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ADVANCED CONTROL AND OPTIMIZATION TECHNIQUES FOR CONNECTED AND AUTONOMOUS VEHICLES IN MULTI-LANE FREEWAYS

Par

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Résumé

Grâce au développement rapide des technologies de communication avancées telles que les technologies de communication sans fil 5G et inter-véhicules (IVC), les véhicules connectés et autonomes (CAV) peuvent être conduits de manière coopérative, améliorant ainsi considérablement la sécurité routière, l'efficacité du trafic et la durabilité environnementale. Bien que des recherches substantielles aient été menées pour étudier l'efficacité des CAV dans les systèmes de transport, peu de tentatives ont été faites pour explorer et améliorer le fonctionnement des CAV sur les autoroutes à plusieurs voies, en particulier la dynamique des CAV aux goulots d'étranglement et aux intersections de baisse de voie.

L'objectif général de ce doctorat consiste à développer de nouvelles méthodes coopératives pour augmenter le débit sur une autoroute à plusieurs voies. De notre recherche, deux contributions majeures à la recherche sont présentées dans deux chapitres principaux correspondants de cette thèse.

Tout d'abord, nous étudions une méthode de distribution spatiale coopérative (CSDM) qui peut non seulement augmenter le débit du goulot d'étranglement de chute de voie, mais également permettre une distribution égale des CAV dans la voie abandonnée vers les autres voies. Cette méthode est détaillée au chapitre 4. Plus spécifiquement, nous proposons un nouveau cadre dans lequel la région d'intérêt sur une autoroute est divisée en trois segments : i) le segment de peloton, ii) le segment d'accélération et iii) le segment de fusion. Dans le premier segment, les CAV circulent ensemble en peloton pour garantir leur sécurité ; ils accéléreront alors pour atteindre la vitesse maximale ainsi que la position déterminée dans le deuxième segment. Enfin, ces CAV changent de voie dans le dernier segment et traversent le goulot d'étranglement avec une vitesse maximale et un écart minimum.

Deuxièmement, nous développons un cadre de contrôleur à deux niveaux pour optimiser le débit des intersections. Les résultats de recherche de cette étude sont abordés au chapitre 5. Dans la méthode que nous proposons, le niveau supérieur (c'est-à-dire le contrôleur d'intersection) est utilisé pour optimiser l'utilisation des voies de chaque approche et les positions des CAV. En revanche, le niveau inférieur (c'est-à-dire les contrôleurs du véhicule) reçoit des informations du niveau supérieur pour contrôler les CAV afin d'atteindre la vitesse maximale. Plus précisément, au niveau supérieur, nous appliquons une nouvelle méthode de distribution spatiale (SDM) pour les CAV afin de maximiser le débit de l'intersection à plusieurs voies où les timings des signaux sont prédéfinis. Après cela, chaque CAV autorisé à traverser l'intersection déterminera sa propre trajectoire et roulera à l'heure prévue sans collision.

Pour chaque cadre proposé, des résultats numériques sont détaillés pour obtenir de plus amples informations et évaluer ses performances réalisables. En particulier, nous montrons que les conceptions proposées surpassent les techniques de pointe dans des scénarios pratiques.

Mots-clés: Véhicules connectés et autonomatisés (CAVs); méthode de distribution d'espac (SDM); méthode coopérative de distribution de l'espace (CSDM); cadre du contrôleur à deux niveaux.

Abstract

Thanks to the rapid development of advanced communication technologies such as 5G wireless and inter-vehicle communication (IVC) technologies, connected and autonomous vehicles (CAVs) can drive cooperatively, thus significantly improving road safety, traffic efficiency, and environmental sustainability. While substantial research has been conducted to investigate the efficiency of CAVs in transportation systems, few attempts have been made to explore and improve the operations of CAVs in multi-lane freeways, especially the dynamics of CAVs at lane-drop bottlenecks and intersections.

The overall objective of this Ph.D. research is to develop novel cooperative methods to increase the throughput in a multi-lane freeway. From our research, two major research contributions are presented in two corresponding main chapters of this dissertation.

Firstly, we study a cooperative space distribution method (CSDM) that can not only increase the lane-drop bottleneck's throughput but also enable equal distribution of CAVs in the dropped lane to other lanes, which is presented in Chapter 4. More specifically, we propose a novel framework where the region of interest in a freeway is divided into three segments: i) platoon segment, ii) acceleration segment, and iii) merging segment. In the first segment, CAVs travel together in the platoon to guarantee their safety; they will then speed up to attain the maximum velocity and reach the determined position in the second segment. Finally, these CAVs change lanes in the last segment and pass through the bottleneck with maximum velocity and minimum gap.

Secondly, we develop a bi-level controller framework to optimize the intersection throughput. The research outcomes of this study are presented in Chapter 5. In our proposed method, the upper level (i.e., the intersection controller) is used to optimize the lane usage of each approach and the CAVs' positions. In contrast, the lower level (i.e., the vehicle controllers) receives information from the upper level to control the CAVs to reach the maximum speed. More specifically, at the upper level, we apply a novel Space Distribution Method (SDM) for the CAVs to maximize the throughput of the multi-lane intersection where signal timings are predefined. After that, each CAV allowed to cross the intersection will determine its own trajectory and travel at the scheduled time without a crash.

For each proposed framework, extensive numerical results are presented to gain further insights and to evaluate its achievable performance. In particular, we show that the proposed designs outperform the state-of-the-art techniques in practically relevant scenarios.

Keywords: Connected and autonomous vehicles (CAVs); Space Distribution Method (SDM); cooperative space distribution method (CSDM); bi-level controller framework.

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Important Abbreviations

Abbreviations	Description
CAVs	Connected and autonomous vehicles
C-ITS	Cooperative intelligent transport systems
CVM	Cooperative velocity method
CSs	Conflict sections
CSDM	Cooperative space distribution method
CSDM-C	Merging at center of CSDM
CSDM-H	Merging at head of CSDM
CSDM-T	Merging at tail of CSDM
HVs	Human-driven vehicles
HVM	Human-driven vehicles method
ITS	Intelligent transportation systems
IC	Intersection controller
IVC	Inter-vehicle communication
PAVs	The CAVs have the same direction in each approach
AAVs	The PAVs have the ability to cross the intersection during their phase
GAVs	The CAVs having permission to cross the intersection
LDB	Lane-drop bottleneck
LM	Lane base method
NCV	Non-cooperative velocity method
NLM	Non-lane base method
NL	North-left
NT	North-Straight
P	Bottleneck point (merging point or intersection point)
SDM	Space distribution method
SMLI	A signalized multi-lane intersection
SL	South-left
ST	South-straight
TMC	Traffic management center
VCs	Vehicle controllers
V2I	Vehicle-to-infrastructure communication
V2V	Vehicle-to-vehicle communication
V2X	V2I and V2V
WL	West-left

Chapter 1

Résumé Long

Ce chapitre est le résumé en français de la thèse intitulée:

"TECHNIQUES AVANCÉES DE CONTRÔLE ET D'OPTIMISATION POUR VÉHICULES CONNECTÉS ET AUTONOMES EN MULTI-VOIES AUTOROUTES"

1.1 Contexte et motivation

Avec l'expansion rapide des réseaux de transport, il y a une demande croissante pour développer et déployer des technologies intelligentes de gestion du trafic dans les grandes et petites villes du monde entier. En effet, l'urbanisation rapide et l'augmentation des taux de possession de véhicules ont causé à de nombreux défis de transport, principalement la congestion et la pollution environnementale. Prenons le Canada comme exemple. La congestion du trafic au Canada s'est intensifiée, en particulier dans ses grandes villes, où les conducteurs sont confrontés à des temps de trajet de plus en plus désagréables. En raison de l'étalement urbain et de décennies de sous-investissement aux transports publics, la congestion routière dans la région de Toronto coûte au Canada 3,3 milliards de dollars en perte de productivité par an.

En réponse à la demande croissante du trafic, des techniques innovantes ont été proposées pour mieux gérer la circulation et améliorer l'efficacité des transports. Notamment, les systèmes de transport intelligents (ITS) ont suscité de l'intérêt afin d'atténuer la congestion des autoroutes. Plus précisément, les ITS désignent l'application des technologies de l'information et de la communication au système de transport. De plus, au cœur de ces vastes systèmes interconnectés se trouve une entité cruciale: le centre de gestion du trafic (TMC). Les données provenant de diverses sources telles que les caméras, les capteurs et les détecteurs sur les routes y sont collectées, traitées et transmises. Ce centre surveille non seulement les conditions, mais les gère également de manière proactive.

De plus, l'évolution des véhicules connectés et automatisés (CAVs) et des technologies sans fil avancées telles que la cinquième génération (5G) de technologies sans fil, couplées à des algorithmes avancés et des techniques du contrôle, jouera un rôle important dans l'amélioration de la sécurité et du flux de trafic. Typiquement, un CAV peut collecter des données de son environnement en utilisant différents types de capteurs/dispositifs et la communication véhicule-à-véhicule (V2V) ou véhicule-à-infrastructure (V2I) (appelée communication V2X). Sur la base des données collectées, le véhicule peut mettre en œuvre une stratégie du contrôle appropriée pour atteindre des objectifs spécifiques, comme minimiser le temps de trajet, assurer un espacement constant entre les véhicules ou permettre des motifs de conduite plus fluides. Cela étend la conscience du conducteur sur l'environnement au-delà de la ligne de vue. De plus, l'échange des données de trafic fiable et en temps opportun permet la collaboration entre les conducteurs et les contrôleurs d'intersection, créant ainsi des systèmes de transport intelligents coopératifs (C-ITS). Des recherches récentes indiquent que l'intégration des CAV dans les flux de trafic peut améliorer de manière notable la sécurité routière, les opérations de trafic et promouvoir la durabilité environnementale.

Cependant, la conduite partiellement automatisée est déjà commercialement disponible pour des tâches de conduite de base telles que l'accélération et le freinage grâce au régulateur de vitesse adaptatif (ACC) [1]. En fait, les systèmes ACC sont les premiers systèmes d'assistance au conducteur ayant le potentiel d'influencer les caractéristiques du flux de trafic. Cependant, les implémentations actuelles des systèmes ACC sont exclusivement conçues pour augmenter le confort de conduite et leur influence sur le trafic environnant n'est ni considérée ni optimisée. Selon l'Administration de la Recherche et de la Technologie Innovante (RITA) du Département des Transports des États-Unis (USDOT), 81% des accidents impliquant des véhicules pourraient être évités ou significativement réduits si l'on exploitait annuellement les technologies des véhicules connectés.

Le concept du contrôle de véhicule entièrement automatisé permet des intervalles de temps très courtes et la formation des pelotons, ce qui est la clé pour augmenter la capacité. De plus, les véhicules connectés et autonomes (CAVs) promettent divers avantages sociétaux. Tout d'abord, ils ont le potentiel de réduire considérablement les accidents, étant donné que les erreurs humaines en représentent plus de 90% [2]. Sur le plan économique, les CAV peuvent entraîner une réduction des dépenses de voyage. Par exemple, leur intégration dans les systèmes de transport public peut réduire les coûts de main-d'œuvre et atténuer des problèmes tels que la congestion du trafic, les coûts liés aux accidents, la dégradation de l'environnement et les préoccupations de sécurité [3]. En plus, ils peuvent résoudre les problèmes de circulation urbaine dus aux comportements de conduite humains inappropriés tels que les infractions au code de la route et les changements de voie inutiles [3]. D'un point de vue environnemental, les CAV peuvent améliorer l'efficacité énergétique et diminuer les émissions de carbone [4]. Enfin, le secteur des véhicules autonomes est estimé à environ 0,2 trillion de dollars par an d'ici 2025 [5]. Cependant, de tels systèmes nécessitent une infrastructure spéciale et des voies correspondantes, ce qui ne peut être justifié que si le pourcentage de véhicules automatisés est suffisamment élevé.

Les opérations des véhicules connectés et automatisés (CAVs) ont suscité un intérêt considérable, conduisant au développement de nombreuses techniques pour coordonner la dynamique des CAVs avec la gestion centralisée des intersections, dans le but de réduire le temps de trajet, le coût énergétique et la pollution environnementale. Dresner *et al.* [6] et Perronnet *et al.* [7] ont introduit une méthode de réservation pour les véhicules dont les itinéraires se croisent. Les intersections sont essentiellement divisées en une grille de carreaux, et les véhicules transmettent leurs besoins en carreaux pour leur itinéraire prévu au contrôleur d'intersection (IC) avant d'entrer. Si l'IC ne détecte aucun conflit avec d'autres véhicules, il attribue les carreaux nécessaires et une plage horaire. Cependant, en cas de conflits, la demande est refusée, ce qui oblige ces véhicules à ralentir et à attendre une nouvelle approbation de réservation. Dans les scénarios de trafic connecté, la communication inter-véhicules (IVC) aide les CAVs en peloton en fournissant des informations d'infrastructure en temps opportun, telles que des données provenant des contrôleurs en bord de route. Ces véhicules utilisent alors un mécanisme du contrôle adapté pour atteindre des objectifs spécifiques, comme maintenir un espacement constant entre les véhicules en peloton [8] ou minimiser le temps total de trajet et les émissions [9].

Bien que de nombreuses recherches se soient concentrées sur l'efficacité des CAV au sein des systèmes de circulation ces dernières années [10], l'étude de la dynamique des CAV sur les autoroutes multi-voies reste limitée. En particulier, les problématiques liées au changement de voie et à la fusion sous les contraintes de sécurité des autoroutes sont complexes en raison des comportements variables des véhicules approchant un goulet d'étranglement. Lorsque les véhicules s'approchent d'un goulet d'étranglement ou d'un point de fusion où une voie se termine, ceux sur cette voie sont contraints de changer de voie. Néanmoins, en l'absence de la coordination avec les véhicules des voies adjacentes, il se peut qu'ils ne trouvent pas d'espace sûr pour effectuer ce changement de voie. Cela pourrait entraîner une réduction de la vitesse, conduisant à la formation potentielle des files d'attente [11], diminuant ainsi le débit ou la capacité du goulet d'étranglement. De telles situations entraînent souvent des collisions ou des embouteillages à ces points de congestion [12].

1.2 Contributions à la recherche

La collaboration entre les véhicules connectés et automatisés (CAVs) et le centre de gestion du trafic (TMC) pour assurer des trajectoires fluides des CAV traversant une intersection multi-voies signalisée (SMLI) ou circulant dans des goulets d'étranglement de voies réduites (LDB) sur des autoroutes multi-voies sans collisions constitue le point central de cette thèse. Pour relever ces défis, nous explorons des stratégies du contrôle facilitant l'intégration de ces systèmes, en particulier une méthode de distribution de l'espace coopérative pour les LDBs et une méthode de distribution de l'espace (SDM) pour les SMLI. Nous discutons ensuite de nos contributions clés, où nous abordons les défis de recherche liés à ces problématiques de conception.

1.2.1 Une méthode de distribution de l'espace coopérative pour les véhicules autonomes dans un goulet d'étranglement de voie réduite sur des autoroutes multi-voies

À notre connaissance, la plupart des recherches existantes se concentrent uniquement sur les problèmes de changement de voie ou de fusion sur des autoroutes à deux voies. Notre présente recherche vise à combler cette lacune dans la littérature en proposant des méthodes coopératives générales qui peuvent augmenter la vitesse moyenne des CAVs dans un goulet d'étranglement de voie réduite sur des autoroutes multi-voies. Plus précisément, la contribution principale de cet article peut être résumée comme suit:

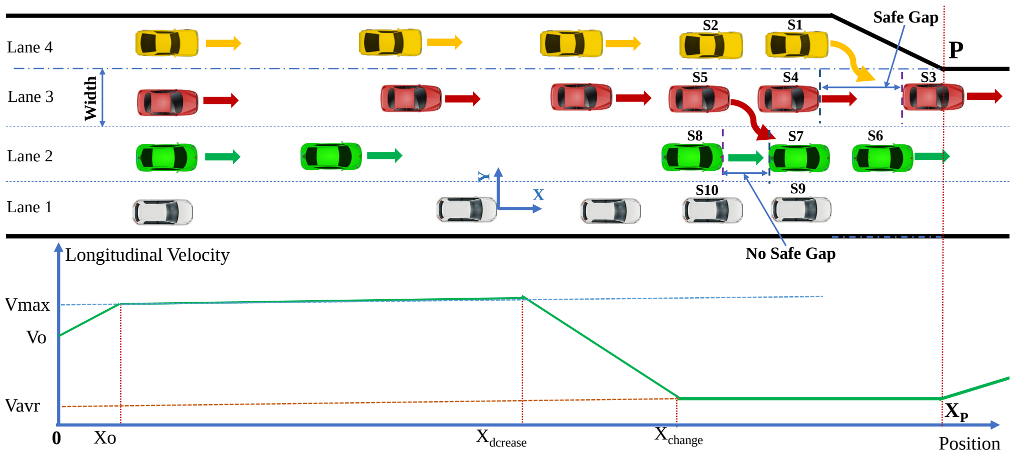


Figure 1.1: La vitesse normale d'un CAV dans un goulet d'étranglement.

- Nous proposons une méthode coopérative de distribution de l'espace (CSDM) pour réduire le temps total de trajet des CAV traversant un goulet d'étranglement de voie sur une autoroute multi-voies (par exemple, des autoroutes à 4 voies). Plus précisément, nous calculons les trajectoires des CAVs (mouvements longitudinaux et latéraux) et les guidons pour changer de voie simultanément tout en maintenant la sécurité. Ainsi, la complexité computationnelle pourrait être réduite car les contraintes de trajectoire longitudinale ne sont pas considérées dans ce processus.

- Nous présentons la manière de répartir les CAV de la voie abandonnée vers d'autres voies pour équilibrer efficacement le flux en aval entre les voies.

- Nous fournissons des mécanismes efficaces pour contrôler la priorité des véhicules se rendant à leur position souhaitée. En particulier, la méthode de fusion en tête pourrait être utilisée si les véhicules de la voie abandonnée ont une priorité plus élevée, comme les ambulances, les camions de pompiers et les bus rapides.

1.2.1.1 Modèle de dynamique de véhicule

Nous envisageons une situation normale pour une autoroute multi-voie où il y a une réduction de voie (par exemple, la voie 4 comme dans la Fig. 1.1), obligeant ainsi tous les véhicules de cette voie à changer de voie. Ensuite, nous décrirons le modèle cinématique du véhicule et présenterons les variables du contrôle et l'objectif de conception du problème lié aux opérations des CAV. La trajectoire générale d'un véhicule est composée de deux mouvements: le mouvement longitudinal et le mouvement latéral. Concernant le problème du contrôle de trajectoire du véhicule, les véhicules sont considérés avec seulement deux manœuvres: le processus de maintien de voie et le processus de changement de voie.

a) Processus de maintien de voie

Si les CAVs se déplacent dans la même voie (maintien de voie), leur trajectoire longitudinale satisfait aux relations suivantes:

$$\dot{x}_{il}(t) = v_{il}(t), \tag{1.1a}$$

$$\dot{v}_{il}(t) = a_{il}(t), \tag{1.1b}$$

$$\dot{y}_{il}(t) = 0, \tag{1.1c}$$

où $x_{il}(t)$ et $x_{(i-1)l}(t)$ représentent respectivement les positions longitudinales des véhicules i et $(i-1)$ dans la voie l; $y_{il}(t)$ est la position latérale du véhicule i dans la voie l; $v_{il}(t)$ et $a_{il}(t)$ sont respectivement la vitesse (longitudinale) et l'accélération du véhicule i.

Les contraintes de sécurité pour leur mouvement peuvent être décrites comme suit:

$$x_{il}(t) - x_{(i-1)l}(t) - L = g_{il}(t) \geq T * v_{il}(t), \tag{1.2a}$$

$$V_{\max} \geq v_{il}(t) \geq V_{\min}, \tag{1.2b}$$

$$a_{x\max} \geq a_{il}(t) \geq a_{x\min}, \tag{1.2c}$$

où L représente la longueur d'un véhicule; $g_{il}(t)$ est l'écart (distance) entre le véhicule i et le véhicule $(i-1)$ dans la voie l; T est l'intervalle de temps pour le véhicule i (suiveur), où $T = 1s$; $V_{\max}$ et $V_{\min}$ sont les limites supérieure et inférieure de la vitesse des véhicules; $a_{x\max}$ et $a_{x\min}$ sont respectivement l'accélération longitudinale maximale et minimale des véhicules.

L'inégalité (1.2a) décrit la limite inférieure de l'écart entre les CAVs adjacents; (1.2b) et (1.2c) représentent respectivement les contraintes concernant la plage de vitesse et d'accélération des CAVs.

b) Processus de changement de voie

Lorsqu'un véhicule particulier M souhaite changer de voie, il doit trouver un espace sûr dans la voie cible ou attendre que ces conditions de sécurité soient remplies. L'espace sûr doit être

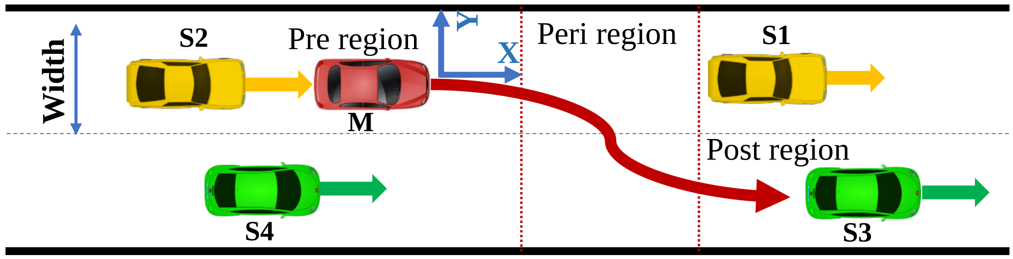


Figure 1.2: Processus de changement de voie.

suffisamment grand pour permettre au véhicule de changer de voie sans entrer en collision avec les autres.

Plus précisément, le processus de changement de voie est divisé en trois régions: Pre, Peri et Post segment, comme illustré dans la Fig. 1.2. Définissons $x_{Sk}(t), v_{xSk}(t), s_m(Sk), \forall k = 1, ..4$ comme étant respectivement la position longitudinale, la vitesse et l'écart de sécurité du véhicule Sk; tandis que $x_M(t), y_M(t), v_{xM}(t), v_{yM}(t), a_{xM}(t), a_{yM}(t)$ décrivent respectivement la position longitudinale et latérale, la vitesse et l'accélération du véhicule M; et LW représente la largeur de la voie.

Tout d'abord, durant l'intervalle de temps $T_{Pre} = (0, t_{Pre})$ de la région Pre de la voie d'origine, le véhicule M doit respecter l'écart de sécurité avec le véhicule de tête $S1$ et le véhicule suiveur $S2$ avant d'initier le mouvement latéral. Ainsi, nous pouvons déterminer les limites supérieure et inférieure des états du véhicule M comme suit:

$$x_{\max}(t) = x_{S1}(t) - s_m(S1) - L, \qquad \forall t \in T_{Pre}, \tag{1.3a}$$

$$x_{\min}(t) = x_{S2}(t) + s_m(S2) + L, \qquad \forall t \in T_{Pre}, \tag{1.3b}$$

$$v_{yM}(t) = 0, \qquad \forall t \in T_{Pre}. \tag{1.3c}$$

Par conséquent, l'ensemble des contraintes dans la région Pre se lit comme suit:

$$x_{\max}(t) \geq x_M(t) \geq x_{\min}(t), \qquad \forall t \in T_{Pre}, \tag{1.4a}$$

$$V_{\max} \geq v_{xM}(t) \geq V_{\min}, \qquad \forall t \in T_{Pre}, \tag{1.4b}$$

$$a_{x\max} \geq a_{xM}(t) \geq a_{x\min}, \qquad \forall t \in T_{Pre}. \tag{1.4c}$$

Ensuite, lorsque le véhicule M se trouve dans l'intervalle de temps $T_{Pe} = (t_{Pre}, t_{Pre} + t_{Pe})$ de la région Peri de la voie d'origine, il doit être limité par les bornes supérieures $S1$ et $S3$, et les bornes inférieures $S2$ et $S4$. Ainsi, nous avons $\forall t \in T_{Pe}$:

$$x_{\max}(t) = \min\left\{x_{S1}(t) - s_m(S1), x_{S3}(t) - s_m(S3)\right\} - L, \tag{1.5a}$$

$$x_{\min}(t) = \max\left\{x_{S2}(t) + s_m(S2), x_{S4}(t) + s_m(S4)\right\} + L, \tag{1.5b}$$

où

$$s_m(Sk) = T * v_{xSk}(t), \forall k = 1, ..., 4. \tag{1.6}$$

L'ensemble des contraintes longitudinales de la région Péri est:

$$x_{\max}(t) \geq x_M(t) \geq x_{\min}(t), \qquad \forall t \in T_{Pe}, \tag{1.7a}$$

$$V_{\max} \geq v_{xM}(t) \geq V_{\min}, \qquad \forall t \in T_{Pe}, \tag{1.7b}$$

$$a_{x\max} \geq a_{xM}(t) \geq a_{x\min}, \qquad \forall t \in T_{Pe}. \tag{1.7c}$$

Les contraintes latérales et la limite de l'angle de dérapage latéral pour $\forall t \in T_{Pe}$ sont:

$$LW \geq y_M(t) \geq 0, \tag{1.8a}$$

$$y_M(t = t_{Pre}) = 0, \tag{1.8b}$$

$$y_M(t = t_{Pre} + t_{Pe}) = LW, \tag{1.8c}$$

$$0.17 * v_{xM}(t) \geq v_{yM}(t) \geq V_{\min}, \tag{1.8d}$$

$$v_{yM}(t = t_{Pre}) = v_{yM}(t = t_{Pre} + t_{Pe}) = 0, \tag{1.8e}$$

$$a_{y\max} \geq a_{yM}(t) \geq a_{y\min}, \tag{1.8f}$$

où $a_{y\max}$ et $a_{y\min}$ représentent respectivement l'accélération latérale maximale et minimale des véhicules. Notez que le coefficient 0.17 dans l'équation (1.8d) reflète l'effet de l'angle de dérapage latéral entre la vitesse longitudinale et latérale.

