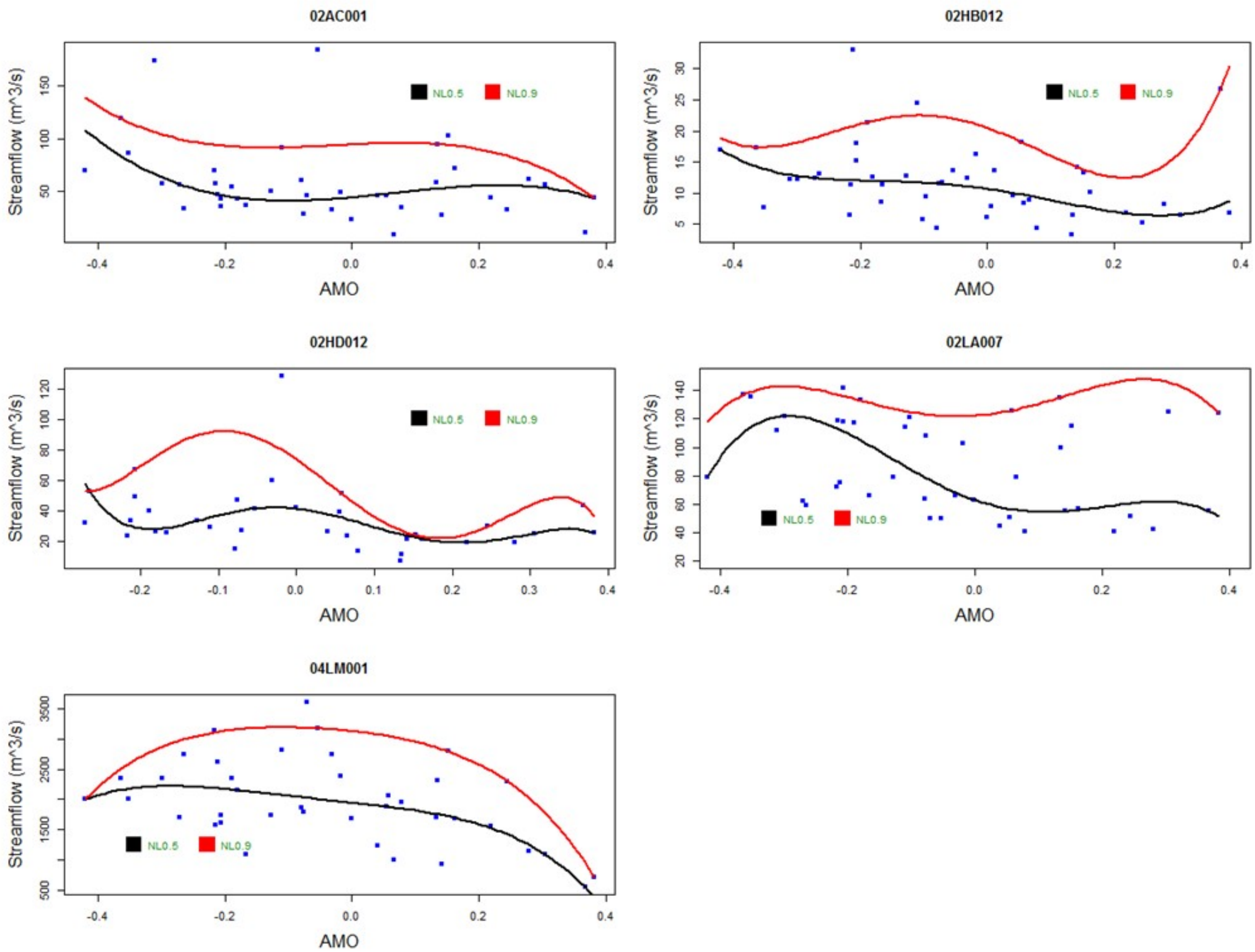

Figure 7: 0.5 and 0.9 quantile results estimated by using the B-spline quantile regression model -Application (a)