Enfin, la région Post présente des équations et contraintes similaires à celles de la région Pre, en remplaçant $(S1, S2)$ par $(S3, S4)$ pour $\forall t \in T_{Po} = (t_{Pre} + t_{Pe}, t_{Pre} + t_{Pe} + t_{Po})$.

Notons que TF est le temps final lorsque le véhicule M termine le processus de changement de voie. Le véhicule M peut attendre pendant un certain temps TW jusqu'à ce qu'un écart de sécurité suffisant lui permette de changer de voie. Ainsi, le temps total nécessaire pour compléter ce processus de changement de voie est:

$$TF = TW + t_{Pre} + t_{Pe} + t_{Po}. \tag{1.9}$$

1.2.1.2 Contraintes de réduction de voie

Dans la Fig. 1.1, P et X_P désignent respectivement le point de fusion et sa position longitudinale. Pour continuer leur trajet, tous les véhicules de la voie 4 doivent se déplacer vers d'autres voies avant d'atteindre le point P. En désignant par $\mathcal{M}_l$ l'ensemble de tous les véhicules dans la voie l, nous avons la contrainte suivante:

$$x_{i4}(t) < X_P, \forall i \in \mathcal{M}_4. \tag{1.10}$$

Le débit Q^P au point de fusion P (c'est-à-dire le débit au goulet d'étranglement) est défini comme suit:

$$Q^P = \int_0^{T_o} Q(t)dt = \sum_{l=1}^{3} \int_0^{T_o} Q_l(t)dt, \tag{1.11}$$

où $Q(t)$ est le taux de flux par seconde (c'est-à-dire le nombre de CAVs traversant le goulet d'étranglement en une seconde); $Q_l(t)$ est le taux de flux par seconde de la voie l; T_o est la période de temps considérée dans le problème.

d) Variables du contrôle et objectif de conception

Considérons maintenant le problème de réduction de voie, comme illustré dans la Fig. 1.1. Soit $\mathcal{V} = \{a_{xil}(t), a_{yil}(t), \forall i, l\}$ l'ensemble des variables du contrôle (accélération longitudinale et latérale) de tous les véhicules. L'objectif de notre conception est de contrôler tous les CAVs (en trouvant leurs trajectoires conçues) pour atteindre un débit maximal (ou un temps total de trajet minimal) au goulet d'étranglement tout en respectant les conditions de sécurité. Nous déterminons plus en détail $\mathcal{V}$ pour chaque CAV afin d'augmenter le débit au goulet d'étranglement Q^P et de satisfaire le modèle cinématique du véhicule (contraintes (1.1a) à (1.9)) ainsi que la contrainte de réduction de voie (contrainte (1.10)).

1.2.1.3 Principe général de conception

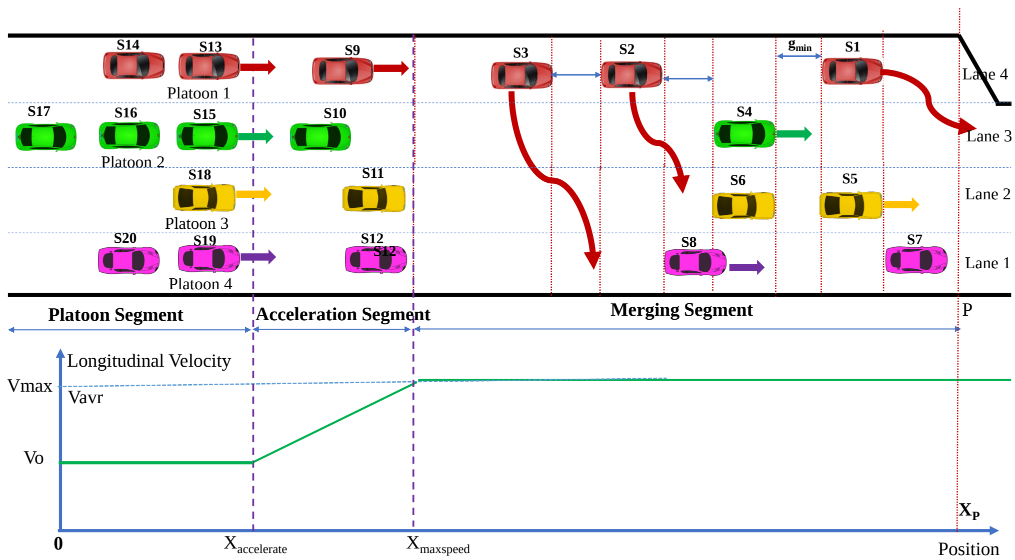


Figure 1.3: Le principe de conception pour le contrôle des CAVs.

Pour résoudre le problème complexe de la réduction de voie, nous créons un segment juste avant le goulet d'étranglement (appelé segment de fusion) dans lequel les véhicules, après avoir changé de voie, peuvent circuler à vitesse maximale et avec des écarts minimaux entre les véhicules adjacents dans la même voie. En réalité, si le nombre de véhicules arrivant au goulet d'étranglement (afflux) est supérieur au nombre pouvant en sortir (flux sortant), cela pourrait entraîner une diminution de la vitesse moyenne des véhicules dans le segment de fusion. Pour cette raison, un segment (tampon) est nécessaire pour stocker le trafic entrant afin de réguler efficacement le trafic sortant. Pour réduire la consommation de carburant et la demande computationnelle, ces véhicules devraient circuler en peloton, d'où le nom de segment de peloton pour cette zone. Il est évident que la vitesse des véhicules dans le segment de peloton est souvent inférieure à celle dans le segment de fusion. Par conséquent, nous avons besoin d'un segment intermédiaire pour permettre aux véhicules du segment de peloton d'accélérer jusqu'à leur vitesse maximale. De plus, les véhicules dans le segment intermédiaire devraient être guidés vers des positions appropriées pour qu'ils puissent changer de voie sans collision dans le segment de fusion. Ce segment intermédiaire est appelé le segment d'accélération.

En conséquence, l'autoroute (zone d'intérêt) est divisée en trois segments: le segment de peloton, le segment d'accélération et le segment de fusion, comme illustrée dans la Fig. 1.3. D'abord, les véhicules S13-S20 approchant du goulet d'étranglement circulent en pelotons dans le segment de peloton. Ensuite, ils (S9-S12) accélèrent pour atteindre la vitesse maximale et se positionnent de manière adéquate dans le segment d'accélération. Enfin, ils (S1-S8) changent de voie sans collisions et traversent le goulet d'étranglement avec une vitesse maximale et des écarts minimaux entre eux dans le segment de fusion.

a) Une solution idéale pour les autoroutes à deux voies

Maintenant, nous montrons un état idéal où tous les véhicules dans la voie abandonnée pourraient changer de voie rapidement (c'est-à-dire, le temps minimal TF) et atteindre la vitesse longitudinale la plus élevée pendant le processus de changement de voie (c'est-à-dire, la vitesse maximale $v_{xM}(t), \forall t \in (0, TF)$).

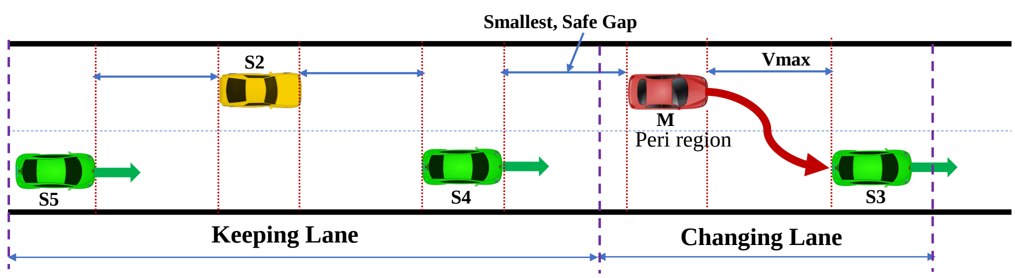


Figure 1.4: La position et la vitesse idéales pour les véhicules CAV dans le segment de fusion sont les suivantes.

Theorem 1.1. *Si les véhicules CAVs sont répartis comme indiqués dans la Fig. 1.4 et circulent à la vitesse maximale, le véhicule M atteindra non seulement le temps minimum TF, et la vitesse longitudinale maximale $v_{xM}(t) = V_{max}, \forall t \in (0, TF)$, mais aura également l'écart optimal avec les véhicules adjacents après avoir terminé le changement de voie.*

b) Solution pour les autoroutes multi-voies

Afin de trouver la solution pour le scénario général décrit à la Figure 1.5 (c'est-à-dire, les autoroutes multi-voies avec une voie abandonnée), nous devons répartir également les véhicules CAVs dans la voie 4 vers les voies restantes. Nous proposons donc trois stratégies différentes basées sur le CSDM pour effectuer une répartition équitable des voies: la fusion en tête, la fusion en queue, la fusion au centre. Nous présentons maintenant la méthode de fusion en tête à titre d'exemple:

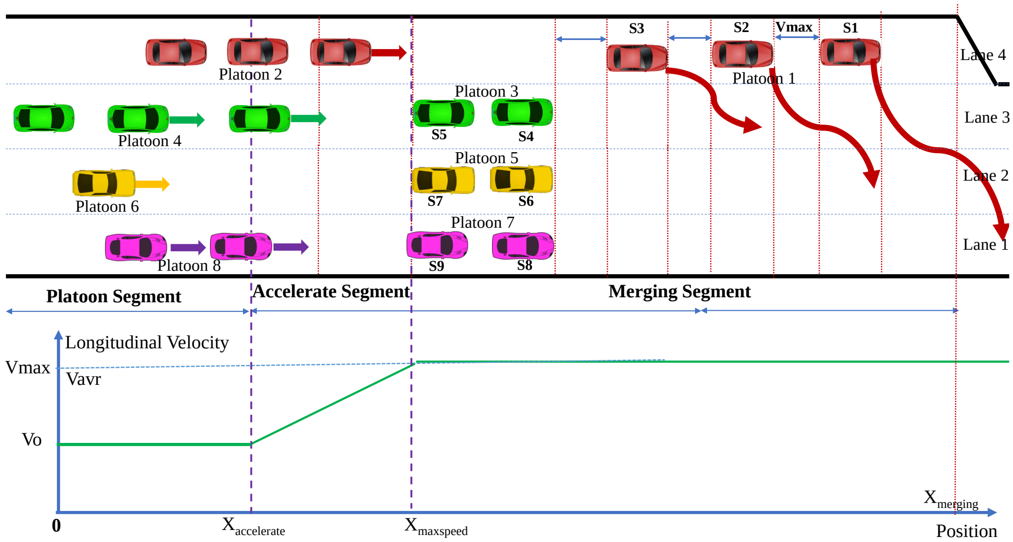


Figure 1.5: CSDM-H: Fusion en tête.

Cette méthode est utilisée i) si les véhicules de la voie abandonnée ont la priorité, tels qu'une ambulance, un camion de pompier, un bus à haut niveau de service. ii) ou si les véhicules se trouvent bien en avant par rapport à la position longitudinale des véhicules dans les autres voies. iii) ou si la densité de la voie abandonnée est beaucoup plus élevée que celle des autres voies. Par exemple, s'il y a six véhicules dans la voie abandonnée (par exemple, la voie 4) et seulement un véhicule dans chacune des autres voies (trois voies sans voie abandonnées), nous devons contrôler les trajectoires (vitesse et position) de trois véhicules dans ces voies au lieu de six véhicules dans la voie 4. Par conséquent, nous permettons à ces CAV dans la voie 4 de fusionner en tête des convois dans les autres voies, comme illustré à la Figure 1.5.

1.2.1.4 Résultats numériques

Maintenant, nous considérons le goulet d'étranglement de la voie dans une autoroute à quatre voies, illustrée à la Fig. 1.1. Les paramètres de conception généraux pour le mouvement des CAVs et de l'autoroute sont donnés respectivement dans les tableaux 1.1 et 1.2. Au début, les CAV dans chaque voie circulent en deux convois (il y a huit convois: convois 1 et 2 dans la voie 4, convois 3 et 4 dans la voie 3, etc.). La position des convois et la distance entre eux sont réglées de manière aléatoire.

Les métriques de performance utilisées pour comparer la performance des différentes méthodes sont le temps de voyage moyen T_{avr} de tous les CAVs, le nombre de véhicules s'arrêtant SVN, et la

Table 1.1: Paramètres de conception généraux pour les CAVs

Paramètres	Valeur	Unité
Longueur	4.5	m
Vitesse longitudinale	$\in \{0, 30\}$	m/s
Accélération longitudinale	$\in \{-3, 2\}$	m/s
Vitesse latérale	$\in \{0, 6\}$	m/s
Accélération latérale	$\in \{-2, 2\}$	m/s

Table 1.2: Paramètres de conception généraux pour une autoroute

Paramètres	Valeur	Unité
Longueur du goulet d'étranglement	1000	m
Largeur de la voie	4	m

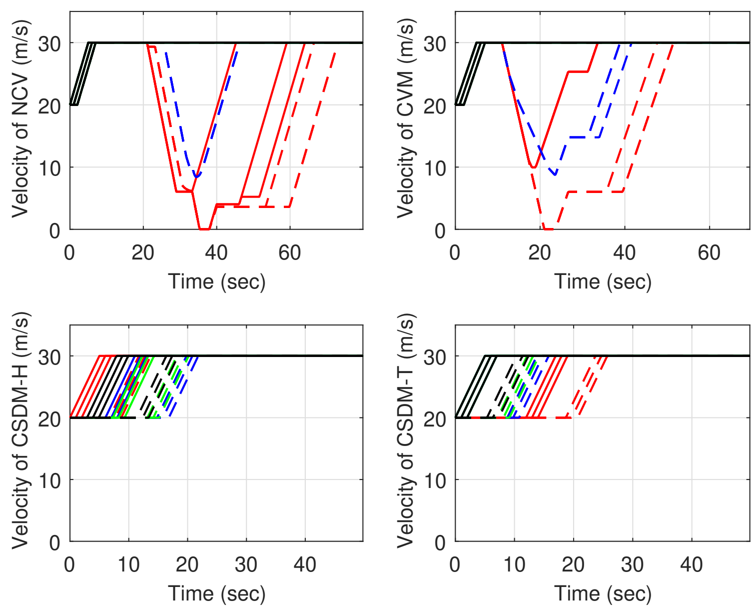


Figure 1.6: Vitesse des CAVs pour NCV, CVM, CSDM-H et CSDM-T.

vitesse moyenne des CAVs changeant de voie V_{mer} depuis la voie 4 et l'efficacité de la solution E_f. Définissons T_m comme le temps minimum total et T_r comme le temps total de tous les CAVs pour traverser le goulet d'étranglement. Ensuite, la métrique E_f est définie comme:

$$E_f = \frac{T_m}{T_r} * 100\%. \tag{1.12}$$

Pour comparer avec l'état de l'art, trois méthodes sont étudiées dans le scénario 1 pour minimiser le temps de voyage moyen comme suit: méthode de vitesse non coopérative (NCV), méthode de vitesse coopérative (CVM) et méthode de distribution de l'espace coopérative (CSDM). En particulier, si un véhicule ne communique pas et ne coopère pas avec les autres, la méthode NCV (ou méthode de véhicule conduit par un humain) est utilisée pour trouver une façon de changer de voie. La CVM est plus intelligente que la NCV car les CAVs dans une voie adjacente ajustent leur vitesse afin de créer un espace sûr. Cependant, il n'est pas permis aux CAVs de changer de voie de manière continue car les espaces sûrs dans les voies 1 à 3 peuvent ne pas être disponibles en continu pour un tel changement de voie continu. Pour surmonter cet inconvénient, le CSDM contrôle la vitesse des CAVs et les guide vers des positions appropriées afin que ces CAVs puissent changer de voie sur plusieurs voies dès que possible. De plus, contrairement à la NCV, la CVM et la CSDM permettent aux CAVs de se déplacer en peloton pour changer de voie simultanément, ce qui réduit considérablement la demande de calcul (car les contraintes d'évitement de collision pour la trajectoire longitudinale ne sont pas impliquées) [13].

La Fig. 1.6 montre la vitesse longitudinale des CAVs en fonction de la NCV, de la CVM et de la CSDM-C. Les lignes pleines rouges et les lignes pointillées rouges correspondent aux CAVs en pelotons 1 et 2 de la voie 4, les lignes pleines bleues et les lignes pointillées bleues représentent les trajectoires des CAVs des pelotons 3 et 4 dans la voie 3, les lignes vertes sont pour les CAVs des pelotons 5 et 6 dans la voie 2, et les lignes noires pour les CAVs des pelotons 7 et 8 dans la voie 1. On peut voir que les vitesses longitudinales des CAVs, qui changent de voie dans la NCV et la CVM, sont très faibles tandis que la vitesse longitudinale dans la CSDM est la plus élevée. Par conséquent, la durée pour changer de voie dans la CSDM est considérablement plus courte que celle dans la NCV et la CVM.

Les résultats des trois méthodes sont présentés dans le tableau 1.3. Le nombre de véhicules s'arrêtant dans la méthode NCV ($SVN = 5$) est le plus élevé car l'espace sûr dans la voie 3 ne peut accueillir qu'un seul véhicule. Les cinq autres CAVs doivent ralentir jusqu'à l'arrêt complet. Par conséquent, le temps de voyage moyen est le plus élevé pour cette méthode. La méthode CVM

Table 1.3: Résultats de performance de trois méthodes dans le SCE.1

Méthode	$T_{avr}(s)$	SVN	$V_{mer}(m/s)$	E_f
NCV	38.93	5	19.43	80.31%
CVM	37.03	3	22.69	84.42%
CSDM-H	34.20	0	28.62	91.41%
CSDM-T	33.77	0	26.23	92.58%
CSDM-C	34.37	0	26.83	90.96%

atteint un temps de voyage moyen inférieur à celui de la méthode NCV car les CAVs des pelotons 3 et 4 dans la voie 3 ont ajusté leur vitesse pour générer suffisamment d'espaces sûrs afin que le peloton 1 dans la voie 4 puisse changer de voie. Tandis que le peloton 2 dans la voie 4 doit s'arrêter pour attendre que le peloton 1 termine le processus de fusion, donc le $SVN = 3$. En utilisant la méthode CSDM, nous pouvons atteindre le temps de voyage moyen le plus court car les CAVs dans la voie 4 n'ont pas besoin de ralentir et d'attendre un espace sûr pour pouvoir changer de voie instantanément et simultanément. De plus, les efficacités des trois types de méthodes CSDM (au-dessus de 90 %) sont meilleures que celles des méthodes NCV (80,31%) et CVM (84,42%) par rapport à la limite inférieure dans ce scénario. Surtout, la CSDM-T atteint le meilleur résultat (92,58%) car le nombre de CAVs (6 VACs) dans la voie abandonnée est bien plus petit que celui (18 CAVs) dans les autres voies restantes.

1.2.2 Méthode de distribution d'espace pour véhicules autonomes à une intersection multivoies signalisée

L'exploitation optimale des CAVs aux intersections multivoies signalisées est un problème difficile en raison de l'interaction complexe des véhicules entre les voies. À notre connaissance, il y a peu de tentatives dédiées au contrôle basé sur les voies des CAVs aux intersections multivoies signalisées. Pour surmonter de tels problèmes, nous avons introduit une nouvelle méthode de coopération permettant de contrôler les véhicules connectés et autonomes afin d'optimiser le débit de l'intersection. Les principales contributions de notre travail peuvent être résumées comme suit:

- Nous développons un cadre du contrôle à deux niveaux pour optimiser le débit de l'intersection.
- Nous proposons une méthode de distribution d'espace (SDM) pour les CAVs afin de maximiser le débit (c'est-à-dire, le nombre de VACs) de l'intersection (multivoies) où les temps de signalisation sont prédéfinis.

1.2.2.1 Modèle cinématique de véhicule

Soit $\mathcal{L} = 1, 2, ..., Lm$, $\mathcal{J} = \{\text{'ouest', 'est', 'nord', 'sud'}\}$, $\mathcal{P} = \{1, 2, 3, 4\}$, désignant respectivement l'ensemble des voies, des approches et des phases. Il est à noter que toutes les approches ont le même nombre maximal de voies, qui est égal à Lm. Soient respectivement l, j et p les éléments de

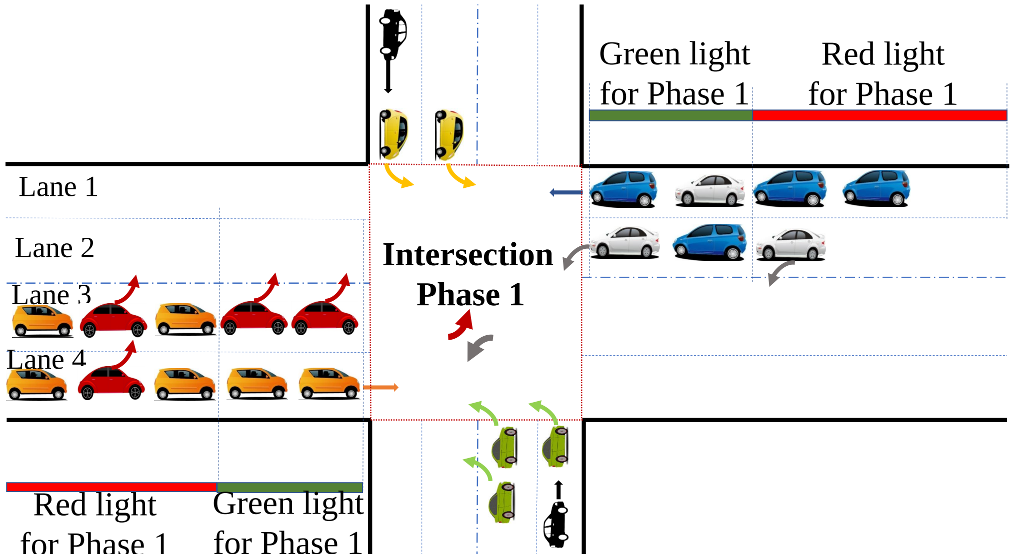


Figure 1.7: L'intersection à 4 voies et 4 approches et 4 phases avec des temps de signalisation prédéfinis pour les véhicules conduits par des humains. Les voitures rouges et blanches tourneront à gauche, les voitures oranges et bleues iront tout droit, etc.

$\mathcal{L}$, $\mathcal{J}$ et $\mathcal{P}$. De plus, l'ensemble des CAVs dans les voies l et les approches j est nommé $\mathcal{I}_{lj}$ dont l'élément est indiqué par i.

Si les CAVs se déplacent dans la même voie de la même approche, les équations dynamiques sont les suivantes:

$$\dot{x}_{ilj}(t) = v_{ilj}(t), \tag{1.13}$$

$$\dot{v}_{ilj}(t) = a_{ilj}(t), \tag{1.14}$$

$$x_{ilj}(t) - x_{(i-1)lj}(t) = g_{ilj}(t) \geq v_{ilj}, \tag{1.15}$$

$$V_{\max} \geq v_{ilj}(t) \geq V_{\min}, \tag{1.16}$$

$$a_{\max} \geq a_{ilj}(t) \geq a_{\min}, \tag{1.17}$$

où $x_{ilj}(t)$ et $x_{(i-1)lj}(t)$ sont respectivement la position du véhicule i et de son suiveur $(i-1)$ dans la voie l, approche j; $v_{ilj}(t)$ et $a_{ilj}(t)$ sont respectivement la vitesse et l'accélération du véhicule i sur l'approche j; $g_{ilj}(t)$ dénote la distance entre le véhicule i et $(i-1)$ dans la voie l, approche j; $V_{\max}$ et $V_{\min}$ sont la limite supérieure et inférieure de la vitesse des véhicules; $a_{\max}$ et $a_{\min}$ sont respectivement l'accélération maximale et minimale.

Il convient de noter que l'inégalité (1.15) est une limite inférieure de l'écart entre les CAVs adjacents (c'est-à-dire, la distance parcourue en 1s). Les inégalités (1.16) et (1.17) sont respectivement les contraintes pour la plage de vitesse et celle d'accélération réalisables du CAV.

1.2.2.2 Formulation du problème

L'objectif du problème d'optimisation dans ce travail est d'atteindre le débit maximal à une intersection multivoies signalisée, qui est défini comme suit:

$$\max \int_0^T \sum_{j\in\mathcal{J}} Q_j(t)dt = \max \sum_{j\in\mathcal{J}} \int_0^T Q_j(t)dt = \max \sum_{j\in\mathcal{J}} \int_0^T \sum_{l=1}^{M} Q_{lj}(t)dt, \tag{1.18}$$

où $Q_j(t)$ est le taux de flux par seconde (c'est-à-dire, le nombre de CAVs traversant l'intersection en une seconde) depuis l'approche j; $Q_{lj}(t)$ est le taux de flux par seconde depuis la voie l de l'approche j; T est l'horizon temporel considéré dans le processus d'optimisation.

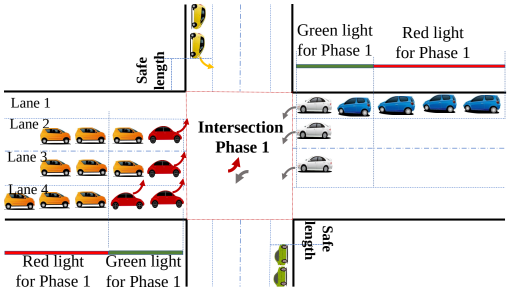
Sans perte de généralité, nous ne considérons que le problème d'optimisation sur un cycle de temps de signalisation prédéfinis. Définissons:

$$NA_j = \int_0^{T_c} Q_j(t)dt = \sum_{p\in\mathcal{P}} \int_0^{t_p} Q_j(t)dt = \sum_{p\in\mathcal{P}} NA_{jp}, \tag{1.19}$$

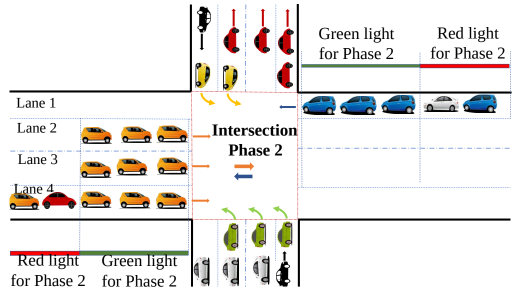
où NA_j est le nombre total de CAVs traversant l'intersection depuis l'approche j pendant un cycle; NA_{jp} est le nombre total de CAVs traversant l'intersection depuis la route j pendant la phase p; T_c est la durée du cycle; t_p est le temps vert prédéfini de la phase p.