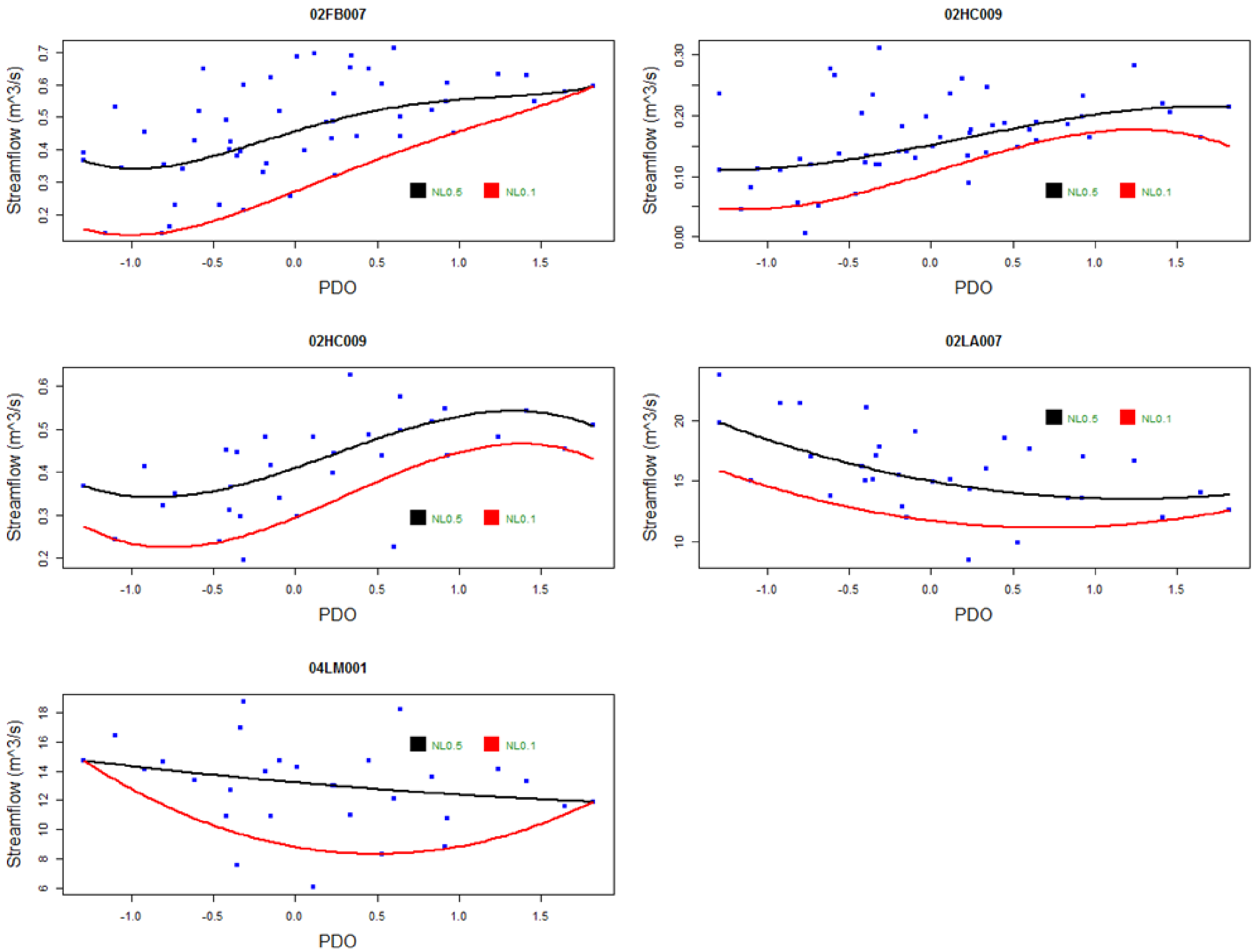

Figure 8: 0.1 and 0.5 quantile results estimated by using the B-spline quantile regression model-Application (b)