Par conséquent, le débit optimal sur un cycle peut être calculé comme suit:

$$\begin{aligned} \max_{v_{lj}^A,\ g_{ilj},\ \sigma_{jp}} \sum_{j\in\mathcal{J}} NA_j &= \max_{v_{lj}^A,\ g_{ilj},\ \sigma_{jp}} \sum_{j\in\mathcal{J}} \sum_{p\in\mathcal{P}} NA_{jp} = \max_{v_{lj}^A,\ g_{ilj},\ \sigma_{jp}} \sum_{p\in\mathcal{P}} \sum_{j\in\mathcal{J}} NA_{jp} \\ &= \max_{v_{lj}^A,\ g_{ilj},\ \sigma_{jp}} \sum_{p\in\mathcal{P}} \sum_{j\in\mathcal{J}} \sum_{l=1}^{\sigma_{jp}} \int_0^{t_p} Q_{lj}(t)dt. \end{aligned} \tag{1.20}$$



(a) Si la Phase 1 est active



(b) Si la Phase 2 est active

Figure 1.8: Méthode de distribution d'espace pour les VACs.

1.2.2.3 Méthode de distribution d'espace pour une intersection multivoie signalisée

Considérons une intersection à 4 voies et 4 approches avec 4 phases: tourner à gauche dans les phases 1 et 3, aller tout droit dans les phases 2 et 4, comme dans la Fig. 1.7. Pour simplifier, nous ignorons la phase de tourner à droite dans cet exemple. Définissons les CAVs qui ont la même direction (par exemple, tourner à gauche ou aller tout droit) dans chaque approche comme des PAVs.

Theorem 1.2. *En supposant que les temps de signalisation sont déterminés de manière à maximiser le nombre total de PAVs traversant l'intersection depuis l'approche j pendant la phase p, noté NA_{jp}, s'ils se déplacent en peloton à la vitesse maximale, avec l'écart le plus petit et le nombre le plus élevé de voies assignées.*

Par conséquent, nous proposons un cadre du contrôleur à deux niveaux utilisant une méthode de distribution d'espace (SDM) pour la régulation du trafic sous l'environnement connecté afin de tirer parti des avantages des CAVs. L'idée principale du cadre proposé est que les positions les plus proches de la barre d'arrêt dans toutes les voies possibles sont attribuées à tous les PAVs ayant la capacité de traverser l'intersection pendant leur phase à la vitesse maximale et avec l'écart le plus petit. Ces CAVs sont appelés AAVs.

a) Méthode du contrôleur à deux niveaux

Comme les informations (c'est-à-dire, la vitesse, la position, etc.) des CAVs sont connues et que les feux de circulation sont prédéfinis, un PAV est considéré comme un AAV s'il satisfait à la condition suivante:

$$t_{rp} \geq \frac{d^s_{ilj}}{V_{\max}}, \tag{1.21}$$

où t_{rp} est le temps restant pour un signal rouge de la phase p, d^s_{ijl} est la distance du véhicule i sur la voie l de l'approche j jusqu'à la barre d'arrêt. Ainsi, nous pouvons estimer le nombre total maximal des AAVs et les considérer comme une file d'attente virtuelle attendant de traverser dans la phase suivante.

La Fig. 1.9 montre la disposition du cadre proposé du contrôleur à deux niveaux et décrit comment maximiser la vitesse et minimiser l'écart des AAVs. Il se compose de deux niveaux: le contrôleur d'intersection (c'est-à-dire, le niveau supérieur) et les contrôleurs de véhicules (c'est-à-dire, le niveau inférieur). Lorsque le contrôleur d'intersection (IC) reçoit des informations indiquant que des CAVs entrent dans l'intersection sur l'approche j, il ordonne à ces CAVs de rejoindre un peloton. Ensuite, le IC vérifie la possibilité pour les CAVs d'atteindre la barre d'arrêt pendant la phase suivante. S'ils le peuvent, le nombre de AAVs sur l'approche j sera augmenté. Puis, le nombre de AAVs pour les deux approches dans cette phase sera utilisé dans l'étape d'optimisation des voies pour maximiser le débit. Après que le problème d'optimisation soit résolu, le IC enverra les informations de position et de voie à chaque AAV attendant la permission de traverser l'intersection.

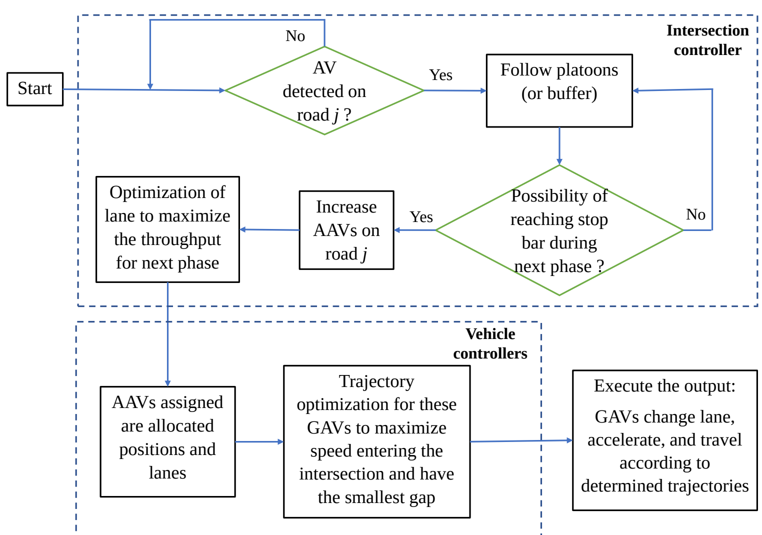
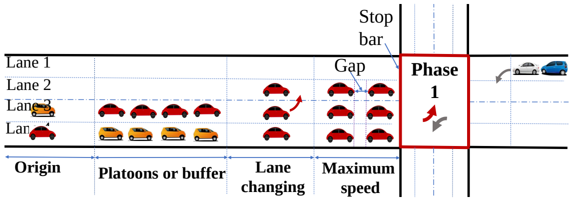
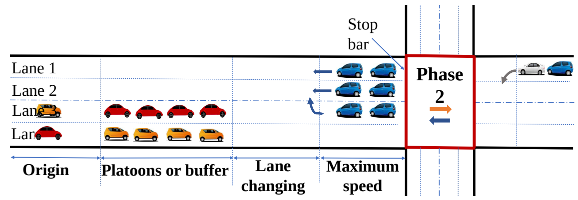


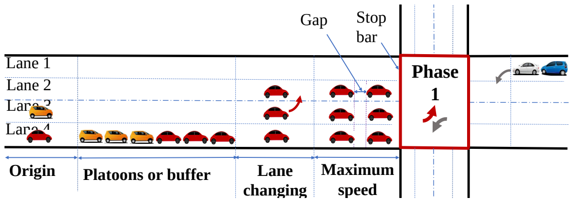
Figure 1.9: La disposition du cadre du contrôleur à deux niveaux.



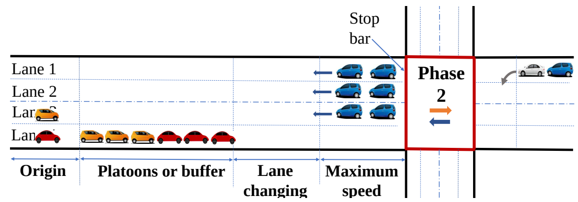
(a) Si l'approche j (amont) est en circulation fluide.



(b) Si l'approche j (aval) est en circulation fluide.



(c) Si l'approche j (amont) est congestionnée.



(d) Si l'approche j (aval) est congestionnée.

Figure 1.10: Distribution des positions des CAVs dans l'approche j.

Au niveau inférieur, les contrôleurs de véhicules (VCs) de ces GAVs calculeront leurs trajectoires par eux-mêmes afin d'atteindre la vitesse conseillée. De plus, ils communiquent et coopèrent entre eux afin d'avoir l'écart le plus petit. Ces trajectoires satisferont aux contraintes de sécurité (1.15-1.16). Enfin, les GAVs exécutent cette étape en suivant les étapes suivantes: changer de voie, accélérer et maintenir la vitesse maximale et l'écart minimal à la barre d'arrêt avec des contraintes de sécurité. D'autre part, si les AAVs ne reçoivent pas d'autorisation, ils doivent continuer à circuler en peloton et attendre le cycle suivant.

b) Méthode de distribution d'espace

Maintenant, nous décrivons comment allouer les positions des véhicules sur l'approche amont afin qu'ils puissent facilement changer de voie et accélérer sans bloquer les véhicules dans l'autre phase. À cette fin, l'approche j est divisée en quatre segments: Origine, peloton (ou tampon), changement de voie, et vitesse maximale (voir Fig. 1.10).

L'origine est le premier segment lorsque les véhicules entrent dans le réseau de manière aléatoire. Ensuite, il y a un segment de formation de peloton, où les véhicules sont organisés en peloton et circulent à la même vitesse sous contraintes de sécurité. Si la circulation sur l'approche j est fluide ou bien inférieure à celle des autres approches, les CAVs doivent être attribués à une seule voie comme indiqué dans la Fig. 1.10a. Les AAVs autorisés à traverser l'intersection lors de la phase suivante seront les plus proches de la barre d'arrêt. Cette allocation permettra aux autres approches congestionnées d'obtenir plus de voies afin d'augmenter leur débit. En revanche, si l'approche j est congestionnée, les CAVs peuvent donc être répartis sur deux voies comme dans la Fig. 1.10c. Dans les deux cas, les CAVs (c'est-à-dire, les véhicules rouges) ne sont pas bloqués par les autres (par exemple, les véhicules jaunes) et peuvent passer facilement au troisième segment. Une fois atteint le dernier segment, la vitesse des GAVs est presque maximale, et ils cherchent à réduire au minimum leurs écarts. Enfin, ils atteignent la barre d'arrêt à temps avec la vitesse maximale et l'écart minimal.

1.2.2.4 Résultats numériques

Revenons à l'intersection à 4 voies et 4 approches comme illustrées dans la Fig. 1.7, et les signaux de circulation pour les 4 phases sont prédéfinis comme dans la Fig. 1.11. L'ordre de séquence des 4 phases est le virage à gauche de l'ouest vers le nord, tout droit de l'ouest vers l'est, le virage à gauche du nord vers l'est, et tout droit du nord vers le sud. Il est important de respecter cet ordre car cela aide à éviter les collisions et donne du temps aux AAV de la phase suivante à se déplacer vers les positions assignées.

Les véhicules se déplacent à la vitesse maximale, qui est de 20 m/s, et à la vitesse de 3 m/s. Les accélérations maximales et minimales des véhicules sont respectivement fixées à 4,0 m/s^2 et $-3,0$ m/s^2. Au total, 36 véhicules sont envisagés dans ce test, dont 17 véhicules venant de l'ouest, 8 véhicules de l'est, 5 véhicules du nord et 6 véhicules du sud. Les CAVs rouges et blancs tournent à gauche à la phase 1, les oranges et les bleus vont tout droit à la phase 2, les jaunes et les verts tournent à gauche à la phase 3, et les noirs vont tout droit à la phase 4.

Pour comparer avec les résultats de l'état de l'art, quatre méthodes sont étudiées pour optimiser le débit comme suit: méthode des véhicules conduits par des humains (HVM), méthode non basée sur les voies (NLM), méthode basée sur les voies (LM), et méthode de distribution d'espace (SDM). En détail, la méthode HVM est uniquement utilisée pour les HVs sans communication entre eux, donc ils essaient seulement d'atteindre rapidement la barre d'arrêt. En comparaison, la NLM et la LM sont adoptées pour les CAVs et tentent d'éviter les vagues de stop-and-go en calculant leurs trajectoires de manière à ce que les CAVs puissent atteindre la barre d'arrêt à la vitesse maximale sous contraintes de sécurité. Cependant, la NLM n'optimise pas le nombre de voies, tandis que le LM le fait. La SDM calcule non seulement le nombre maximum de voies mais estime alloue également les positions des AAVs ainsi que de minimiser leurs écarts.

Table 1.4: Le nombre de voies attribuées et le débit de l'intersection pour chaque phase avec 4 méthodes: HVM, NLM, LM, et SDM

	Phase 1		Phase 2		Phase 3		Phase 4		NA
	WL	EL	WT	ET	NL	SL	NT	ST	
HVM (L)	1	1	1	1	1	1	1	1	
NA_{jp}	1	1	2	1	1	1	0	1	8
NLM (L)	1	1	2	2	1	1	1	1	
NA_{jp}	2	1	4	4	1	1	1	1	15
LM (L)	2	2	3	1	2	2	2	1	
NA_{jp}	3	2	7	2	2	2	2	1	21
SDM (L)	3	3	3	1	2	3	2	2	
NA_{jp}	4	3	9	3	2	3	2	2	28

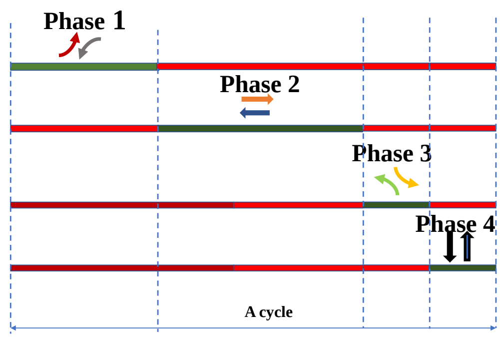
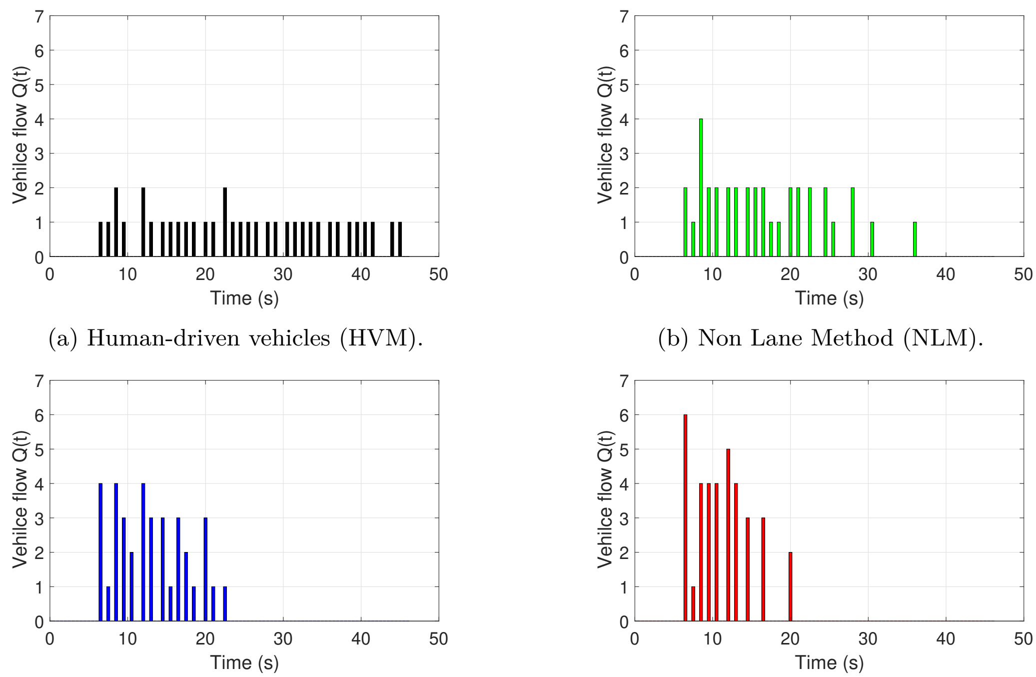


Figure 1.11: Un cycle prédéterminé pour les signaux de circulation.

Le tableau 1.4 affiche les meilleurs résultats des quatre méthodes dans chaque phase, où WL, ET, NL, etc., désignent l'approche amont dans chaque phase. C'est-à-dire que dans WL (ouest-gauche), les véhicules traversent l'intersection depuis l'ouest dans la phase de virage à gauche (phase 1); dans NT (nord-tout droit), les véhicules traversent l'intersection depuis le nord dans la phase de tout droit (phase 3). Il n'y a que deux approches amont qui permettent aux véhicules de traverser l'intersection dans cette phase, telles que l'ouest-gauche (WL) et l'est-gauche (EL) en phase 1. Dans chaque méthode, la première ligne décrit le nombre de voies attribuées aux approches amont à chaque phase. Par exemple, la SDM permet 3 voies pour WL et 3 voies pour EL de circuler en phase 1, mais il attribue 3 voies pour WT et seulement 1 voie pour ET en phase 2. D'autre part, la deuxième ligne présente le débit de l'approche amont j pendant la phase p (NA_{jp}) ainsi que le débit total (c'est-à-dire, dans la dernière colonne). Le tableau 1.4 montre que la SDM obtient toujours le débit le plus élevé ($NA = 28$) ainsi que le nombre de voies attribuées dans les quatre méthodes. Dans certains cas, le débit la SDM peut être presque deux ou trois fois supérieur à celui la NLM, notamment dans la phase de virage à gauche. La LM ($NA = 21$) est meilleur que la NLM



(a) Human-driven vehicles (HVM).

(b) Non Lane Method (NLM).

(c) Lane Method (LM).

(d) Space Distribution Method (SDM).

Figure 1.12: Débit de 4 méthodes: HVM, NLM, LM, SDM.

($NA = 15$) car il permet aux CAVs de circuler dans plus de voies. La vitesse moyenne, l'écart et les voies de la HVM ne peuvent pas être contrôlés, c'est pourquoi il a le débit le moins élevé ($NA = 8$).

En outre, la Fig. 1.12 illustre le débit $Q(t)$ au fil du temps dans quatre méthodes, à savoir: HVM, NLM, LM et SDM. Au cours de la première période (de 6 s à 16 s), la SDM a toujours le débit le plus élevé et le temps le plus court ($T_e = 21$ s) pour que le dernier véhicule sorte de l'intersection. La LM nécessite un peu plus de temps ($T_e = 23$ s) que la SDM, tandis que le NLM et la HVM nécessitent beaucoup plus de temps, c'est-à-dire respectivement $T_e = 36$ s et $T_e = 45$ s. La raison en est que la NLM et la LM n'exploitent pas l'avantage des CAV, tandis que la HVM n'est pas capable de contrôler la vitesse et la position des véhicules.

1.3 Remarques conclusives

Dans cette thèse de doctorat, nous avons développé plusieurs méthodes novatrices pour le contrôle des véhicules connectés et autonomes (CAVs) et leur coopération avec un centre de gestion du trafic (TMC). En particulier, nous avons apporté deux contributions de recherche importantes.

Tout d'abord, nous avons proposé la méthode de distribution d'espace coopérative (CSDM) pour réduire le temps de trajet moyen des CAVs changeant de voie à un goulet d'étranglement à voies multiples sur les autoroutes multi-voies. En utilisant cette méthode, les CAVs peuvent changer de voie simultanément et rapidement, ce qui permet de réduire le temps de trajet moyen à travers un goulet d'étranglement sans perdre l'équilibre de la densité entre les voies. Deuxièmement, nous avons développé une nouvelle méthode de distribution spatiale pour les opérations des CAVs à une intersection multi-voies signalisée avec des horaires de signaux prédéterminés. La conception proposée améliore considérablement le débit et réduit remarquablement le temps de trajet total.

Chapter 2

Introduction

2.1 Background and Motivation

With the rapid expansion of transportation networks, there is an increasing demand to develop and deploy intelligent traffic management technologies in many cities around the world. In fact, rapid urbanization and increasing vehicle ownership rates have contributed to many transportation challenges, primarily congestion and environmental pollution. Let us take Canada as an example. Traffic congestion in Canada is escalating, especially in its major cities, where drivers face more unfavorable travel times. Due to urban sprawl and decades of under-investment in public transit, it is reported that traffic congestion in the Toronto region costs Canada \$3.3 B in lost productivity a year. [1] In response to the rising traffic demand, innovative techniques have been studied to manage traffic better and enhance transportation efficiency. Notably, Intelligent Transportation Systems (ITS) have been gaining attraction in mitigating freeway congestion as illustrated in Fig. 2.1. Specifically, ITS refers to the application of information technology and communication technologies to the transportation system. It encapsulates various technologies and strategies — from advanced traffic signal control algorithms to real-time traffic reporting, from adaptive signal controls that respond to traffic conditions to dynamic message signs that provide vehicles with current traffic information. However, at the heart of these vast interconnected systems is a key entity: the Traffic Management Center (TMC). Here, data from various sources, such as cameras, sensors,

[1] https://www.sharetheroad.ca/the-economics-of-traffic-p135678

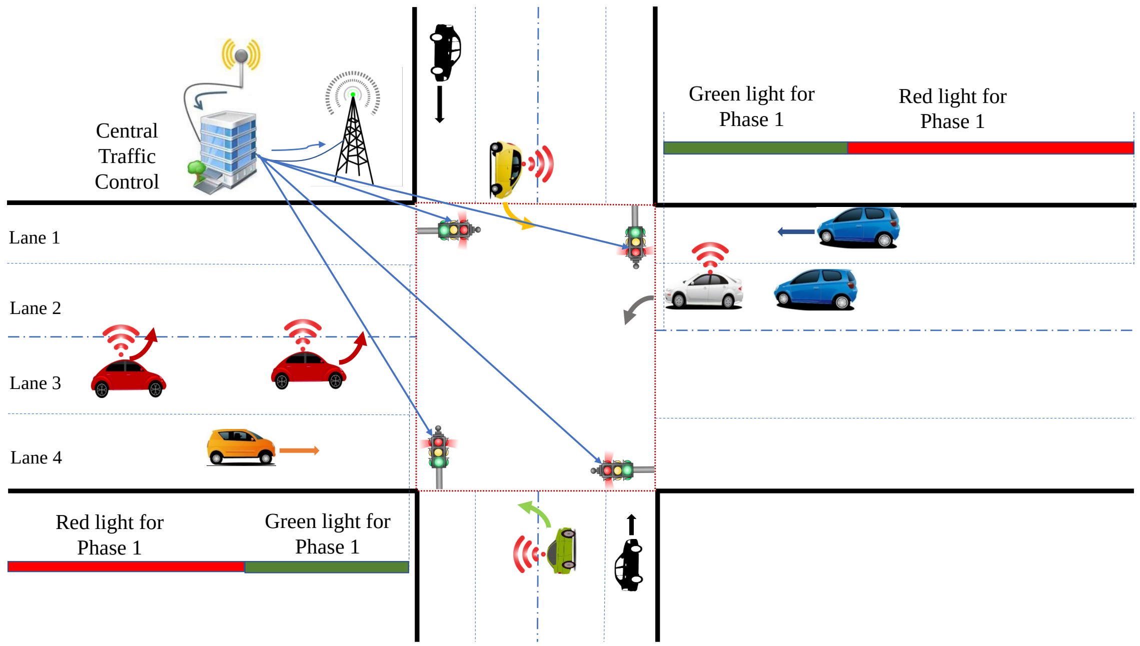


Figure 2.1: Intelligent transport system and architecture.

and detectors on roadways, is collected, processed, and transmitted. This center not only monitors traffic conditions but also proactively manages them.

Furthermore, the evolution of Connected and Automated Vehicles (CAVs) and advanced wireless technologies such as the fifth generation (5G) wireless technologies coupled with advanced algorithm controls will play important roles in enhancing safety and traffic flow. Typically, a CAV can gather data from its surroundings through vehicle-to-vehicle (V2V) or vehicle-to-infrastructure (V2I) communication (referred to as V2X communication). Based on the collected data, the vehicle can implement an appropriate control strategy to achieve specific goals, like minimizing travel time, ensuring constant spacing between vehicles, or enabling smoother driving patterns. This expands the driver's awareness of the environment beyond the line of sight. Moreover, the reliable and timely traffic data exchange enables collaboration between drivers and intersection controllers, creating cooperative intelligent transport systems (C-ITS). To support the development of C-ITS, IEEE 1609 standards, leveraging IEEE 802.11-based Dedicated Short Range Communication (DSRC), have been established [14]. This has given rise to various novel signal control strategies, as referenced in [15] and [16]. Recent research indicates that integrating CAVs into traffic flows can notably enhance road safety, optimize traffic operations, and promote environmental sustainability.

2.1.1 From Human-Driven Vehicles to Autonomous Vehicles in Various Congestion Scenarios

The escalating issues of traffic jams and accidents have become societal concerns, leading to significant consequences. It has been observed that drivers react differently to varying levels of road congestion. In less crowded situations, they typically adhere to traffic signals. However, in denser traffic conditions, many drivers tend to bypass signals, seizing any opportunity to maneuver through intersections. Such actions can create bottlenecks, further intensifying congestion and reducing the overall flow of traffic. A fundamental limitation of human-driven vehicles (HVs) is their reliance on visual signals for lane guidance and advisories. Therefore, it is difficult to force or guide human-driven vehicles to obey instructions far from a distance to mitigate congestion.

2.1.2 Control Methods for Connected Automated Vehicles on Multi-Lane Freeways

However, partly automated driving has already been commercially available for basic driving tasks such as accelerating and braking by means of adaptive cruise control (ACC) [1]. In fact, ACC systems are the first driver assistance systems with the potential to influence traffic flow characteristics. However, current implementations of ACC systems are exclusively designed to increase driving comfort, and the influence on the surrounding traffic is neither considered nor optimized. According to the Research and Innovative Technology Administration (RITA) of the U.S. Department of Transportation (USDOT), 81% of all vehicle-involved crashes can be avoided or significantly mitigated by leveraging connected vehicle technologies annually.

The concept of fully automated vehicle control allows for very small time gaps and platoon driving, which is the key to greater capacity. Moreover, Connected and Autonomous vehicles (CAVs) promise a lot of societal advantages. First of all, they have the potential to dramatically decrease accidents, given that human errors account for over 90% of them [2]. Economically, CAVs can lead to reduced travel expenses. For instance, their integration into public transportation systems can cut labor costs and alleviate issues like traffic congestion, accident-related costs, environmental degradation, and security concerns [3]. Besides, they can also solve urban traffic issues due to human driving misbehaviors such as traffic rule violations and needless lane switches [3]. From an environmental perspective, CAVs can enhance fuel efficiency and reduce carbon emissions [4]. Lastly, the AV industry is estimated to be around 0.2 trillion dollars annually by 2025 [5]. However, such systems need special infrastructure and dedicated lanes, something which can only be justified if the percentage of automated vehicles is sufficiently high.

The operations of CAVs have received substantial interest, leading to the development of numerous techniques to coordinate the dynamics of CAVs with central intersection management, aiming to decrease travel time, energy cost, and environmental pollution as shown in Fig. 2.1. Dresner *et al.* [6] and Perronnet *et al.* [7] introduced a reservation method for vehicles crossing two intersecting routes. Essentially, intersections are segmented into a tile grid, and vehicles send their tile requirements for their intended route to the Intersection Controller (IC) before entry. If the IC identifies no conflicts with other vehicles, it allocates the necessary tiles and a time slot. However, if conflicts arise, the request is denied, these vehicles are forced to slow down and wait for a new reservation approval. In connected traffic scenarios, Inter-Vehicle Communication (IVC) aids CAVs in platoons by providing timely infrastructure information, such as data from roadside controllers. These vehicles then utilize a suitable control mechanism to meet specific goals, like keeping a constant vehicle headway in platoons [8, 17] or minimizing total travel time and emissions [9, 18].

While extensive research has focused on the efficacy of CAVs within traffic systems in recent years [10], there has been limited investigation of CAV dynamics on multi-lane freeways. Specifically, the complications of lane changing and merging under freeway safety constraints remain challenging due to the complex lane-changing behaviors of vehicles approaching the bottleneck. As vehicles approach a lane-drop bottleneck or merging point, those in the terminating lane are forced to change lanes. However, due to a lack of coordination with vehicles in adjacent lanes, they might not find a safe gap during the lane changing. It can result in lowered speeds, leading to potential queue formation [11], subsequently decreasing the bottleneck's throughput or capacity. Such scenarios often result in collisions or traffic jams at these bottlenecks [12].

Cooperating CAVs with the Traffic Management Center (TMC) to enable smooth CAV trajectories crossing a signalized multi-lane intersection (SMLI) or traveling lane-drop bottleneck on multi-lane freeways (LDB) without collisions are the research focal points of this dissertation. To address these challenges, we study control strategies that enable the integration of these systems, specifically, a Space Distribution Method (SDM) for SMLI and a Cooperative Space Distribution Method for LDBs. In the following, we discuss research challenges related to these design problems.

2.2 Research Challenges

Advanced ITSs exploiting CAVs and TMC require sophisticated cooperation methods and efficient control strategies to guide vehicles traveling smoothly and avoiding collisions. However, various research challenges must be tackled to effectively control and cooperate CAVs at multi-lane freeways. We discuss some major challenges in the following.

2.2.1 Controlling CAVs at a Signalized Multi-lane Intersection

There has been quite extensive research on intersection control methods for CAVs, which encompasses the combined control of signal timings and trajectory planning, focusing specifically on lane-based control at signalized multi-lane intersections. It is worth mentioning that the problem of lane-based control has been introduced in [19], [20], and [21] for human-driven vehicles (HVs) at an isolated signal-controlled intersection. However, these approaches come with their technical challenges.

Firstly, the speeds and positions of human-driven vehicles (HVs) are out of our control, so their lane-based optimization methods should be based on passive control. Secondly, most existing techniques can only be applied for one-directional movements on the road, such as straightforward or left/right turning. Thus, these techniques may not be suitable for scenarios with multi-directional movements, such as a combination of straightforward and left-turning traffic.

The final issue is that most existing infrastructure may not be ready for intelligent intersection control of the CAVs. Essential information such as queue information, stop bar positions, conflict zones, and obstructions from leading cars with different intentions are often absent. For instance, in scenarios where one direction might have more CAVs than others, the green light time for this direction should be longer. Moreover, a specific vehicle could be stopped crossing the intersection due to the preceding left-turning vehicles. Such issues can result in lower throughput, potentially escalating to serious traffic congestion in certain worst cases, such as spill-back scenarios. Consequently,

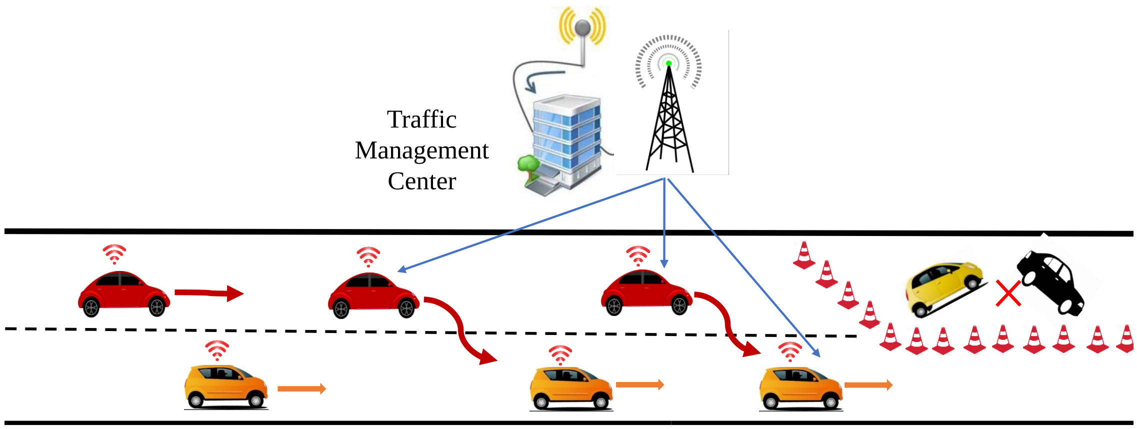


Figure 2.2: A land-drop bottleneck.

an efficient control strategy that appropriately integrates CAV control and TMC cooperation is necessary to allow the vehicles to travel smoothly and safely.

2.2.2 Cooperating CAVs at a Lane-Drop Bottleneck on Multi-Lane Freeways

Most existing research has primarily addressed the lane-changing or merging issues on two-lane freeways. As a result, solving these problems for multi-lane freeways with more than two lanes demands further studies. We outline some major challenges related to these research problems in the following.

The first challenge is related to the optimization of the lane-changing and merging processes considering freeway safety constraints at lane-drop/merging bottlenecks. This is non-trivial because of the complex lane-changing behaviors of the vehicles upstream of the bottleneck, as illustrated in Fig. 2.2. In fact, when vehicles travel to the (lane-drop) bottleneck (i.e., the merging point), those in the dropped lane must change lanes. However, due to the lack of cooperation with other vehicles in other lanes, there may not be an available safe gap during the lane-changing process. Hence, the speeds of these vehicles might reduce, and queues might be formed [11], reducing the throughput or capacity of the bottleneck. Car crashes and/or congestion are also critical problems at such bottlenecks [12].

The second challenge concerns the computational aspect. Specifically, deployed control methods must be able to rapidly and precisely compute the CAVs' trajectories (i.e., longitudinal and lateral trajectories) while maintaining safety conditions. As the numbers of vehicles and lanes increase, the computational complexity of these calculation tasks typically grows exponentially. Therefore, the development of a computation-efficient technique to determine control decisions for a large-scale freeway control problem is challenging. Furthermore, these methods should not only optimize cooperative merging and lane-changing processes but also ensure a balanced traffic flow downstream.

2.3 Literature Review

In this section, we survey the existing literature related to our research studies. Firstly, we describe related works on control strategies for connected and autonomous vehicles at an isolated intersection. Secondly, we present control methods for connected and autonomous vehicles at a lane-drop bottleneck. Thirdly, existing works on bi-level controls of multi-lane freeways are surveyed.

2.3.1 Control Strategies for Connected and Autonomous Vehicles at an Isolated Intersection

The development of effective operation strategies for CAVs has received significant attention. In particular, different methods have been developed to coordinate the dynamics of the CAVs and the central intersection management to reduce the total travel time, energy consumption, and pollution. Dresner *et al.* [6] and Perronnet *et al.* [7] proposed a reservation scheme to guide vehicles traveling on two directions of two approaches. Specifically, an intersection is divided into a grid of tiles, and vehicles need to send the requested tiles on their planned route to the IC before crossing the intersection. Once the request for the vehicle is accepted, if there is no conflict with other vehicles, the IC will allocate the tiles and the time slot for it. Otherwise, the request is denied, so the vehicle might need to decelerate and wait until a new reservation request is accepted.

Jin *et al.* [22] studied a platoon formation problem for intersection control by coordinating adjacent CAVs. In the proposed method, the leader and followers exchange the trajectory information (e.g., speed and position) with the IC to calculate the time to cross the intersection. After that, the platoon leader and followers shall optimize their trajectories to obey the assigned schedule under safety constraints. Generally, the platoon formation provides better traffic operations and leads to more efficient control strategies, as reported in [17, 23].

In the platoon-based approach developed in [18], a receding horizon model predictive control method was proposed to minimize the fuel consumption for platoons and drive the platoons to pass the intersection during a green phase. The method is then extended to dynamic platoon splitting and merging rules for the cooperation of CAVs and conventional vehicles in response to the high variation of urban traffic flow. In [24] and [25], parsimonious shooting heuristic algorithms were proposed to optimize the trajectories of a stream of CAVs, considering multiple objective functions such as fuel consumption and travel time.

To prevent a car accident, Alonso *et al.* [26] authorized CAVs themselves to decide a suitable time interval to cross the intersection. The CAVs must share information about their state, such as position, velocity, driving direction, and order. After that, they use the priority tables to determine whether individual vehicles should keep traveling or stop until the junction is clear. In the next phase, the look-up table is renewed when each vehicle updates its own priority. The decentralized method was proposed in [27], which determines the best vehicle sequence to go through the intersection. It is based on predicting the arrival times of the cars in the queue, then allowing the one having the shortest arrival time to cross and prohibiting the others from crossing by sending messages to them. Avoiding collision among vehicles simultaneously crossing a multi-lane intersection could be achieved by adding an additional logic.

Lin and Liu [28] proposed a coordination method in which the road is divided into three sections, and the vehicles pass through the intersection at maximum speed at exactly the calculated time. Moreover, these vehicles follow the planned trajectories to minimize fuel consumption. Markarem *et al.* [29] proposed an alternative control method in which CAVs coordinate with one another at the intersection to reduce energy consumption and pollution. Kamal *et al.* [30] developed a vehicle-intersection coordination scheme without using any traffic lights where all CAVs can pass through the intersection safely and smoothly. Yan [31], Lee [32], and J. Wu [33] presented optimization formulations to minimize the total travel time considering safety constraints. They applied the dynamic programming (DP) method to find sub-optimal solutions for the problems. Because the control optimization problem for multi-lane intersections is very difficult to solve with the DP method, the authors utilized Petri nets to model the system and then optimize the sum of the two queues' lengths.

2.3.2 Control Methods for Connected and Autonomous Vehicles at A Lane-Drop Bottleneck

There are several control strategies proposed for regular vehicles at a bottleneck such as Variable Speed Limit (VSL) [34], [35], [36], Ramp Metering (RM) [37], [38], feedback control strategy [39], [40], [41]. Carlson *et al.* [42] used VSL to prevent traffic breakdown by regulating the mainline flow, which is smaller than the bottleneck capacity. Similarly, RM can maintain the high outflow by limiting the inflow to bottleneck [43]. Besides, Han *et al.* [44] proposed another method called Variable Speed Release (VSR) to decrease the probability of traffic breakdown. Im addition, they controlled vehicle trajectories to increase the buffer headway so that the system throughput can be improved. Furthermore, Jula *et al.* [45] studied the kinematics of vehicles changing/merging lanes and calculated the minimum longitudinal spacing in specific scenarios to avoid traffic collision.

It has been recognized that cooperative Adaptive Cruise Control [46], [47], [48], [49] is a promising method for cooperation among vehicles and allows to optimize vehicles' maneuver based on the conditions of the adjacent vehicles. Various methods have been proposed in [50], [51], and [52] to coordinate the CAVs and a management center to cut down the total travel time. Besides, a model predictive control (MPC) based method has been proposed to optimize the vehicle trajectories through the coordinated merging algorithm for CAVs, and it has been tested in specific cases [53]. It is worth noting that avoiding stop-and-go at the bottleneck can significantly reduce the travel time [54].

However, most existing research only solves the lane-changing or merging problems on two-lane freeways. Desiraju *et al.* [55] proposed an approach for large-scale freeways by solving the problem in a distributed manner with affordable complexity. Hu *et al.* [56] developed an algorithm that not only optimizes the cooperative merging and lane-changing operations but also balances the downstream outflow.

There have been other optimization or control methods for vehicle trajectory planning, such as the one developed in [57] for which the computational demand usually increases exponentially with the number of vehicles. Li *et al.* [13] proposed a two-stage motion planning framework in which the connected and automated vehicles could simultaneously change lanes without any collision. Therefore, the collision-avoidance constraints for the longitudinal trajectory are not imposed, reducing the computation demand. Another way to reduce the computational burden is to group CAVs in platoons. Rajamani *et al.* [58] designed an integrated longitudinal and lateral control system for the leading CAV while the followers try to maintain minimal inter-vehicle spacing and travel at high velocity. A similar platoon-based approach was investigated in [23].

2.3.3 Bi-level Controller Algorithms for Vehicles on Multi-lane Freeways

A bi-level control framework for a crossroad was described in [59], in which the lower-level controller collects and evaluates the information of traffic flow to specify a control policy that stabilizes vehicle flows. The upper-level controller first utilizes a consensus algorithm to calculate the desired traffic density and then determines the speed of each vehicle. The adjacent intersections then share states of their traffic density with one another to boost the system's throughput. Along this line, Zhao *et al.* [60] formulated the intersection control and vehicle trajectory planning problems as bi-level programs, in which the upper-level mixed integer linear program aims to minimize the total travel time. In contrast, the lower-level linear program maximizes the total speed of vehicles entering the intersection. The two-level problems are coupled by the arrival time and the terminal speed. Schmidt *et al.* [61] developed a similar bi-level control method, in which the upper layer assumes that the speeds of the vehicles are equal and estimates their time and order to merge in the control section. By using heuristic rules in the second layer, the acceleration for each vehicle is optimized to avoid crashes during the merging process.

Tlig *et al.* [62] used a centralized controller to cooperate with the vehicles crossing the intersection alternately. Each vehicle calculates its own trajectory that coincides with three segments: a deceleration segment, a constant speed segment, and an acceleration segment. The deceleration and acceleration of the vehicle are determined; thus, each vehicle only optimizes the speed for the constant speed segment to minimize the total travel time and energy consumption. The authors extended the work and studied a bi-level control framework in which intersections share information and adjust their phases to improve the traffic flow efficiency [63]. Besides, Ntousakis *et al.* [64] used the bi-level control method to enable each vehicle to move through the bottleneck smoothly and safely without any congestion interference. Specifically, the task of the roadside controller (RC) at the upper level is to update the merging instructions (e.g., acceleration, sequence, time), while the lower-level controller makes sure that the vehicle is following the instructions from the upper-level one.

2.4 Research Objectives and Contributions

The general objective of my Ph.D. research is to develop effective control and collaboration methods for CAVs to minimize their overall travel time and ensure safety with affordable computational complexity. Specifically, the main contributions of our research can be outlined as follows:

1. A Cooperative Space Distribution Method for Connected and Autonomous Vehicles at A Lane-Drop Bottleneck on Multi-Lane Freeways

   We develop a cooperative space distribution method (CSDM) to improve the traffic flow of CAVs at a lane-drop bottleneck on general multi-lane freeways. A general cooperative method is proposed to increase the average velocity of CAVs on general multi-lane freeways. Specifically, we develop control strategies by using our CSDM for CAVs crossing the lane-drop with the following advantages: significantly increasing throughput, efficiently balancing the downstream outflow among lanes, allowing vehicles to change lanes simultaneously, and reducing the lane-changing time.

   More specifically, we propose a novel framework where the region of interest of the freeway is divided into three segments: i) platoon segment, ii) acceleration segment, and iii) merging segment. In the first segment, CAVs travel together in the platoon to guarantee their safety; they will then speed up to attain the maximum velocity and reach the determined position in the second segment. Finally, these CAVs change lanes in the last segment and pass through the bottleneck with maximum velocity and minimum gap. Simulation results demonstrate the performance of the proposed method, where we show that the proposed framework can significantly decrease the average travel time of all vehicles.

2. Space Distribution Method for Connected and Autonomous Vehicles at A Signalized Multi-lane Intersection

   We propose a bi-level control framework to optimize the intersection throughput. In our proposed method, the upper level (i.e., the intersection controller) is used to optimize the lane usage of each approach and the CAVs' positions. The lower level (i.e., the vehicle controllers) receives information from the upper level to control the CAVs to reach the maximum speed. More specifically, in the upper-level, we apply a novel Space Distribution Method (SDM) for the CAVs to maximize the throughput of the multi-lane intersection where signal timings are predefined. The SDM is divided into three steps: platoon formulation, lane-mode optimization, and CAVs' position distribution. To maximize the throughput, the intersection controller receives information about the states of the CAVs (e.g., the trajectories), then optimizes the lane usages for each approach, the desired speed, and the gap of the CAVs as well as the CAV's position along the approach. After that, each CAV that is allowed to cross the intersection determines its own trajectory and travels at the scheduled time without a crash. Numerical simulations are conducted to show that the throughput increases significantly, even more than twice the throughput obtained from other methods in some circumstances.

2.5 Dissertation Outline

The remainder of this dissertation is organized as follows. Chapter 3 provides some fundamental backgrounds and important dynamic models used in the dissertation. In Chapter 4, we present the cooperative space distribution method for connected and autonomous vehicles at a lane-drop bottleneck on multi-lane freeways. Then, we study the space distribution method for connected and autonomous vehicles at a signalized multi-lane intersection in Chapter 5. In Chapter 6, we summarize the main contributions of the dissertation and make some recommendations for future research.

Chapter 3

Background

This chapter presents some fundamentals of vehicle dynamic models and explores two pivotal ITS problems: intersection and merging control on multi-lane freeways. Specifically, we discuss various trajectory planning methodologies, which are used to address these two problems in this dissertation.

3.1 Kinematic Dynamic Models

3.1.1 Global Kinematic Vehicle Model

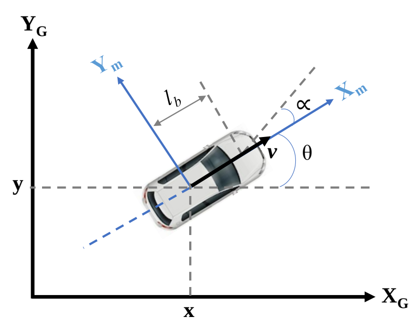


Figure 3.1: The variables in the global reference frame.

The global kinematic model of vehicles is based on the well-known tricycle model [65]. In Fig. 3.1, we illustrate the variables associated with a particular vehicle in the global reference frame

X_G, Y_G. In more detail, the global kinematic vehicle model is presented as follows:

$$\dot{x}(t) = v(t)\cos(\theta(t)), \tag{3.1}$$

$$\dot{y}(t) = v(t)\sin(\theta(t)), \tag{3.2}$$

$$\dot{\theta}(t) = v(t)\sin(\alpha(t))/l_b, \tag{3.3}$$

where $(x(t),\ y(t),\ \theta(t))$ are the vehicle positions in the global reference frame; $v(t)$ and $\alpha(t)$ are the linear velocity and orientation of the vehicle front wheel, respectively; and l_b is the wheelbase of the vehicle.

3.1.2 Lane-Changing Model

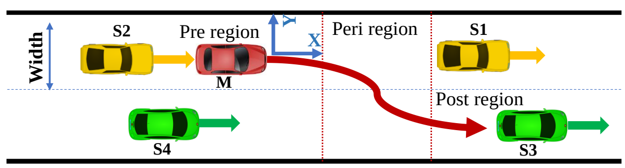


Figure 3.2: Lane-changing process.

To ensure collision avoidance, vehicle M's trajectory needs to be carefully planned to maintain appropriate safety distances from other vehicles in its vicinity. Regarding its longitudinal movement, this means that vehicle M should remain within a defined safety corridor, setting both upper and lower boundaries on its longitudinal position during the execution of specific maneuvers, like a lane change.

In more detail, the authors of [66] propose to arrange the lane-changing process into three regions: Pre, Peri, and Post segment as shown in Fig. 3.2. Let us define $x_{Sk}(t), v_{xSk}(t), s_m(Sk), \forall k = 1, ..., 4$, as the longitudinal position and velocity, and safe gap of vehicle Sk, respectively. In addition, $x_M(t), y_M(t), v_{xM}(t), v_{yM}(t), a_{xM}(t), a_{yM}(t)$ represent the longitudinal, lateral position and velocity, and acceleration of the vehicle M, respectively; and LW denotes the lane width.

Firstly, in the time interval $T_{Pre} = (0, t_{Pre})$ of the Pre region in the original lane, vehicle M must maintain the safe gap with the leading $S1$ and following $S2$ before initializing the lateral movement. Then, it is in the time interval $T_{Pe} = (t_{Pre}, t_{Pre} + t_{Pe})$ of the Peri region in the original lane, it must be bounded by upper bound $S1$ and $S3$, and lower bound $S2$ and $S4$. Finally, it has an upper bound $S3$ and a lower bound $S4$ in the Post region. So, the upper and lower bound on M's longitudinal position in each region can be determined as

$$x_{\max}(t) = x_{S1}(t) - s_m(S1), \qquad \forall t \in T_{Pre}, \tag{3.4a}$$

$$x_{\min}(t) = x_{S2}(t) + s_m(S2), \qquad \forall t \in T_{Pre}, \tag{3.4b}$$

$$x_{\max}(t) = \min\left\{x_{S1}(t) - s_m(S1), x_{S3}(t) - s_m(S3)\right\}, \qquad \forall t \in T_{Pe} \tag{3.4c}$$

$$x_{\min}(t) = \max\left\{x_{S2}(t) + s_m(S2), x_{S4}(t) + s_m(S4)\right\}, \qquad \forall t \in T_{Pe} \tag{3.4d}$$

$$x_{\max}(t) = x_3(t) - s_m(S1), \qquad \forall t \in T_{Po}, \tag{3.4e}$$

$$x_{\min}(t) = x_{S4}(t) + s_m(S4), \qquad \forall t \in T_{Po}, \tag{3.4f}$$

where $s_m(Sk)$, T_{Sk}, and LM_{Sk} respectively denote the safety margin, the time gap, and the minimum distance that vehicle M must maintain for each Sk. So, the safety margin $s_m(Sk)$ can be defined as

$$s_m(Sk) = \max\left\{T_{Sk} v_{xSk}, LM_{Sk}\right\}. \tag{3.5}$$

Besides, the set of constraints for the lane-changing model is as

$$x_{\max}(t) \geq x_M(t) \geq x_{\min}(t), \qquad \forall t, \tag{3.6a}$$

$$V_{\max} \geq v_{xM}(t) \geq V_{\min}, \qquad \forall t, \tag{3.6b}$$

$$a_{x\max} \geq a_{xM}(t) \geq a_{x\min}, \qquad \forall t. \tag{3.6c}$$

According to the model in [67], the lateral constraints and the limit of the side-slip angle for $\forall t \in T_{Pe}$ can be written as:

$$y_{\max} \geq y_M(t) \geq y_{\min}, \qquad \forall t, \tag{3.7a}$$

$$v_{y\max}(t) \geq v_{yM}(t) \geq V_{\min}, \qquad \forall t, \tag{3.7b}$$

$$a_{y\max} \geq a_{yM}(t) \geq a_{y\min}, \qquad \forall t, \tag{3.7c}$$

$$0.17 * v_{xM}(t) \geq v_{yM}(t) \geq -0.17 * v_{xM}(t), \qquad \forall t, \tag{3.7d}$$

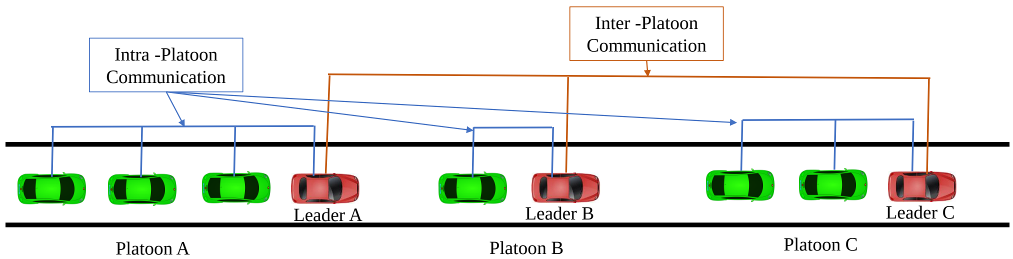


Figure 3.3: Inter-vehicle communication.

where $a_{y\,\max}$ and $a_{y\,\min}$ are the maximum and minimum lateral accelerations of a vehicle, respectively. Note that the coefficient 0.17 in equation (3.7d) reflects the side-slip angle effect between the longitudinal and lateral velocity.