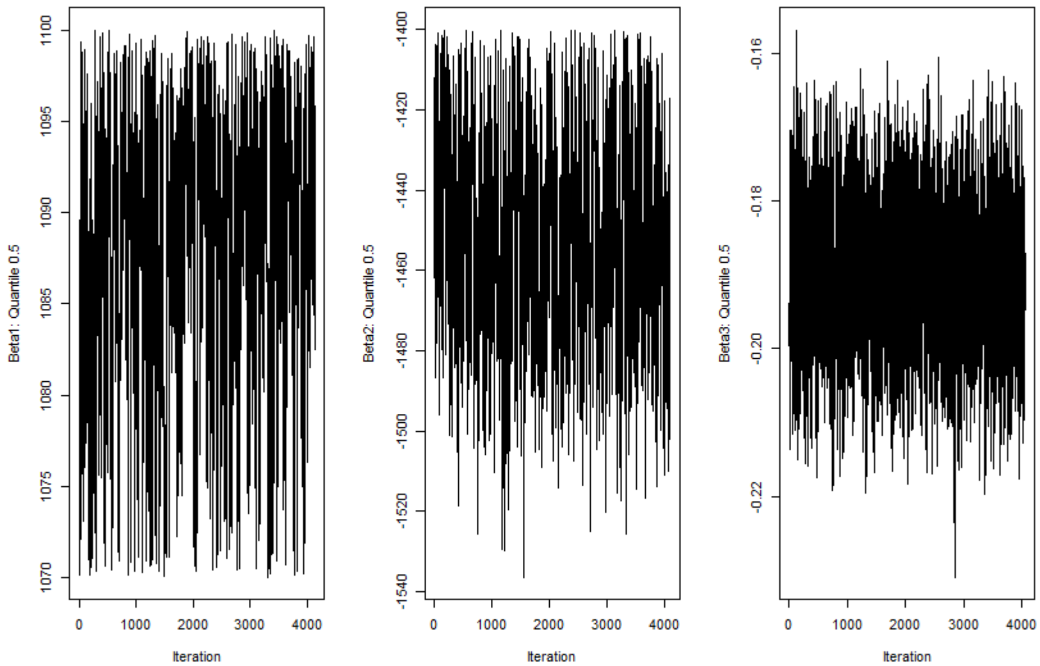

Figure 9: MCMC results for 04LM001 station for $p = 0.5$

APPENDIX A.

The *p*th linear regression quantile (0 < p < 1) is defined as any solution, $\alpha(p), \widehat{\alpha(p)}$ to the quantile regression minimization problem

$$\arg\min_{\alpha,\alpha_0 \in \mathbb{R}} \sum_{i=1}^{n} \rho_p\left(y_i - x_i'\alpha - \alpha_0\right).$$

where ρ_p (z) is a loss function defined as:

$$\rho_p(z) = \frac{|z| + (2p-1)z}{2}$$

Minimization of the loss function ρp (z) is equivalent to the maximization of a likelihood function formed by combining independently distributed asymmetric Laplace densities

$$f(y, x, \alpha, \alpha_0, p) = p(p-1)\exp\{-\rho_p(y - x'\alpha - \alpha_0)\}$$

$$\arg\min_{\alpha,\alpha_0 \in \mathbb{R}} \sum_{i=1}^{n} \rho_p(y_i - x_i'\alpha - \alpha_0) \Leftrightarrow p(p-1)\arg\max_{\alpha,\alpha_0 \in \mathbb{R}} \exp\{-\rho_p(y - x'\alpha - \alpha_0)\}$$

This equivalence is due simply to the fact that the exponential function is strictly increasing.

APPENDIX B.

.1. North Atlantic Oscillation (NAO)

NAO is an irregular fluctuation of atmospheric pressure over the North Atlantic Ocean that has a strong effect on winter weather in Europe, northeastern North America, North Africa, and northern Asia (Hurrell and Van Loon, 1997).

.2. El Nino Southern Oscillation (ENSO)

ENSO is a naturally occurring phenomenon that involves fluctuating ocean temperatures in the equatorial Pacific. For North America and much of the globe, the phenomenon is known as a dominant force causing variations in regional climate patterns (Bjerknes, 1969).

.3. Pacific Decadal Oscillation (PDO)

PDO is a pattern of Pacific climate variability similar to ENSO in character, but which varies over a much longer time scale. The PDO can remain in the same phase for 20 to 30 years, while ENSO cycles typically only last 6 to 18 months (Nathan and Hare, 2002).

.4. Atlantic Multi-decadal Oscillation (AMO)

AMO is a fluctuation in the sea surface temperature in the North Atlantic Ocean. It seems to occur with a period of roughly 70 years (Teegavarapu et al., 2013).

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