3.2 Merging Control Problem on Freeways

Merging on freeways, especially under heavy traffic or at high velocities, can be challenging for drivers. Merging control on freeways is an important area of research for connected and automated vehicles (CAVs). Algorithms dedicated to merging control in CAVs aim to enhance both the safety and efficiency of these maneuvers, particularly in high-traffic zones or bottlenecks. They typically rely on a combination of inter-vehicle communication and control algorithms to coordinate the movement of vehicles during merging maneuvers. By enabling real-time information exchanges about position, speed, and acceleration, CAVs can make informed decisions, predicting and responding to the actions of surrounding vehicles. In the following, we briefly present the control algorithms for merging vehicles on freeways.

3.2.1 Platooning with Merging Control Algorithm

A platoon of connected autonomous vehicles (CAVs) is a group of vehicles that travel close to each other, with a small gap between vehicles [68]. This cooperative driving approach primarily aims to increase traffic flow efficiency and mitigate congestion, especially in bottleneck zones and high-traffic regions [69]. The key advantage of platooning lies in its potential to minimize aerodynamic drag, leading to notable improvements in fuel consumption rates and decreased emissions. Besides, platooning can reduce the space required for each vehicle, so road capacity and congestion can be improved.

The smooth functioning of platooning relies on a combination of communication systems and control algorithms. In a typical platoon, the lead vehicle acts as the primary communicator and communicates with other vehicles using wireless communication technologies, such as Dedicated Short-Range Communications (DSRC) or Cellular Vehicle-to-Everything (C-V2X). The lead vehicle transmits speed and trajectory information, enabling the following vehicles to adapt their speed and position to maintain the safe gap throughout the platoon.

To reduce the channels between CAVs, the merging control algorithm [70] has two communication layers. The first layer focuses on intra-platoon communication, which involves information exchange among vehicles within the same platoon. The second layer, known as inter-platoon communication, facilitates communication between the lead vehicles of different platoons, as illustrated in Fig. 3.3.

3.2.2 Two-Stage Multi-vehicle Motion Planning - 2SMVMP

In practical scenarios, computational complexity typically grows exponentially with the number of vehicles due to the complexity of the two-dimensional optimization problem (i.e., longitudinal

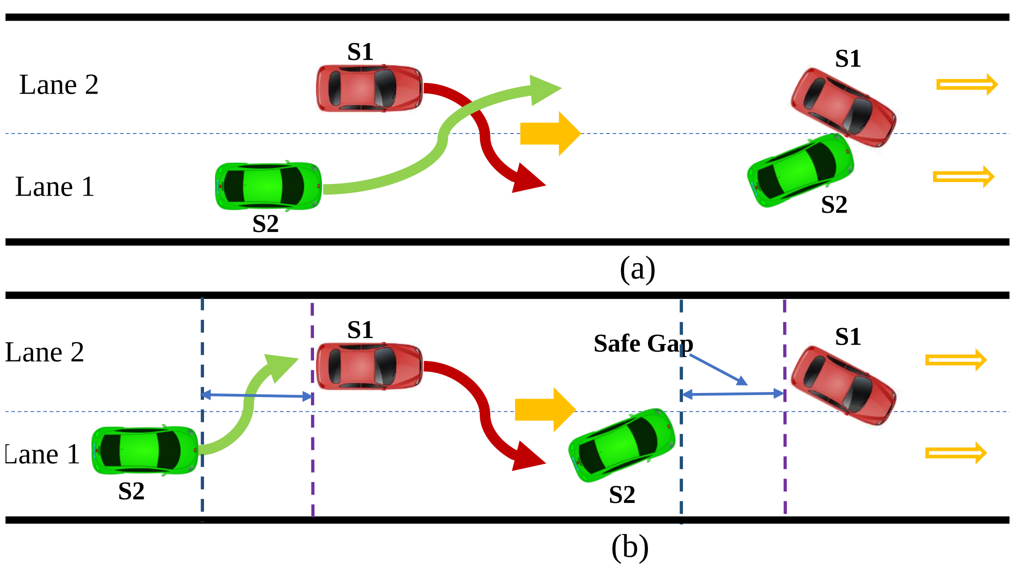


Figure 3.4: Schematics of MVMP method to achieve simultaneous lange changes: (a) the lane-change trajectories of both vehicles are conflicting; and (b) if the mutual gap is widened, then both vehicles can change lanes safely and simultaneously.

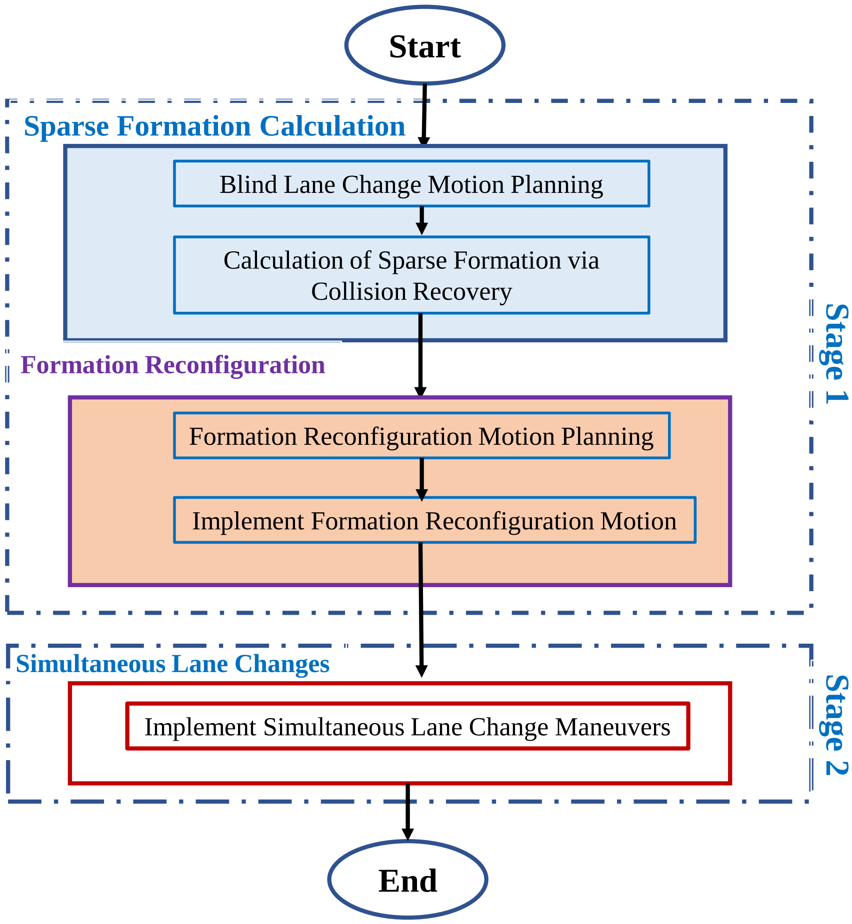


Figure 3.5: The layout of the two-stage MVMP method.

and lateral). Various methods have been proposed to transform the underlying optimization problem and solve it in a decentralized manner to swiftly find solutions in real-time. The two-stage multi-vehicle motion planning (2SMVMP) method developed in [13] ensures that connected and automated vehicles can change lanes simultaneously without collisions. As a result, there is no need to incorporate collision-avoidance constraints for the longitudinal trajectory, which in turn reduces computational demands.

In this method, the entire lane-changing process is divided into two distinct stages. In the first stage, vehicles do not change lanes. Instead, they adjust their speeds to expand the gaps between them, as shown in Fig. 3.4. These gaps should be large enough so that in the second stage, vehicles can execute simultaneous lane-change maneuvers without considering collision-avoidance constraints. The structure of the two-stage MVMP method can be visualized in Fig. 3.5. The kinematic system to a longitudinal system of CAVs can be represented as

$$\dot{x}_i(t) = v_i(t),, \tag{3.8}$$

$$\dot{y}_i(t) = 0, \tag{3.9}$$

$$\dot{v}_i(t) = a_i(t), \tag{3.10}$$

$$\dot{a}_i(t) = jerk_i(t). \tag{3.11}$$

The collision-avoidance constraints among all the CAVs are reduced to

$$x_{i,l}(t) - x_{i+1,l}(t) \geq L, \forall t \in [0, t_{sp}], \tag{3.12}$$

where $jerk_i(t)$ denotes the time derivative of acceleration; L is the length of vehicle; t_{sp} is the time when the sparse formation is reconfigured.

Then, the formation reconfiguration problem can be formulated as:

$$\begin{aligned} \min \quad & t_{sp} \\ \text{s.t} \quad & \text{constraints } (3.8), (3.10), (3.11) \\ & \text{and } (3.12). \end{aligned} \tag{3.13}$$

3.3 Intersection Control Problem

Intersection control plays a pivotal role in transportation systems, facilitating vehicles' and pedestrians' safety and orderly flow. Moreover, traffic lights have traditionally been an effective means of managing traffic in areas with human-driven vehicles. Connected and autonomous vehicles have the potential to improve intersection control by enabling more precise and coordinated movement of vehicles than that due to human-driven vehicles. The underlying control algorithms use shared information to generate optimal trajectories for individual vehicles based on their intended speeds, positions, and the dynamics of surrounding traffic. Such algorithms prioritize safety and efficiency, emphasizing collision avoidance, safe following distances, and minimizing disruptions to overall traffic flow. In the following, we introduce the bi-level model framework and the buffer-assignment mechanism as a fundamental method to coordinate CAVs at intersections.

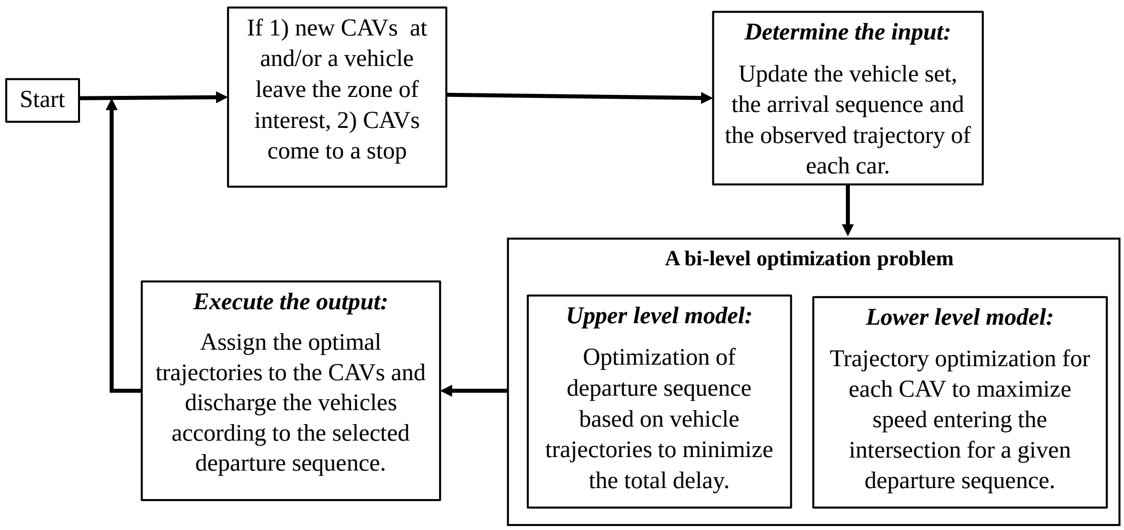


Figure 3.6: The layout of the bi-level model framework.

3.3.1 Bi-Level Model Framework

The structure of the bi-level controller model [1] is illustrated in Fig. 3.6. The algorithm's input comprises the vehicle set N, which includes all connected and automated vehicles (CAVs) within a zone of interest. This zone of interest is defined as the area where CAVs can exchange information regarding the intersection. The algorithm outputs an optimal departure sequence for all vehicles in the vehicle set and customized trajectories for each CAV in this zone. The objective of the algorithm is to calculate the optimal departure sequence and devise optimized trajectories for all vehicles in real-time.

In each decision step, a bi-level optimization model is employed. The upper level focuses on minimizing total delay by optimizing the departure sequence based on current vehicle trajectories. The lower level model then optimizes the trajectory of each CAV within the given departure sequence, aiming to maximize their speed as they approach the intersection. These two levels are interdependent: the lower-level model receives the departure sequence and provides vehicle trajectory feedback to the upper-level model.

In the final stage, vehicles are discharged according to the determined departure sequence, with automated vehicles following their designated trajectories. The traffic signal for an approach is assumed to turn green just before the first vehicle in that sequence enters the intersection.

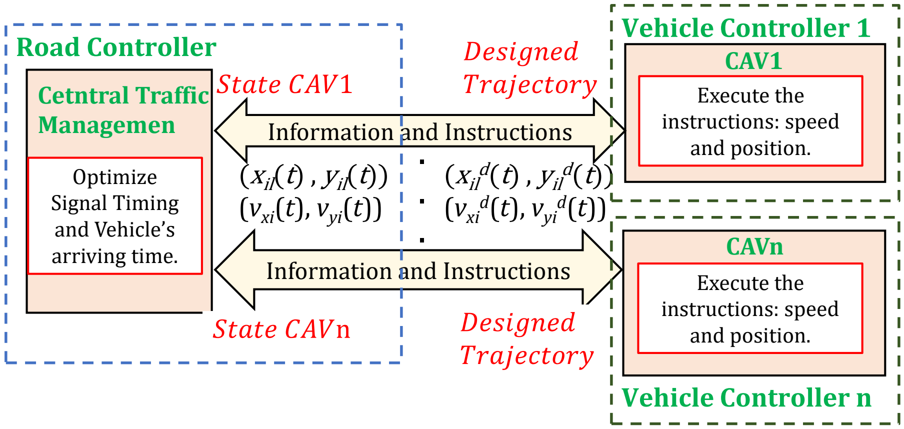


Figure 3.7: Overview of the cooperation of traffic signal and vehicles.

Similarly, [71] described the cooperative method, including two-level controllers, as in Fig.3.7. This system gathers speed and position data from approaching vehicles and computes the optimal traffic signal timings. Its goal is to plan the vehicles' arrival times to minimize the total travel time of all vehicles at the macro-intersection level. Once these optimized arrival times are determined, they are sent to each vehicle. Subsequently, each vehicle's onboard control system chooses the optimal speed trajectory, focusing on optimizing engine power and braking force to minimize fuel consumption in the whole vehicle trip at the micro-vehicle level.

3.3.2 Buffer-Assignment Mechanism

The fundamental objective of the buffer-assignment mechanism [28] is to ensure CAVs smoothly cross through intersections without compromising safety. At its core, the buffer-assignment mechanism strategically allocates time and space within certain conflict sections (CSs) that are potential hotspots for accidents or traffic congestions. In more detail, a vehicle's trajectory within an intersection can be visualized as a fixed-width strip region. This strip region is further segmented into identical units along its longitudinal direction. The overall layout of the intersection with these designations is illustrated in Fig. 3.8.

At each entrance of the intersection, two distinct stop lines are established to handle assignment failures and human-driven vehicles (HVs). In particular, the first stop line aligns with the actual traffic lights, which are governed by the traffic management center (TMC) and cater exclusively to HVs. The second stop line corresponds with virtual traffic lights, specifically for CAVs. In addition,

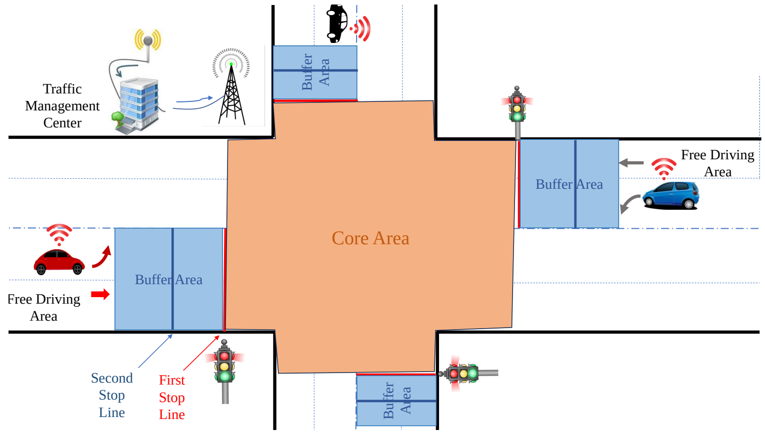


Figure 3.8: Buffer assignment mechanism for an isolated intersection.

the vicinity of the intersection is strategically sectioned into several areas, such as the Core Area, Buffer Area, and Free Driving Area. Firstly, the Core Area encompasses the intersection's interior, where CAVs can establish a connection to the TMC and travel at a constant speed. Secondly, the Buffer Area (BA) lies directly outside the intersection, serving as an approach zone. Here, CAVs can still connect with the TMC, which coordinates their entry timing and speed into the Core Area, but they are not allowed to change lanes within the BA. Finally, the Free Driving Area is a zone where either human-driven vehicles or connected autonomous vehicles travel without coordination.

To ensure CAVs cross through the intersection smoothly and safely, two key tasks are essential. First, the TMC allocates a specific crossing span to each CAV and transmits the respective instructions. Second, the CAV is expected to obey these instructions to encounter a given crossing span. The synchronization of these tasks relies on the time occupancy model for conflict sections and the trajectory control model designed for connected autonomous vehicles. Under the buffer-assignment strategy, CAVs' movements are coordinated in the following manner. In the BA, each CAV adjusts its acceleration to match the predetermined entry time and speed set for the core area. As a result, vehicles can avoid the traditional stop-and-go cycle and eliminate any potential conflicts.

Chapter 4

A Cooperative Space Distribution Method for Autonomous Vehicles at A Lane-Drop Bottleneck on Multi-Lane Freeways

The content of this chapter was published in IEEE Transactions on Intelligent Transportation Systems in the following paper:

Tung Thanh Phan, Long Bao Le, and Dong Ngoduy “A cooperative space distribution method for autonomous vehicles at a lane-drop bottleneck on multi-lane freeways,” *IEEE Trans. Intell. Transp. Syst,* vol. 23, no. 4, pp. 3710–3723, Apr. 2022.

4.1 Abstract

With the help of inter-vehicle communication (IVC), connected and autonomous vehicles (CAVs) can drive cooperatively, thus significantly improving road safety, traffic efficiency, and environmental sustainability. While substantial research has been conducted to investigate the efficiency of CAVs in transportation systems, few attempts have been carried out to explore the lane-changing operations

of CAVs in multi-lane freeways, especially the dynamics of CAVs at lane-drop bottlenecks. To this end, this paper aims to develop a cooperative space distribution method (CSDM) not only to increase the lane-drop bottleneck's throughput but also to equally distribute the CAVs in the dropped lane to other lanes by efficiently coordinating the dynamics of CAVs upstream of the bottleneck in a multi-lane freeway. More specifically, we propose a novel framework where the freeway (region of interest) is divided into three segments: i) platoon segment, ii) acceleration segment, and iii) merging segment. In the first segment, CAVs travel together in the platoon to guarantee their safety; they will then speed up to attain the maximum velocity and reach the determined position in the second segment. Finally, these CAVs change lanes in the last segment and pass through the bottleneck with maximum velocity and minimum gap (i.e., gap distance). Simulation results are tested to demonstrate the performance of the proposed method, where we show that the proposed framework can significantly decrease the average travel time of all vehicles.

4.2 Introduction

In the connected traffic flow, with the help of the IVC, CAVs in the platoon can timely obtain information from the infrastructure (e.g., roadside controllers), and then adopt a suitable control strategy to achieve a specific objective such as maintaining a constant vehicle headway within the same platoon [8, 17] or minimizing the total travel time and emissions [9, 18]. Despite intense research devoted to studying the efficiency of CAVs in traffic systems over the last few years [10], very few attempts have been conducted to study the dynamics of CAVs on multi-lane freeways. Notably, the process of changing lanes and merging under freeway safety constraints at lane-drop/merging bottlenecks is a challenging problem due to the complex lane-changing behaviors of the vehicles upstream of the bottleneck. When the vehicles travel to the (lane-drop) bottleneck (i.e., the merging point), those in the dropped lane must change lanes. Due to a lack of cooperation with other vehicles in other lanes, however, there may not be an available safe gap during the lane-changing process. Hence, their speeds might reduce, and the queues might be formed [11]; consequently, the throughput or capacity of the bottleneck decreases. Car crashes and/or congestion are usually the main traffic problems at such bottlenecks [12].

4.2.1 Related Works

To overcome these issues, there are several control strategies proposed for regular vehicles such as Variable Speed Limit (VSL) [34], [35], [36], Ramp Metering (RM) [37], [38], feedback control strategy [39], [40], [41], etc. Carlson *et al.* [42] used VSL to prevent traffic breakdown by regulating the mainline flow, which is smaller than bottleneck capacity. Similarly, RM keeps maintaining the high outflow by limiting the inflow to bottleneck [43]. Besides, Han *et al.* [44] proposed another method called Variable Speed Release (VSR) to decrease the probability of traffic breakdown. They controlled vehicle trajectories to increase the buffer headway so that the system throughput could be improved. Furthermore, the study by Jula *et al.* [45] mentioned the kinematics of vehicles changing/merging lanes and calculated the minimum longitudinal spacing in specific scenarios to avoid traffic collision.

Cooperative Adaptive Cruise Control [46], [47], [48], [49] system is a promising method for cooperation among vehicles to optimize their maneuver based on the conditions of the adjacent vehicles. Many methods [50], [51], and [52] suggested to coordinate the CAVs and a management center to cut down the total travel time. Besides, a model predictive control (MPC) method, based on the coordinated merging algorithm for CAVs, has been proposed to optimize the vehicle trajectories and has been tested in specific cases [53]. It is worth noting that avoiding stop-and-go at the bottleneck significantly reduces the travel time [54]. Therefore, Ntousakis *et al.* [64] used the bi-level controller method to allow each vehicle to move through the bottleneck smoothly and safely without any congestion interference. Specifically, the task of the roadside controller RC (upper level) is to update the merging instructions (e.g., acceleration, sequence, time), while the lower level makes sure that the vehicle is following the instructions from the upper level. Furthermore, Phan *et al.* [72] developed the upper-level controller of this method by optimizing the lane usages for each road and CAV's position to significantly improve the throughput at the intersections.

To the best of our knowledge, most existing research only solves the problems of lane-changing or merging on two-lane freeways. Desiraju *et al.* [55] proposed an approach for large-scale freeways by solving the problem in a distributed manner, which could bring down the complexity of the problem. Hu *et al.* [56] developed an algorithm that not only optimizes the cooperative merging and lane-changing problem but also balances the downstream outflow. Furthermore, there are other optimization or control methods for vehicle trajectory planning such as the one developed in [57].

In such methods, the computational demand usually increases exponentially with the number of vehicles. Li *et al.* [13] proposed the two-stage motion planning framework in which the connected and automated vehicles could simultaneously change lanes without any collision. Therefore, the collision-avoidance constraints for the longitudinal trajectory are not involved; the computation demand is thus reduced. Another way to reduce the computational burden is by grouping the CAVs in platoons. Rajamani *et al.* [58] designed the integrated longitudinal and lateral control system for the leading CAV, while the followers try to maintain minimal inter-vehicle spacing and travel at high velocity. A similar approach (i.e., platoon-based) was investigated in [23].

4.2.2 Contributions and Organization of the Paper

This paper departs from the above literature by proposing a cooperative space distribution method (CSDM) to maximize the average velocity of CAVs (which will also decrease the total travel time) at a lane-drop bottleneck on general multi-lane freeways (e.g., 4-lane freeways). To manage such a complex lane-changing process upstream of the lane-drop bottleneck, our proposed CSDM will control the velocity and the position of these CAVs and then actively generate safe gaps for the vehicles in the dropped lane to change lanes safely and quickly. More specifically, the freeway is divided into three segments: platoon segment, acceleration segment, and merging segment. Then, this paper proposes the following steps to deal with the CAVs operations of the lane-drop problem: i) In the first segment, CAVs travel together in the platoon under safety constraints, ii) they then will speed up to maximize their velocity and reach the determined position in the second segment. Finally, iii) they change lanes in the last segment and pass through the bottleneck with maximum velocity and minimum gaps.

In particular, we develop control strategies by using our CSDM for CAVs crossing the lane drop with the following advantages:

- Aiming to achieve as high throughput as possible (or equivalently as small total travel time of all CAVs as possible) by attempting to maximize the CAVs' velocity and minimize their gaps when crossing the bottleneck;
- Performing explicit planning and control of CAVs' trajectories (both longitudinal and lateral movements) to enable CAVs to change lanes while maintaining safety constraints;

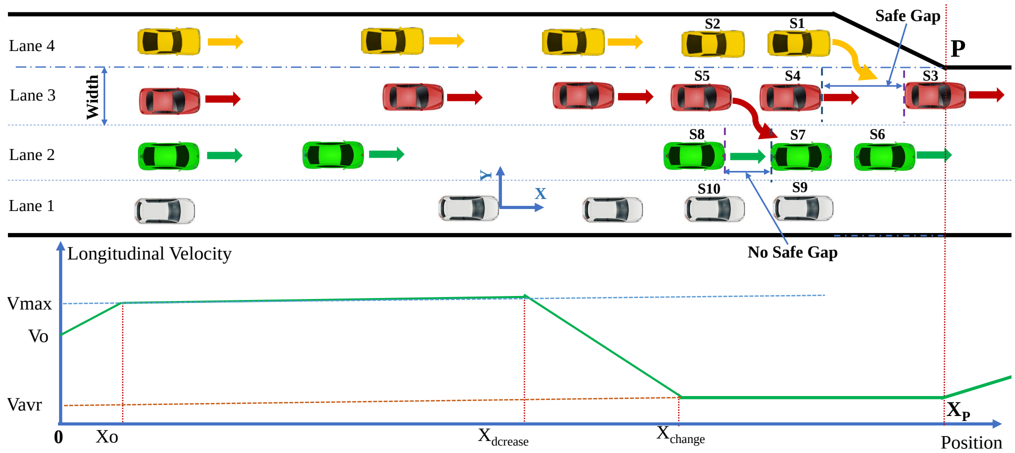


Figure 4.1: The normal CAV's velocity in a bottleneck.

- Distributing CAVs from the dropped lane to other lanes to efficiently balance the downstream outflow among lanes;
- Providing efficient mechanisms to control the priority vehicles traveling to their desired position;
- Reducing the lane-changing time and allowing vehicles to change lanes simultaneously without considering their longitudinal trajectory constraints in this process, allows us to reduce the computational complexity.

The rest of this paper is organized as follows. Section 4.3 describes a vehicle dynamic model and our design objective considering both the longitudinal and lateral trajectory of the vehicles. Section 4.4 presents an overview of a general design principle for our problem. The main parts of this paper are Section 4.5 and 4.6 in which we propose the CSDM for two-lane freeways and three different control strategies for multi-lane freeways, respectively. Section 4.7 presents simulation results in several typical scenarios. Finally, we conclude the paper in Section 4.8.

4.3 Vehicle Dynamic Model

We consider a normal situation for a multi-lane freeway in which there is a lane drop (e.g., lane 4 as in Fig. 4.1) so that all vehicles in this lane must change lanes. In the ensuing section, we will describe the kinematic vehicle model and show the control variables and design objective of the problem dealing with the CAV operations.

The general trajectory of a vehicle is composed of two movements: longitudinal movement and lateral movement. In the vehicle trajectory control problem, the vehicles are considered with only two maneuvers: the lane-keeping process and the lane-changing process.

4.3.1 Lane-Keeping Process

If CAVs travel in the same lane (lane-keeping), their longitudinal trajectory satisfies the following relations:

$$\dot{x}_{il}(t) = v_{xil}(t), \tag{4.1a}$$

$$\dot{v}_{xil}(t) = a_{xil}(t), \tag{4.1b}$$

$$\dot{y}_{il}(t) = 0, \tag{4.1c}$$

where $x_{il}(t)$ and $x_{(i-1)l}(t)$ are longitudinal positions of vehicle i and $(i-1)$ in lane l, respectively; $y_{il}(t)$ is lateral position of vehicle i in lane l; $v_{xil}(t)$ and $a_{xil}(t)$ are (longitudinal) velocity and acceleration of vehicle i, respectively.

The safety constraints for their movement can be described as:

$$x_{il}(t) - x_{(i-1)l}(t) - L = g_{il}(t) \geq T * v_{xil}(t), \tag{4.2a}$$

$$V_{\max} \geq v_{xil}(t) \geq V_{\min}, \tag{4.2b}$$

$$a_{x\max} \geq a_{xil}(t) \geq a_{x\min}, \tag{4.2c}$$

where L is a vehicle length; $g_{il}(t)$ is the gap (distance) between vehicle i and $(i-1)$ in lane l; T is a time gap for vehicle i (follower), where $T = 1s$; $V_{\max}$ and $V_{\min}$ are upper and lower bounds of vehicles' velocity; $a_{x\max}$ and $a_{x\min}$ are maximum and minimum longitudinal acceleration of vehicles, respectively.

The inequality (4.2a) describes the lower limit of the gap between the adjacent CAVs; (4.2b) and (4.2c) are the constraints about the range of velocity and acceleration of the CAVs, respectively.

4.3.2 Lane-Changing Process

When a particular vehicle M wants to change lanes, it must find a safe gap in the target lane or keep waiting until these safety conditions are met. The safe gap must be long enough to allow the vehicle to change lanes without colliding with the others.

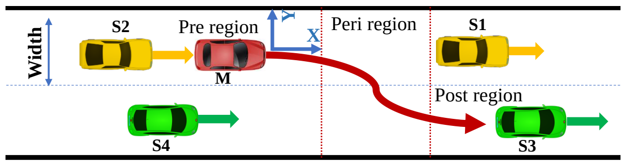


Figure 4.2: Lane-changing process.

In more detail, [66] proposes to arrange the lane-changing process into three regions: Pre, Peri, and Post segment as in Fig. 4.2. Let us define $x_{Sk}(t), v_{xSk}(t), s_m(Sk), \forall k = 1, ..4$ the longitudinal position and velocity, and safe gap of vehicle Sk, respectively; while $x_M(t), y_M(t), v_{xM}(t), v_{yM}(t), a_{xM}(t)$, $a_{yM}(t)$ describe the longitudinal, lateral position and velocity, and acceleration of the vehicle M, respectively; and LW is the lane width.

First, in the time interval $T_{Pre} = (0, t_{Pre})$ of the Pre region of the original lane, vehicle M must satisfy the safe gap to the leading $S1$ and following $S2$ before initializing the lateral movement. So we can determine the upper and lower bounds of states of vehicle M as:

$$x_{\max}(t) = x_{S1}(t) - s_m(S1) - L, \qquad \forall t \in T_{Pre}, \tag{4.3a}$$

$$x_{\min}(t) = x_{S2}(t) + s_m(S2) + L, \qquad \forall t \in T_{Pre}, \tag{4.3b}$$

$$v_{yM}(t) = 0, \qquad \forall t \in T_{Pre}. \tag{4.3c}$$

Therefore, the set of constraints in the Pre region reads:

$$x_{\max}(t) \geq x_M(t) \geq x_{\min}(t), \qquad \forall t \in T_{Pre}, \tag{4.4a}$$

$$V_{\max} \geq v_{xM}(t) \geq V_{\min}, \qquad \forall t \in T_{Pre}, \tag{4.4b}$$

$$a_{x\max} \geq a_{xM}(t) \geq a_{x\min}, \qquad \forall t \in T_{Pre}. \tag{4.4c}$$

Second, when vehicle M is in the time interval $T_{Pe} = (t_{Pre}, t_{Pre} + t_{Pe})$ of the Peri region of the original lane, it must be bounded by upper bound $S1$ and $S3$, and lower bound $S2$ and $S4$. Hence, we have $\forall t \in T_{Pe}$:

$$x_{\max}(t) = \min\left\{x_{S1}(t) - s_m(S1), x_{S3}(t) - s_m(S3)\right\} - L, \tag{4.5a}$$

$$x_{\min}(t) = \max\left\{x_{S2}(t) + s_m(S2), x_{S4}(t) + s_m(S4)\right\} + L. \tag{4.5b}$$

The set of longitudinal constraints of the Peri region is:

$$x_{\max}(t) \geq x_M(t) \geq x_{\min}(t), \qquad \forall t \in T_{Pe}, \tag{4.6a}$$

$$V_{\max} \geq v_{xM}(t) \geq V_{\min}, \qquad \forall t \in T_{Pe}, \tag{4.6b}$$

$$a_{x\max} \geq a_{xM}(t) \geq a_{x\min}, \qquad \forall t \in T_{Pe}. \tag{4.6c}$$

To follow the model in [66], the lateral constraints and the limit of the side-slip angle for $\forall t \in T_{Pe}$ are:

$$LW \geq y_M(t) \geq 0, \tag{4.7a}$$

$$y_M(t = t_{Pre}) = 0, \tag{4.7b}$$

$$y_M(t = t_{Pre} + t_{Pe}) = LW, \tag{4.7c}$$

$$0.17 * v_{xM}(t) \geq v_{yM}(t) \geq V_{\min}, \tag{4.7d}$$

$$v_{yM}(t = t_{Pre}) = v_{yM}(t = t_{Pre} + t_{Pe}) = 0, \tag{4.7e}$$

$$a_{y\max} \geq a_{yM}(t) \geq a_{y\min}, \tag{4.7f}$$

where $a_{y\max}$ and $a_{y\min}$ are the maximum and minimum lateral acceleration of vehicles, respectively. Note that the coefficient 0.17 in equation (4.7d) reflects the side-slip angle effect between the longitudinal and lateral velocity [66].

Last, the Post region has similar equations and constraints as the Pre region by replacing $(S1, S2)$ with $(S3, S4)$ for $\forall t \in T_{Po} = (t_{Pre} + t_{Pe}, t_{Pre} + t_{Pe} + t_{Po})$.

To simplify the safety constraints, we set:

$$s_m(Sk) = T * v_{xSk}(t), \forall k = 1, ..., 4. \tag{4.8}$$

Denote TF the final time when vehicle M completes the lane-changing process. Vehicle M may wait for some time TW until there is sufficient safe gap to allow it to change lanes. So, the total time required to complete this lane-changing process is:

$$TF = TW + t_{Pre} + t_{Pe} + t_{Po}. \tag{4.9}$$

4.3.3 Lane-Drop Constraints

In Fig. 4.1, P and X_P are the merging point and its longitudinal position, respectively. In order to keep traveling, all vehicles in lane 4 must move to other lanes before reaching point P. Denote $\mathcal{M}_l$ the set of all vehicles in lane l, we have the following constraint:

$$x_{i4}(t) < X_P, \forall i \in \mathcal{M}_4. \tag{4.10}$$

The throughput Q^P at the merging point P (i.e., bottleneck throughput) is defined as follows:

$$Q^P = \int_0^{T_o} Q(t)dt = \sum_{l=1}^{3} \int_0^{T_o} Q_l(t)dt, \tag{4.11}$$

where $Q(t)$ is the flow rate per second (i.e., the number of CAVs crossing the bottleneck in a second); $Q_l(t)$ is the flow rate per second from lane l; T_o is the time period considered in the problem.

4.3.4 Control Variables and Design Objective

Consider the lane-drop problem, as illustrated in Fig. 4.1. Let $\mathcal{V} = \{a_{xil}(t), a_{yil}(t), \forall i, l\}$ denote the set of control variables (longitudinal and lateral acceleration) of all vehicles. The objective of our design is to control all CAVs (by finding their designed trajectories) to achieve a maximum throughput (or minimum total travel time) at the bottleneck while meeting the safety conditions. In more detail, we will calculate $\mathcal{V}$ for each CAV to increase throughput at the bottleneck Q^P and satisfy the kinematic vehicle model (constraints (4.2a) to (4.7f)) and the lane-drop constraint (constraint (4.10)).

4.4 General Design Principle

In a normal situation, unequipped vehicles often travel with maximum velocity when they are far from the bottleneck; then, they may slow down and change lanes when seeing a roadblock [73], so the queue could be formed. In consequence, the average velocity of these vehicles crossing the bottleneck decreases, and the flow rate $Q(t)$ (or throughput Q^P) is low.

It is worth noting that i) increasing flow rate $Q(t)$ as soon as possible would decrease the total travel time of all vehicles; ii) maximum throughput is achieved when all vehicles cross through the bottleneck with maximum velocity and minimum gaps [72]:

$$v_l^A(t) = V_{\max}, \tag{4.12a}$$

$$g_l^{\max} = g_l^{\min} = T * v_l^A(t), \tag{4.12b}$$

where v_l^A is the average velocity of vehicles crossing the merging point P from lane l; $g_l^{\min}$ and $g_l^{\max}$ are minimum and maximum gaps between adjacent vehicles in lane l, respectively.

Therefore, the flow rate $Q(t)$ (or Q^P) depends on the trajectories of all vehicles (i.e., the velocity and position) and how to coordinate them. To solve this vehicle trajectory control problem, we should control all vehicles so that they cross the bottleneck with maximum velocity and minimum gaps.

To this end, we create the segment right before the bottleneck (called the merging segment) in which the vehicles, after changing lanes, could travel with maximum velocity and minimum gaps between adjacent vehicles in the same lane. In fact, if the number of vehicles coming to the bottleneck (inflow) is larger than the number that can get out of the bottleneck (outflow), it could lead to a decrease in the average velocity of the vehicles in the merging segment. For this reason, a (buffer) segment is necessary to store the incoming traffic so that we can efficiently regulate the outflow traffic. To reduce fuel consumption and the amount of computational demand, these vehicles should travel in platoons, so this segment is called the platoon segment. It is obvious that the velocities of vehicles in the platoon segment are often smaller than theirs in the merging segment; therefore, we need a middle segment to allow the vehicles in the platoon segment to accelerate to maximum velocity. Besides, the vehicles in the middle segment should be guided to

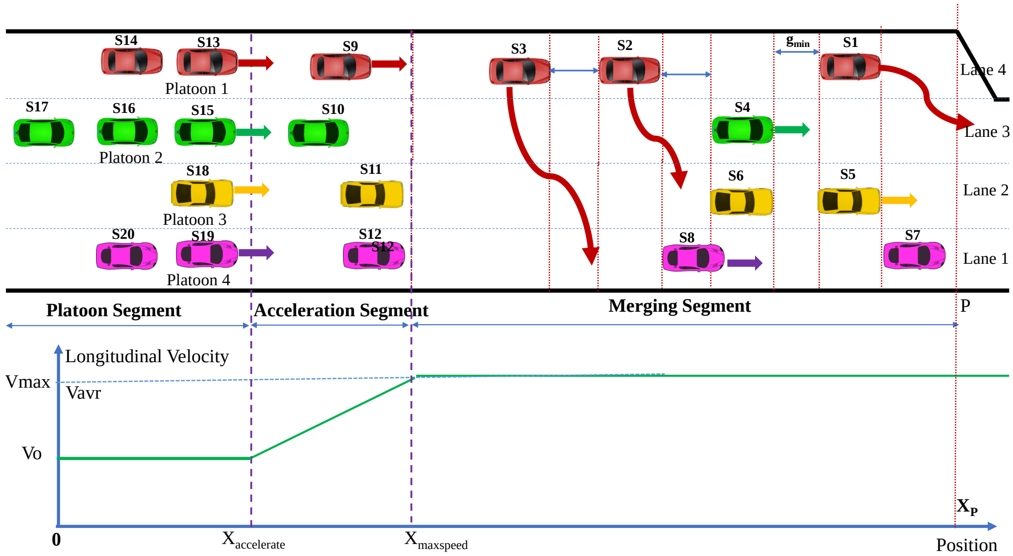


Figure 4.3: The design principle for controlling CAVs.

suitable positions so that they can change lanes without collision in the merging segment. The middle segment is called the acceleration segment.

Consequently, the freeway (region of interest) is divided into three segments: platoon segment, accelerations segment, and merging segment, as in Fig. 4.3. First, the vehicles S13-S20 approaching the bottleneck travel in platoons in the platoon segment. They (S9-S12), then, accelerate to attain the maximum velocity and reach the suitable positions in the acceleration segment. Finally, they (S1-S8) change lanes without collisions and cross the bottleneck with maximum velocity and minimum gaps between them at the merging segment.

Therefore, major design challenges are how to determine the suitable positions for vehicles at the end of the acceleration segment so that they can change lanes without collision, and then achieve maximum velocity in the last segment. In the next section, therefore, we will propose a CAVs' trajectory control solution for a two-lane-drop scenario (i.e. two-lane freeways) by determining the best positions and velocity (ideal situation) for CAVs in the merging segment. Moreover, for multi-lane highways with more than two lanes, balancing outflow traffic density among the lanes is important to achieve good throughput efficiency [56]. Therefore, we apply this design principle to tackle the more general multi-lane-drop scenario (e.g., four-lane freeways).

Remark 4.1. *In fact, our design focuses on the system control strategy to plan the trajectories of all vehicles crossing the bottleneck. In practice, the proposed framework can be realized by the bi-level* *controller method (i.e., the system controller of the higher level is deployed at the management center, and the vehicle controllers of the lower level are deployed at CAVs). However, detailed designs of such bi-level controllers are not the focus of our paper.*

4.5 Solution for the Two-Lane Freeways

The main goal of our design is to efficiently control the trajectories of all CAVs to achieve the maximum throughput (or minimum total travel time) at the bottleneck while meeting the safety constraints. Toward this end, we first describe our proposed method (CSDM) that guides vehicles in the last segment to change lanes safely and reach the maximum velocity and minimum gaps between them before crossing the bottleneck for the two-lane freeway problem. Then, we show that our proposed method indeed allows CAVs to meet these conditions perfectly.

4.5.1 CSDM for the Two-Lane Freeways

As described in Section 4.4: if we could divide a two-lane freeway into three segments and guide CAVs to a suitable situation (i.e., suitable position and velocity) in the third segment (merging segment), then both the average velocity and the flow rates will increase remarkably, and the total travel time will decrease.

Therefore, we will present a novel cooperative space distribution method (CSDM) for the CAVs to control their average velocity and minimize their gaps at the merging point, as in Fig. 4.4. Accordingly, the freeway is divided into three segments: platoon segment, acceleration segment, and merging segment. In the platoon segment, CAVs travel in a platoon to reduce fuel consumption and computational demand. In the acceleration segment, CAVs will speed up and then be allocated at the positions, as in Fig. 4.4, where their gaps are minimum. All of them must reach the maximum velocity before the red CAVs change lanes in the last segment.

Now, we will explain why reaching certain desirable states (position and velocity) of CAVs in the third segment could bring advantages such as improving throughput and allowing them to change lanes simultaneously, quickly, and safely.

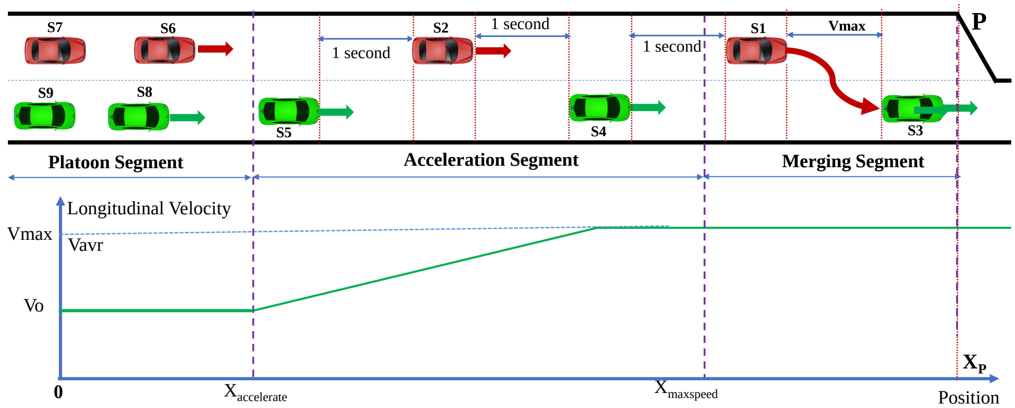


Figure 4.4: The CSDM method for merging two lanes.

4.5.2 Lane-Changing Problem

Note that most of the existing papers in the literature do not consider the final time TF. This is because most papers consider the two-lane highway or assume that the lane width is small. However, if vehicles change multiple lanes then the distance corresponding to this lateral movement is large, and it cannot be neglected. Besides, when vehicles traverse many lanes, the velocity of their following vehicles could be bounded to maintain safety constraints. Therefore, we will show the impacts of two variables TF and $v_{xM}(t)$ of the lane-changing process of each vehicle M (in the dropped lane) on the throughput. Then, we present two propositions in which we describe why certain ideal positions and velocities of CAVs in the merging segment could allow these vehicles to change lanes simultaneously, quickly, and safely.

4.5.2.1 Impacts of TF and $v_{xM}(t)$

Let us consider the lane-changing process, as in Fig. 4.2. We note that the TF and $v_{xM}(t)$ for $\forall t \in (0, TF)$ of vehicle M have negative impacts on the travel time of its following vehicles (e.g., S2 and S4) because of the safety constraints (4.2a - 4.7f). Specifically, if vehicle M wants to change lanes, vehicles $S2$ and $S4$ must maintain their safe gaps with it to avoid a collision. If the velocity of $S2$ is higher than that of M ($v_{xS2}(t) \geq v_{xM}(t)$), or the gap between vehicle $S4$ and M is not safe, $S2$ or $S4$ has to decrease its velocity, as in Fig. 4.5. Similarly, the followers of vehicle $S2$ (e.g. $S5$ and $S7$) and vehicle $S4$ (e.g. $S6$ and $S8$) may also have to reduce their velocities to maintain safe gaps. Therefore, the velocity of M is the upper bound of the velocities of its followers (i.e., $S2$, $S4$-$S8$) during the lane-changing process of vehicle M. Consequently, the larger final time TF and

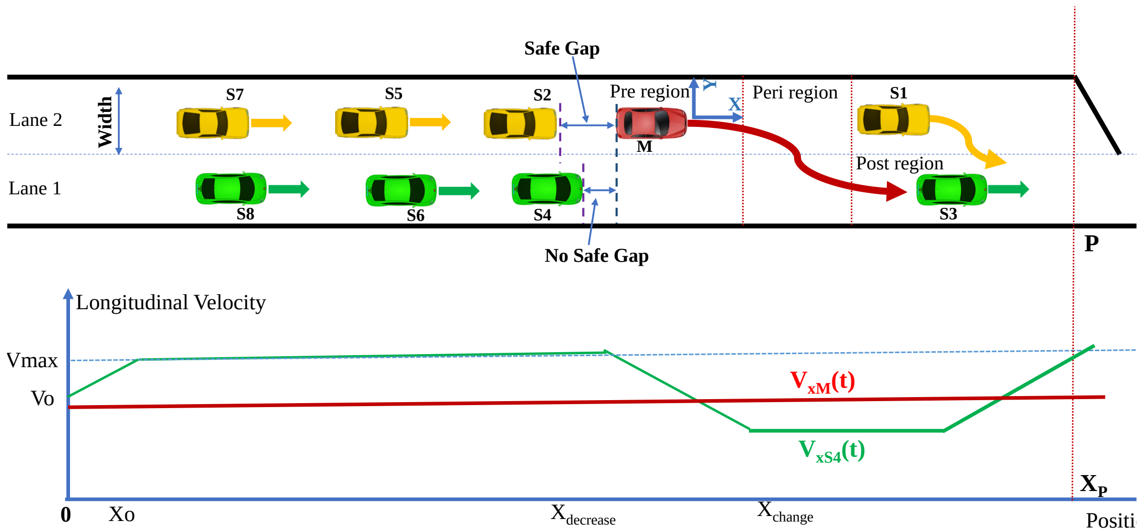


Figure 4.5: Velocity of M and $S4$ in the lane-changing process.

smaller velocity $v_{xM}(t)$ of vehicle M will imply the smaller speed upper bound (or, the longer travel time) for its following vehicles. Besides, the larger the value of TF, the higher the probability of a car crash between vehicle M and its surrounding ones, or the longer the required length of the merging segment in the freeway region of interest.

Therefore, we would want to coordinate and control all CAVs to allow vehicle M to change lanes quickly (i.e., minimizing TF) and achieve the highest velocity (i.e., maximizing $v_{xM}(t), \forall t \in (0, TF)$) when completing the lane-changing process.

4.5.2.2 Minimum Time t_{Pe}

In the lane-changing process, the time t_{Pe} in the Peri region often corresponds to a major portion of TF (i.e., $TF \geq t_{Pe} \geq t_{Pe}^{min}$). Therefore, it is necessary to minimize t_{Pe} in order to minimize TF. We will study potential factors affecting t_{Pe} of vehicle M during the lane-changing process.

Proposition 4.1. *If vehicle M travels in the Peri region, the minimum time t_{Pe} depends on the lane width LW, the maximum lateral acceleration a_{ymax}, the longitudinal velocity $v_{xM}(t)$, and the maximum longitudinal velocity $V_{\max}$.The highest probability of minimizing t_{Pe} could be achieved if vehicle M travels with maximum longitudinal velocity.*

It is challenging to find all ideal solutions to satisfy two requirements (i.e., minimizing TF and maximizing $v_{xM}(t)$) for all possible lane-changing scenarios. However, it can be shown that if CAVs

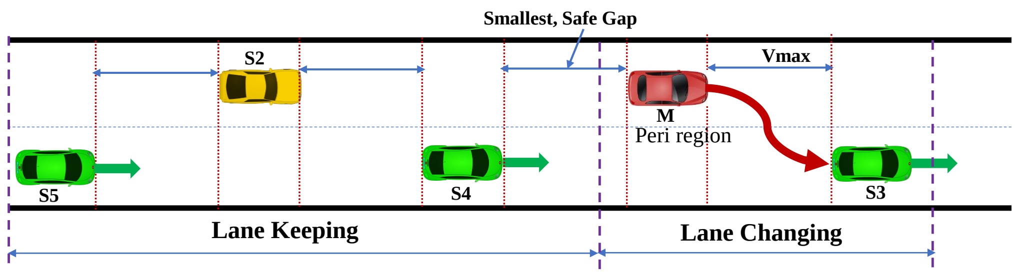


Figure 4.6: The ideal position and velocity for CAVs in the Merging segment.

are controlled to reach an ideal state (ideal position and velocity), they can achieve both desirable conditions: minimizing the final time TF and maximizing the velocity $v_{xM}(t), \forall t \in (0, TF)$.

4.5.2.3 An Ideal Solution

Now, we show an ideal state where all vehicles in the dropped lane could change lanes quickly (i.e., minimum TF) and achieve the highest longitudinal velocity during the lane-changing process (i.e., maximum $v_{xM}(t), \forall t \in (0, TF)$).

Proposition 4.2. *If CAVs are allocated as in Fig. 4.6 and travel at maximum velocity, vehicle M will not only achieve the minimum time TF, and the maximum longitudinal velocity $v_{xM}(t) = V_{\max}, \forall t \in (0, TF)$, but also have the optimum gap with the adjacent vehicles after finishing the lane-changing process.*

It is worth mentioning that after vehicle M finishes the lane-changing process, $S2$ will replace M and prepare to repeat this lane-changing process. Therefore, all vehicles in the dropped lane (i.e., vehicles M and S2) always achieve the minimum time TF and maximum velocity $v_{xM}(t), \forall t \in (0, TF)$ during this process. Therefore, reaching the derived ideal position and velocity of CAVs in the merging segment allows them to have minimum gaps and change lanes simultaneously, quickly, and safely. So, achieving this ideal state allows improving the throughput significantly.

4.6 Solution for the Multi-Lane Freeways

In the previous section, we presented a novel CSDM for two-lane freeways. It is more complex to apply it to the general scenario described in Fig. 4.1 (i.e., multi-lane freeways with one dropped

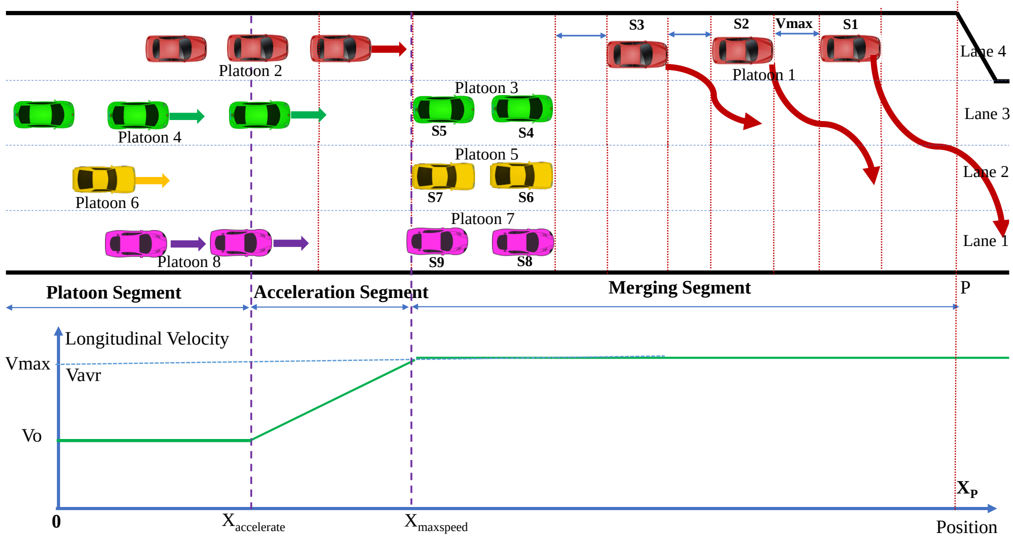


Figure 4.7: CSDM-H: Merging at head.

lane). If we only control CAVs in lane 4 to move to lane 3, the density in lane 3 can be much higher. This could lead to a decrease in the flow rate in lane 3 (this is because of the well-known flow-density relationship), whereas the density in lanes 1 and 2 does not change (i.e., at low density). Therefore, we should equally distribute the CAVs in lane 4 to the remaining lanes (i.e., balancing the downstream outflow for a multi-lane freeway). Now, we propose three different strategies based on the CSDM to perform this equal lane distribution: Merging at head, Merging at tail, Merging at center.

4.6.0.1 Design 1- Merging at Head

It is used i) if the vehicles in the dropped lane have higher priority, such as ambulance, fire truck, bus rapid transit. ii) Or the vehicles go far front longitudinal position with respect to vehicles in the other lanes. iii) Or the density of the dropped lane is much higher than that in the other lanes. For example, there are six vehicles in the dropped lane (e.g., lane 4), and only one vehicle in each of the other lanes (three non-dropped lanes), so we should control the trajectories (velocity and position) of three vehicles in these lanes instead of six vehicles in the lane 4. Therefore, we let these CAVs in lane 4 merge at the head of the platoons on the other lanes, as in Fig. 4.7.

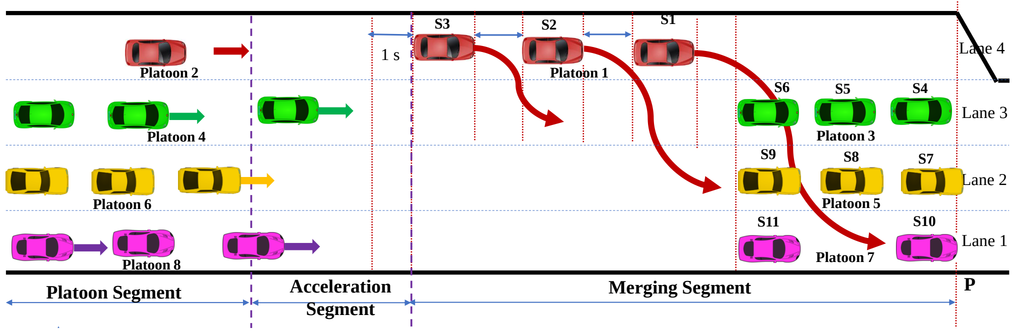


Figure 4.8: CSDM-T: Merging at tail.

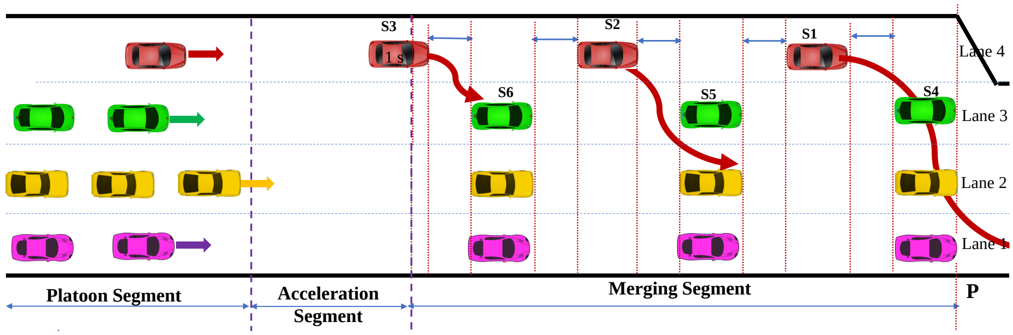


Figure 4.9: CSDM-C: Merging at center.

4.6.0.2 Design 2 - Merging at Tail

Similarly, it is used when i) the vehicles in the other lanes have higher priority than those in the dropped lane (i.e., the vehicles in lane 4 are from the non-priority lane). ii) their longitudinal positions are far ahead of the others in the dropped lane. iii) the densities of other lanes are much higher than that in the dropped lane. For example, there is only one vehicle in the dropped lane (lane 4), but three vehicles in each of the three remaining lanes, so we should control the trajectory of the vehicle in the dropped lane instead of the trajectories of nine vehicles in the other lanes. Therefore, we let those CAVs in the dropped lane merge at the tail of the platoons on the other lanes, as in Fig. 4.8.

4.6.0.3 Design 3 - Merging at Center

It is used when all vehicles have the same priority, are in lanes with similar density, and the vehicles have very close longitudinal positions in each lane. Therefore, we could apply the First-In-First-Out law and let those CAVs merge at the center of the platoon on the other lanes, as in Fig. 4.9.

Table 4.1: General design parameters for CAVs

Parameters	Value	Unit
Length	4.5	m
Longitudinal velocity	$\in \{0, 30\}$	m/s
Longitudinal acceleration	$\in \{-3, 2\}$	m/s
Lateral velocity	$\in \{0, 6\}$	m/s
Lateral acceleration	$\in \{-2, 2\}$	m/s

Table 4.2: General design parameters for a freeway

Parameters	Value	Unit
Length of bottleneck	1000	m
Width of lane	4	m

Table 4.3: Initial condition for three scenarios

	Numbers CAV (in each platoon)		
Lane	Scenario 1	Scenario 2	Scenario 3
1, 2, and 3	3	1	3
4	3	3	1

4.7 Numerical Studies

We have developed a MATLAB-based simulation platform to evaluate the proposed CSDM. The general design parameters for the movement of the CAVs and the freeway are given in Tables 4.1 and 4.2, respectively.

Now, we consider the lane-drop bottleneck in a four-lane freeway, as illustrated in Fig. 4.1. In particular, all CAVs in lane 4 must change lanes before reaching the merging point. To avoid the unbalance of the density for the three remaining lanes, these CAVs will be distributed equally. The goal of the CSDM for the considered lane drop situation is to control the CAVs' velocities and locations to minimize their average time or to increase the bottleneck throughput.

Three scenarios are considered, referred to as scenario 1, scenario 2, and scenario 3, in which the numbers of CAVs in the lanes are different, as summarized in Tables 4.3. In the beginning, the CAVs in each lane travel in two platoons (there are eight platoons: platoons 1 and 2 in lane 4, platoons 3 and 4 in lane 3, etc.). The position of the platoons and the distance between them are randomly set up.

The performance metrics used to compare the performance of different methods are the average travel time T_{avr} of all CAVs, the number of stopping vehicles SVN, and the average velocity of the CAVs changing lanes V_{mer} from lane 4 and the efficiency of the solution E_f. Specifically, V_{mer} is used in some situations in which we want to maximize the CAVs' performance in lane 4, such as for priority vehicles: bus, ambulance, etc. Moreover, SVN is the critical metric that captures the number of vehicles that must stop at all lanes before coming to the bottleneck, or in other words, showing the level of congestion (the larger the SVN, the higher the level of congestion). Finally, the most important metrics are T_{avr} and E_f, which reveal the effectiveness of the considered methods.

Assume that we know the initial states (position, velocity) of each CAV and all the parameters (length of the freeway) and key parameter limits (e.g., maximum velocity, maximum acceleration). It is easy to calculate the minimum time for each CAV to cross through the bottleneck without considering safety conditions. Define T_m as the total minimum time, and T_r as the total time of all CAVs to cross through the bottleneck. Then, the metric E_f is defined as

$$E_f = \frac{T_m}{T_r} * 100\%. \tag{4.13}$$

It is worth noting that T_m is the lowest bound of T_r, and it is fixed for each scenario. Moreover, T_m may not be achievable because it is not guaranteed that vehicles can cross through the bottleneck with safety conditions.

To compare with the state of the art, three methods are studied in scenario 1 to minimize the average travel time as follows: non-cooperative velocity method (NCV), cooperative velocity method (CVM), and cooperative space distribution method (CSDM). In particular, if a vehicle does not communicate and cooperate with others, the NCV (or human-driven vehicle method) is used to find a way to change lanes. The CVM is more intelligent than NCV because CAVs in an adjacent lane adjust their velocity in order to create a safe gap. However, it is not allowed for the CAVs to change many lanes continuously because the safe gaps in lanes 1-3 may not be available continuously for such a continuous lane change to occur. To overcome this drawback, CSDM controls CAVs' velocity and guides them to suitable positions so that these CAVs can change across many lanes as soon as possible. Besides, contrary to the NCV, the CVM and CSDM allow CAVs to move in the platoon to change lanes simultaneously, so it significantly reduces the computational demand (because the collision-avoidance constraints for the longitudinal trajectory are not involved [13]).

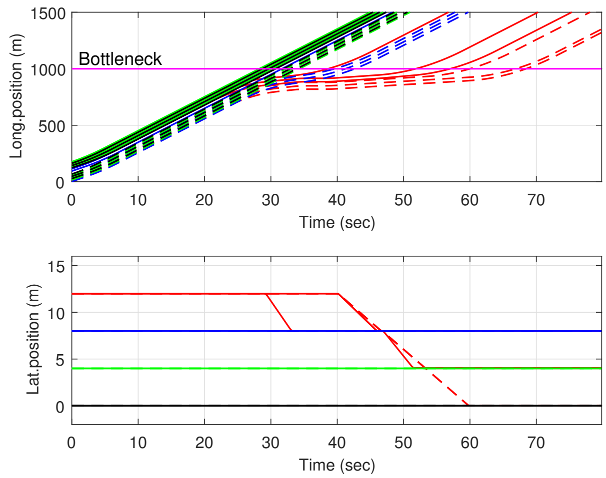


Figure 4.10: Scen.1. Top: Longitudinal trajectory, Bottom: Lateral trajectory of CAVs for NCV.

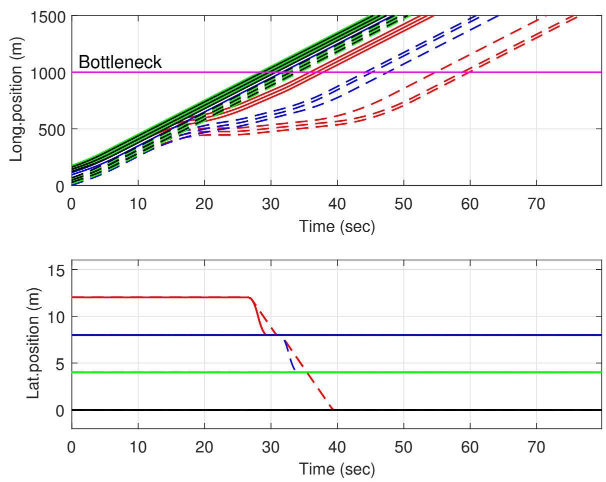


Figure 4.11: Scen.1. Top: Longitudinal trajectory, Bottom: Lateral trajectory of CAVs for CVM.

In this scenario, suppose there are three CAVs in each platoon of each lane, and their positions are randomly allocated. With the NCV, first, the vehicles speed up to reach the maximum velocity, then CAVs in lane 4 must slow down at the merging segment to find the safe gap to change lanes. Because they do not cooperate with each other in the adjacent lane (i.e., lane 3), only one or two vehicles of the first platoon can change lanes, and the rest need to slow down and wait for another safe gap. It is the reason the queue is formed in lane 4. Due to the limitation of the side-slip angle (constraint (4.7d)), the low longitudinal velocity leads to a longer time to change lanes and thus decreases the velocity of the followers.

Fig. 4.10, Fig. 4.11, and Fig. 4.12 display the longitudinal (top) and lateral (bottom) trajectories of the CAVs due to the NCV, CVM, and CSDM-C, respectively. The solid red lines and the red dash lines correspond to the CAVs in platoons 1 and 2 of lane 4, the solid blue lines and the blue

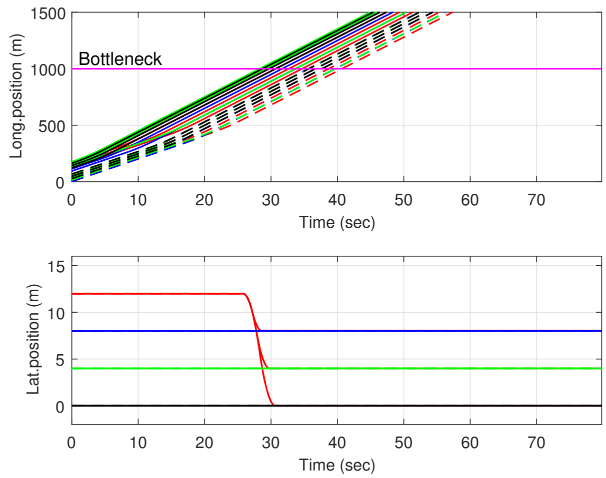


Figure 4.12: Scen.1. Top: Longitudinal trajectory, Bottom: Lateral trajectory of CAVs for CSDM-C.

dash lines present the CAVs' trajectories of platoons 3 and 4 in lane 3, the green lines are for the CAVs of platoons 5 and 6 in lane 2, and the black lines for the CAVs of platoons 7 and 8 in lane 1. We can see that the longitudinal velocities of the CAVs, which are changing lanes in the NCV and CVM, are very low, whereas the longitudinal velocity in the CSDM is the highest. Hence, the duration for changing lanes in the CSDM is considerably shorter than that in the NCV and CVM.

4.7.1 Scenario 1

Fig. 4.13 presents the velocity of CAVs for four methods: NCV, CVM, CSDM-H, and CSDM-T. There are some CAVs stopping in front of the bottleneck for both NCV and CVM because these methods fail to timely create actively safe gaps for all CAVs to change lanes sequentially. However, CSDM could do this and allow CAVs to change lanes simultaneously and sequentially. In most cases, if we want to merge CAVs in lane 4 at the head, these CAVs in platoons 1 and 2 must accelerate, and the other platoons in the other lanes must slow down to create enough safe gaps. Hence, the number of vehicles accelerating (i.e., 6 CAVs in 2 platoons) is smaller than the number of vehicles

Table 4.4: Performance results of three methods in Sce.1

Method	$T_{avr}(s)$	SVN	$V_{mer}(m/s)$	E_f
NCV	38.93	5	19.43	80.31%
CVM	37.03	3	22.69	84.42%
CSDM-H	34.20	0	28.62	91.41%
CSDM-T	33.77	0	26.23	92.58%
CSDM-C	34.37	0	26.83	90.96%

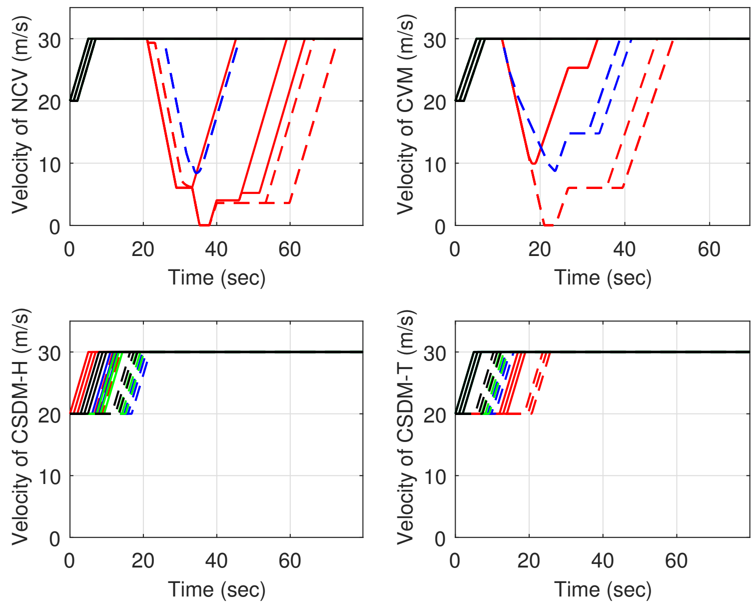


Figure 4.13: Scen.1. Velocity of CAVs for NCV, CVM, CSDM-H, and CSDM-T.

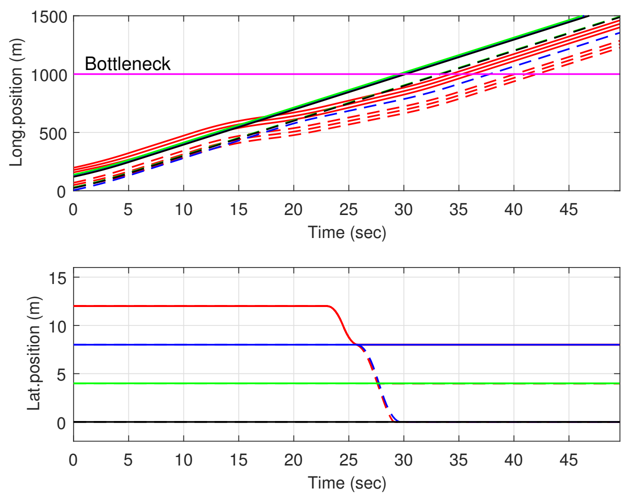


Figure 4.14: Scen.2. Top: Longitudinal trajectory, Bottom: Lateral trajectory of CAVs for CVM.

decelerating (i.e., 18 CAVs in 6 platoons), so the CSDM-H has a lower average velocity than the CSDM-T and CSDM-C. Although the CSDM-T and CSDM-C attain similar average travel time, the CSDM-T (or CSDM-H) only needs to control precisely the leader of platoons 3 to 8, while the CSDM-C must exactly control the velocity and the position of all CAVs in platoons 3 to 8 to avoid a collision.

The results of the three methods are presented in Table 4.4. The number of vehicles stopping in the NCV ($SVN = 5$) is the highest because the safe gap in lane 3 can fit only one vehicle, the

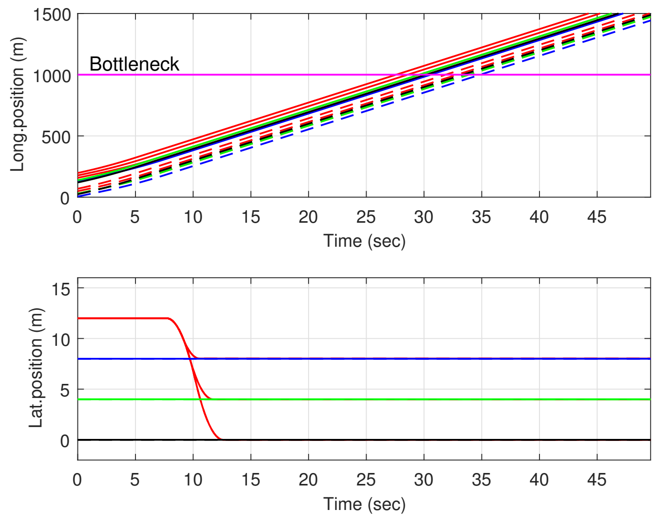


Figure 4.15: Scen.2. Top: Longitudinal trajectory, Bottom: Lateral trajectory of CAVs for CSDM-H.

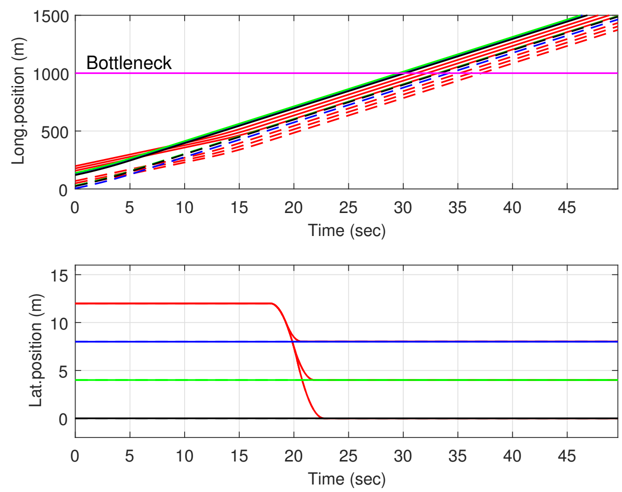


Figure 4.16: Scen.2. Top: Longitudinal trajectory, Bottom: Lateral trajectory of CAVs for CSDM-T.

other five CAVs must slow down to a full stop; therefore, the average travel time is the highest for this method. The CVM achieves a lower average travel time than the NCV because the CAVs of platoons 3 and 4 in lane 3 adjusted their velocity to generate enough safe gaps for platoon 1 in lane 4 to change lanes. Whereas, platoon 2 in lane 4 must stop to wait until platoon 1 can finish the merging process, so the $SVN = 3$. By using CSDM, we can achieve the shortest average travel time because the CAVs in lane 4 do not need to slow down and wait for a safe gap so that they can change lanes instantly and simultaneously. Furthermore, the efficiencies of the three types of CSDM methods (above 90%) are better than the NCV (80.31%) and CVM (84.42%) with respect to the lower bound in this scenario. Especially, the CSDM-T achieves the best result (92.58%) because

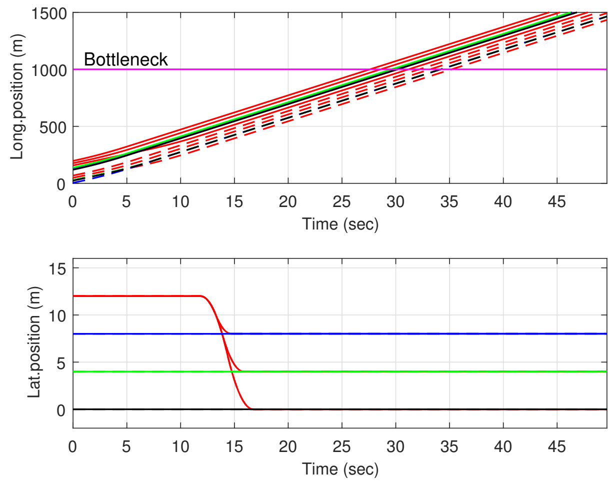


Figure 4.17: Scen.2. Top: Longitudinal trajectory, Bottom: Lateral trajectory of CAVs for CSDM-C.

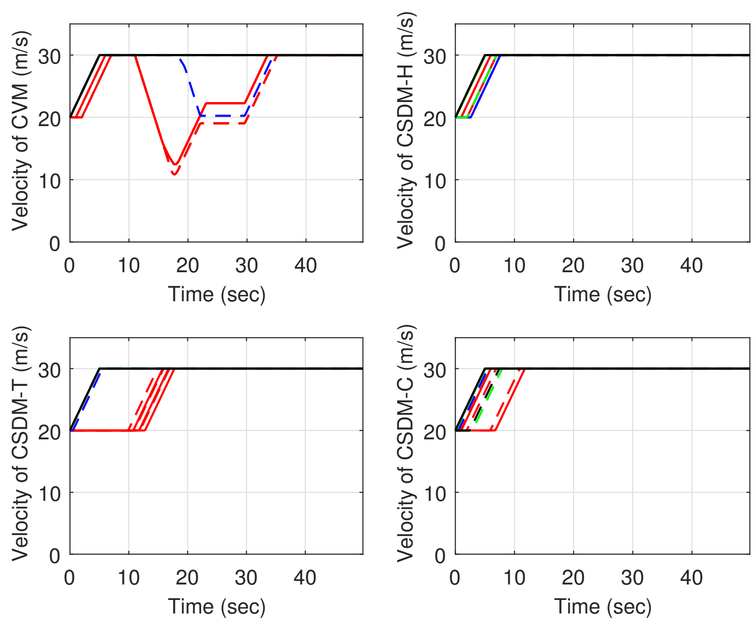


Figure 4.18: Scen.2. Velocity of CAVs for CVM, CSDM-H, CSDM-T and CSDM-C.

the number of CAVs (6 CAVs) in the dropped lane is much smaller than the ones (18 CAVs) in the other remaining lanes.

Because of the poor performance of the NCV, we only compare the performance of CVM with the three types of CSDM in scenarios 2 and 3 in the ensuing section.

4.7.2 Scenario 2

In this scenario, lane 4 has three CAVs in each platoon (or six CAVs total), while there is only one CAV in each platoon in the other lanes. Figs. 4.14, 4.15, 4.16, and 4.17 show the longitudinal and lateral trajectories of the vehicles due to the CVM and three types of the CSDM, respectively. Because the density of the three remaining lanes is low and the gap between platoons is large, the CAVs in lane 4 do not need to wait for a safe gap (i.e., $SVN = 0$) in all methods. The performance results for scenario 2 are presented in Table 4.5.

Fig. 4.18 displays the velocities of CAVs for four methods: CVM, CSDM-H, CSDM-T, and CSDM-C. The CVM has the lowest average velocity and the highest average travel time because platoons 1 and 2 in lane 4 must slow down and cooperate with platoons 3 and 4 in lane 3 to have enough safe gaps for lane-changing operations. The CSDM-H achieves the best performance in this scenario (the lowest average travel time and highest efficiency) because the number of CAVs which are accelerating in lane 4 (six CAVs) is larger than the number of CAVs that are decelerating in the other lanes (two CAVs). This is demonstrated by the average velocity of the CAVs in lane 4 (V_{mer} = 29.3 m/s), which is the highest in the CSDM-H. Besides, the efficiencies of CSDM-T (94.19%) and CSDM-C (98%) methods show that they outperform the CVM method (88.02%) in this case. The CSDM-H method reaches nearly 100% efficiency (i.e., without time delay) with respect to the lower bound due to the low density in the three remaining lanes.

4.7.3 Scenario 3

Lane 4 has the smallest number of CAVs in each platoon (one CAV in each platoon or two CAVs total). Similarly, the longitudinal and lateral trajectories of the CAVs using the CVM and CSDM are shown in Figs. 4.19, 4.20, 4.12, and 4.22, respectively. Table 4.6 presents all the simulation results in this scenario.

Table 4.5: Performance results of three methods in Sce.2

Method	$T_{avr}(s)$	SVN	$V_{mer}(m/s)$	E_f
CVM	35.25	0	24.78	88.02%
CSDM-H	31.41	0	29.3	98.79%
CSDM-T	32.95	0	27.23	94.19%
CSDM-C	31.67	0	28.95	98%

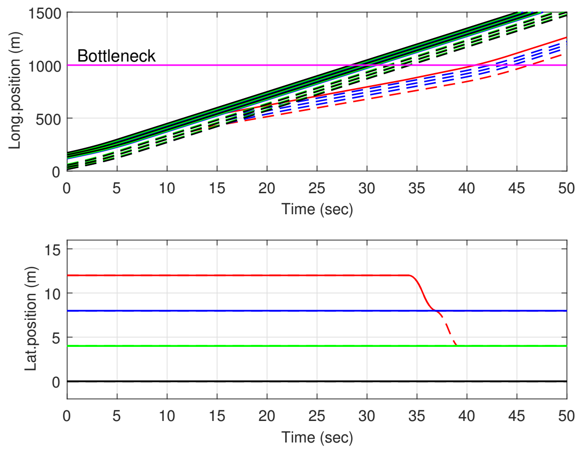


Figure 4.19: Scen.3. Top: Longitudinal trajectory, Bottom: Lateral trajectory of CAVs for CVM.

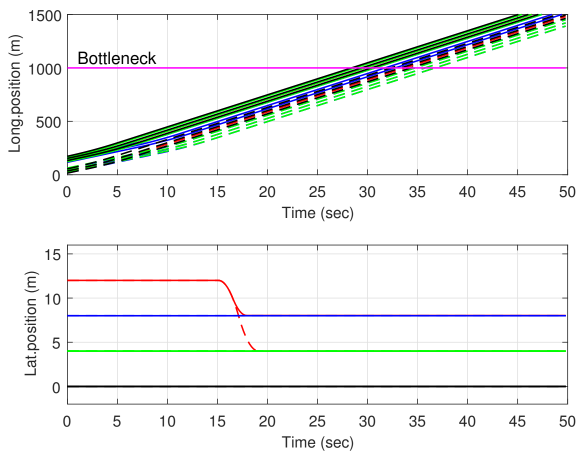


Figure 4.20: Scen.3. Top: Longitudinal trajectory, Bottom: Lateral trajectory of CAVs for CSDM-H.

In this situation, the performance in terms of E_f of the CVM method (91.08%) is slightly worse than that in the CSDM-H method (95.75%). This is because there are fewer vehicles that need to change lanes, and the gap between the platoons in lane 3 is large enough, so they do not need to adjust the velocity significantly as shown in Fig. 4.23. The CSDM-T method (97.92%) and CSDM-C method (97.63%) achieve better performance than the CSDM-H. The reason is that the number of CAVs in three non-obstacle lanes is larger than that in lane 4 and the priority of those CAVs is higher due to their far front position with respect to those in lane 4. They thus do not need to decelerate; as a consequence, their average velocities are greater, and their travel time is shorter than that due to the CSDM-H method.

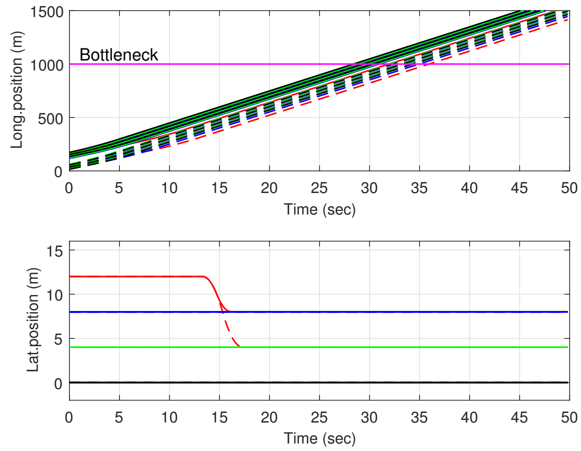


Figure 4.21: Scen.3. Top: Longitudinal trajectory, Bottom: Lateral trajectory of CAVs for CSDM-T.

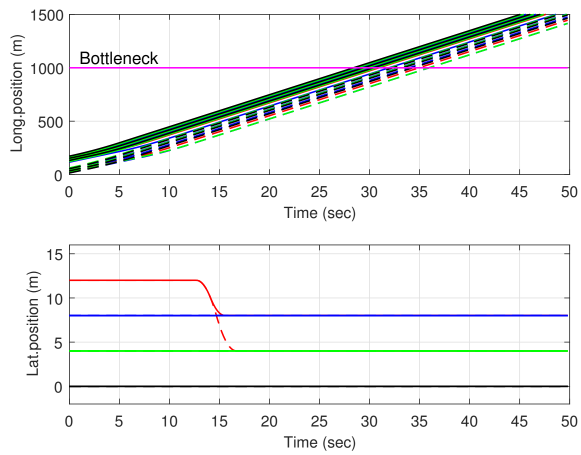


Figure 4.22: Scen.3. Top: Longitudinal trajectory, Bottom: Lateral trajectory of CAVs for CSDM-C.

4.8 Conclusion

In this paper, we have proposed the cooperative space distribution method (CSDM) to decrease the average travel time for the CAVs changing lanes at a lane-drop bottleneck on multi-lane freeways. In more detail, the freeway region of interest is divided into three segments: i) platoon segment, ii) acceleration segment, and iii) merging segment. In the first segment, CAVs travel together in the platoon to guarantee their safety; they will then speed up to attain the maximum velocity and

Table 4.6: Performance results of three methods in Sce.3

Method	$T_{avr}(s)$	SVN	$V_{mer}(m/s)$	E_f
CVM	34.21	0	24.72	91.08%
CSDM-H	32.54	0	28.34	95.75%
CSDM-T	31.82	0	28.63	97.92%
CSDM-C	31.92	0	28.61	97.63%

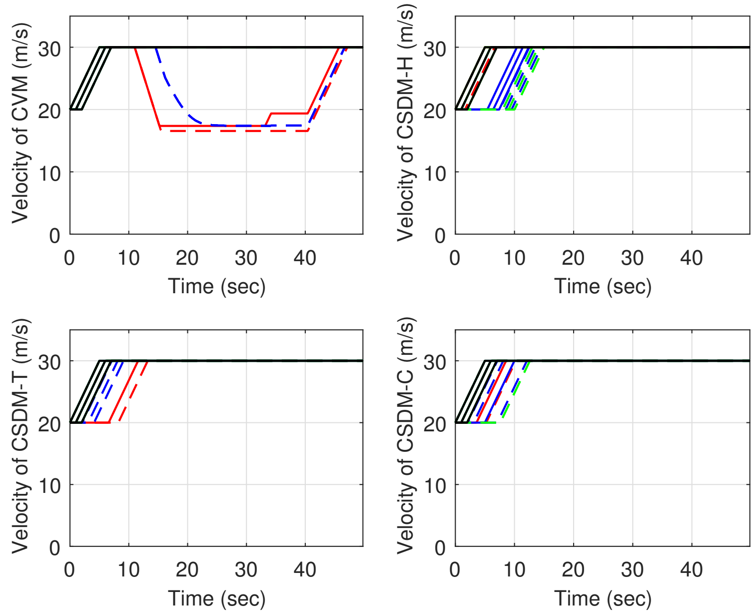


Figure 4.23: Scen.3. Velocity of CAVs for CVM, CSDM-H, CSDM-T and CSDM-C.

reach the determined position in the second segment. Finally, these CAVs change lanes in the last segment and pass through the bottleneck with maximum velocity and minimum gap. Moreover, three types of CSDM have been recommended to apply for the multi-lane freeways, in which the CAVs can change lanes simultaneously and quickly, leading to the reduction of the average travel time through a bottleneck without losing balance in density among lanes.

In all studied scenarios, the CSDM always achieves a significant improvement (higher average velocity and lower travel time) compared to the CVM and NCV. Since the CAVs approaching the bottleneck using the NCV and CVM need to slow down to find the safe gap to change lanes, whereas it is not the case with the CSDM. Cooperating the velocity and reaching the exact position are crucial points to allow the CAVs to pass through the bottleneck with maximum velocity and minimum gap. Three types of CSDM achieve similar results when the density in each lane is the same (scenario 1). However, if there is a higher priority for certain CAVs in changing lanes (scenario 2), then we should use the CSDM-H to allow them to travel faster. Otherwise, the CSDM-C and CSDM-T should be used when there are fewer CAVs in lane 4 in comparison with the other lanes. In fact, the efficiencies of the three types of CSDM methods are more than 90% with respect to the lower bound in all considered scenarios. Moreover, we can achieve nearly 100% efficiency (without delay time) in some cases (low density), for example, CSDM-H achieves 98.79% of the lower bound in scenario 2, and CSDM-C 97.92% of the lower bound in scenario 3.

This paper only considered fully autonomous vehicles, which are expected to become a reality in the long term. Besides, we assume that there is no delay or interruption in communication between the CAVs and between the CAVs and the RC. Therefore, our near-future research puts forward to dealing with the combination of human-driven vehicles and autonomous vehicles in traffic flow. Besides, we assume that there are small disturbances that insignificantly affect CAVs' trajectories, and the vehicle controllers can regulate quickly the designed trajectories. These assumptions motivate future work in considering the major disturbances when the CAVs suddenly decrease to low velocity due to accidents or loss of wireless signal.

Note that we could reduce the maximum acceleration and limit the jerk in the acceleration segment (the second segment) to improve the smooth and comfortable maneuvers of the vehicles' drivers and passengers. However, reducing the maximum acceleration and the jerk could lead to the longer length of this acceleration segment and therefore longer communication range between the management centre and CAVs. Due to the space constraint, we do not consider this interesting comfort issue in this current paper, which is left for our future work. This allows us to focus on developing CAVs' trajectory control strategy to achieve maximum flow rate or minimum total travel time for CAVs.

4.9 Appendices

4.9.1 Proof of Proposition 4.1

Proof. Assume that the longitudinal trajectory of vehicle M satisfies all constraints in the Peri region, we only consider the lateral trajectory of M:

$$\begin{aligned} y_M(t) &= y_M(0) + v_{yM}(0)t + \int_0^t \int_0^\lambda a_{yM}(\tau) d\tau d\lambda \\ &= 0 + 0 + \int_0^t \int_0^\lambda a_{yM}(\tau) d\tau d\lambda, \forall t \in (0, t_{Pe}). \end{aligned} \tag{4.14}$$

With the lateral position (4.7b, 4.7c) and the velocity constraints (4.6b), the minimum time t_{pe} is achieved when the lateral velocity is quickly increased to achieve the maximum value. If the side-slip angle constraint (4.7d) is satisfied, then we choose:

$$a_{yM}(t) = a_{ymax}, \qquad \forall t \in (0, t_{Pe}^{min}/2), \tag{4.15a}$$

$$a_{yM}(t) = -a_{ymax}, \qquad \forall t \in (t_{Pe}^{min}/2, t_{Pe}^{min}). \tag{4.15b}$$

Hence, the lateral trajectory can be calculated as:

$$\begin{aligned} y_M(t) &= \frac{1}{2} a_{ymax} t^2, \forall t \in (0, t_{Pe}^{min}/2), \\ \rightarrow y_M(t_{Pe}^{min}/2) = LW/2 &= \frac{1}{8} a_{ymax} \left(t_{Pe}^{min}\right)^2. \end{aligned} \tag{4.16}$$

And the minimum time t_{Pe} could be written as:

$$t_{Pe}^{min} = 2\sqrt{LW/a_{ymax}}. \tag{4.17}$$

Now consider again the constraints (4.6b), (4.7d) and (4.15a,4.15b), we must have the following constraint for $\forall t \in (0, t_{Pe}/2)$:

$$V_{\max} \geq v_{xM}(t) \geq v_{yM}(t)/0.17 = a_{ymax} t/0.17. \tag{4.18}$$

From Eq. 4.17, we must have

$$\begin{aligned} V_{\max} &\geq v_{yM}(t_{Pe}^{min}/2)/0.17 = \sqrt{LW * a_{ymax}}/0.17, \\ \rightarrow V_{\max} &\geq 5.88\sqrt{LW * a_{ymax}}. \end{aligned} \tag{4.19}$$

Therefore, if the constraints (4.18) and (4.19) are not satisfied, it takes a longer time to complete the Peri region or:

$$t_{Pe} \geq t_{Pe}^{min} = 2\sqrt{LW/a_{ymax}}. \tag{4.20}$$

Proposition 4.1 thus holds. □

Remark 4.2. *Since LW, $V_{\max}$ and a_{ymax} are constants and predetermined, we could not put any effort into constraint (4.19). However, constraint (4.18) depends on the longitudinal velocity $v_{xM}(t)$, so the probability of satisfying constraint (4.18) is the highest if $v_{xM}(t) = V_{\max}$. This means that the highest probability of minimizing t_{Pe} could be achieved if vehicle M travels with maximum longitudinal velocity (*).*

4.9.2 Proof of Proposition 4.2

Proof. First, we will show that vehicle M satisfies all conditions for the lane-changing process from Eq (4.4) - (4.8). Indeed, vehicle M is surrounded by vehicles $S2, S3, S4$ (without the leading vehicle $S1$), so:

$$\begin{aligned} v_{xM}(t) = v_{S2}(t) = v_{S3}(t) = v_{S4}(t) = V_{\max}, \\ \Longrightarrow s_m(S2) = s_m(S3) = s_m(S4) = T * V_{\max} = V_{\max}, \\ \Longrightarrow x_{\max}(t) \geq x_M(t) \geq x_{\min}(t). \end{aligned} \tag{4.21}$$

If all these vehicles keep maximum longitudinal velocity, vehicle M always satisfies all constraints (4.4) - (4.8) of the Pre, Peri, and Post regions. As a result, the velocity and gap between vehicle M and the leading vehicle $S3$ (or following vehicle $S4$) reach the optimum values in the Post region (i.e., maximum velocity and minimum gap). (**)

Moreover, there is a safe gap to allow vehicle M to change lanes immediately, so $TW = t_{Pre} = t_{Po} = 0$. Hence, the final time can be expressed as

$$TF = TW + t_{Pre} + t_{Pe} + t_{Po} = t_{Pe}. \tag{4.22}$$

Thanks to Proposition 4.1 and Remark 4.1, we could get the highest probability of minimizing t_{Pe}, or achieve minimum time TF. (***)

From (*), (**) and (***), Proposition 4.2 holds. □

Chapter 5

Space Distribution Method for Autonomous Vehicles at A Signalized Multi-lane Intersection

The content of this chapter was published in the IEEE Communications Letter in the following paper:

Tung Thanh Phan, Dong Ngoduy, and Long Bao Le "Space distribution method for autonomous vehicles at a signalized multi-lane intersection," *IEEE Trans. Intell. Transp. Syst,* vol. 21, no. 12, pp. 5283-5294, Dec. 2020.

Abstract

Under the connected vehicle environment, connected and autonomous vehicles (CAVs) could bring numerous advantages including improving the traffic flow, enhancing safety, and alleviating air pollution. However, optimally operating CAVs at signalized multi-lane intersections is a challenging problem due to the complex interaction of vehicles between lanes. It is thus a desire to manage and control the dynamics of CAVs at signalized multi-lane intersections. To this end, this paper puts forward a bi-level controller framework to optimize the intersection throughput. In our proposed method, the upper level (i.e. the intersection controller) is used to optimize the lane usage of each approach and the CAVs' positions. In contrast, the lower level (i.e. the vehicle controllers) receives information from the upper level to control the CAVs to get the maximum speed. More specifically, in the upper level, we apply a novel Space Distribution Method (SDM) for the CAVs to maximize the throughput (i.e. a number of CAVs) of the (multi-lane) intersection where signal timings are predefined. The SDM is divided into three steps: i) platoon formulation, ii) lane-mode optimization, and iii) CAVs' position distribution. To maximize the throughput, the intersection controller receives information about the states of the CAVs (e.g. the trajectories), then optimizes the lane usages for each approach, the desired speed, and the gap of the CAVs as well as the CAV's position along the approach. After that, each CAV that is allowed to cross the intersection will determine its own trajectory and travel at the scheduled time without crash. Numerical simulations are set up to show that the throughput increases significantly, even more than twice the throughput obtained from other methods in some circumstances.

5.1 Introduction

Traffic congestion is one of the major transportation challenges that the world is facing in the coming years. For example, it has been reported that between 2013 and 2030, traffic congestion will cost the UK economy a staggering £307bn, with the annual cost of congestion set to rise by 63% to £21.4bn over the same period[1]. This research will investigate a potential traffic control solution to address such problems by introducing emerging technologies in connected and autonomous vehicles (CAVs). In general, a CAV can obtain neighboring information via vehicle-to-vehicle (V2V) communication and/or vehicle-to-infrastructure (V2I) communication (hereafter, V2X communication for short), and then adopt a suitable control law to achieve a certain objective, such as maintaining constant inter-vehicle spacing within vehicles or smooth driving patterns. Recent studies have shown that the use of CAVs in a traffic stream can significantly improve road safety, traffic efficiency, and environmental sustainability.

Due to the V2X capability, the global environmental perception of drivers is broadened beyond the line of sight. Besides, the high-fidelity traffic data exchange sets up connections and cooperation between drivers and intersection controllers, forming cooperative intelligent transport systems (C-ITS). Furthermore, IEEE 1609 standards [14] using IEEE 802.11 based Dedicated Short Range Communication (DSRC) is established to assist the development of C-ITS, so many emerging signal control strategies were developed, such as in [15] and [16].

5.1.1 Related Works

Particularly, the operations of CAVs are getting significant attention and many methods have been developed to coordinate the dynamics of the CAVs and the central intersection management to reduce the total travel time, energy consumption, and pollution. Dresner *et al.* [6] and Perronnet *et al.* [7] proposed a reservation scheme to guide vehicles traveling on two directions of two approaches. In more detail, an intersection is divided into a grid of tiles, and vehicles need to send the requested tiles on their planned route to the IC before crossing the intersection. Once the requirement of the vehicle is accepted if there is no conflict with other vehicles, the IC will allocate the tiles and

[1] http://highwaysmagazine.co.uk/traffic-congestion-could-cost-uk-300bn

the time slot for it. Otherwise, its request is denied, so it might decelerate and wait until a new reservation is accepted.

Jin *et al.* [22] studied a platoon formation for intersection control by coordinating the adjacent CAVs. In this method, the leader and the followers exchange the trajectory information (e.g. speed and position) with the IC to calculate the time to cross the intersection. After that, the platoon leader and the followers shall optimize their trajectories to obey the assigned schedule under safety constraints. Generally, the platoon formation provides better traffic operations and leads to more efficient control strategies, as reported in [17, 23]. A bi-level controller for a crossroad was described in [59], in which the lower level collects and evaluates the information of traffic flow to specify a control policy that stabilizes vehicle flows. Whereas, the upper level first utilizes a consensus algorithm to calculate the desired traffic density and then determines the speed of each vehicle. The adjacent intersections then share states of their traffic density among them to boost the system's throughput. Along this line, Zhao *et al.* [60] integrated an intersection control problem with a vehicle trajectory planning method as a bi-level programming problem, in which the upper level is designed to minimize the total travel time by a mixed integer linear programming model. In contrast, the lower level is a linear programming model with an objective function to maximize the total speed of entering the intersection. The two levels are coupled by the arrival time and the terminal speed. Schmidt *et al.* [61] suggested a similar bi-level controller method, in which the upper layer assumes that the speed of each car is equal, and estimates its time and order to merge in the control section. By using heuristic rules in the second layer, the acceleration for each car is optimized to avoid crashes when merging.

To prevent a car accident, Alonso *et al.* [26] authorized CAVs themselves to decide a suitable time to cross the intersection. The CAVs must share information about their state, such as position, velocity, driving direction, and order. After that, they use the priority tables to determine whether the vehicle should keep traveling or stop until the junction is clear. In the next phase, the look-up table is renewed when each vehicle updates its own priority. The decentralized method was proposed in [27], which determines the best vehicle sequence to go through the intersection. It is done by predicting the arrival time among the cars in the queue, then allowing the one having the shortest arrival time to cross, as well as prohibiting the others from crossing by sending messages to them. Avoiding the collision of vehicles simultaneously crossing a multi-lane intersection could be solved by adding an additional logic.

Lin and Liu [28] proposed a coordination method in which the road is divided into three sections, and let the vehicles pass through the intersection at maximum speed at exactly the calculated time. Moreover, these vehicles are following the planned trajectories so that the fuel consumption is minimized. Markarem *et al.* [29] proposed an alternative control method in which CAVs coordinate at the intersection in order to reduce energy consumption and pollution. Kamal *et al.* [30] developed a vehicle-intersection coordination scheme without using any traffic lights so that all CAVs can pass through the intersection safely and smoothly. Yan [31], Lee [32], and J. Wu [33] managed to formulate an optimization problem in order to minimize the total travel time under safety constraints. They applied a dynamic programming method (DP) to find the heuristic solutions of the problems. Because the multi-lane intersection is very difficult to solve with the DP, the authors utilized Petri nets to model the system, and then optimized the sum of the two queues' length. Tlig *et al.* [62] used a centralized controller to cooperate with the vehicles crossing the intersection alternately. Each vehicle calculates its own trajectory that coincides with three segments: a deceleration segment, a constant speed segment, and an acceleration segment. The deceleration and acceleration of the vehicle are determined, thus they only optimize the speed for the constant speed segment to minimize the total travel time and the energy consumption. To widen the issue of interconnected intersections, the authors studied a bi-level controller framework in which intersections share information and adjust their phases to improve the traffic flow efficiency [63]. In the platoon-based approach of [18], a receding horizon model predictive control method was proposed to minimize the fuel consumption for platoons and drive the platoons to pass the intersection during a green phase. The method is then extended to dynamic platoon splitting and merging rules for cooperation among CAVs and conventional vehicles in response to the high variation of urban traffic flow. In [25], a parsimonious shooting heuristic algorithm is proposed to optimize the trajectories of a stream of CAVs and consider multiple objective functions such as fuel consumption and travel time.

To the best of our knowledge, in the vast literature of intersection control methods for CAVs (i.e. joint control of signal timings and trajectory planning), there are few attempts dedicated to the lane-based control of the CAVs at signalized multi-lane intersections. It is worth mentioning that the lane-based control problem has been proposed in [19], [20] and [21] for human-driven vehicles (HVs) at an isolated signal-controlled intersection. However, with these methods, we cannot control the speeds and the positions of the HVs, so their lane-based optimization method is a passive control. Whereas, the focus of our paper is on the multi-lane intersection control problems for CAVs in which we can influence the trajectories of the CAVs (i.e. a lane-based control with a trajectory planning method). Moreover, Matthew's method in [25] could only be applied for one-directional movement in the roads (e.g. either straightforward or left/right turning), so it is impossible to be applied when there are more than two directional movements (e.g. both straightforward and left turning). Although some recent studies in [74], [71], and [75] concern the capacity for mixed HVs and CAVs and the optimal lane allocation for connected vehicles, these are not ready for intersection control problems of the CAVs. This is because they do not include information on the queue, stopping at a stop bar, conflict area (CAVs from different approaches), blocking by the preceding cars having other directions, etc. For example, in many situations, there are more CAVs in one direction than in other directions, or a vehicle near the stop bar cannot pass the intersection because of being blocked by the left-turning vehicles. As a result, the throughput will be reduced, and this can lead to congestion in the worst cases (e.g. spill-back problems).

5.1.2 Contributions and Organization of the Paper

To overcome such issues, this paper proposes a bi-level controller framework to optimize the throughput at a signalized multi-lane intersection where the signal timings are predefined. In our proposed method, the upper level (i.e. the intersection controller) is used to optimize the lane usage of each approach and the CAVs' positions. In contrast, the lower level (i.e. the vehicle controllers) receives information from the upper level to control the CAVs to get the maximum speed. The main challenging problem is in the upper level, where the intersection controller (TMC) estimates how many CAVs could pass the intersection in their right of way for each phase. Because the number of lanes is a small positive integer, we could solve these problems easily by considering all combinations and choosing the case to get the maximum throughput. In order to do that, we apply a novel space distribution method (SDM) for CAVs at a signalized multi-lane intersection with three steps: i) platoon formulation, ii) lane-based optimization, and iii) position distribution.

Formulating platoons is the first step when the CAVs enter the network and move at the same speed as the preceding ones (see [17]). These platoon-based operations make it easier and more efficient for traffic control strategies [18]. Meanwhile, the vehicles send information about their states (e.g. position and speed) to the TMC to maximize the throughput in the second step. Specifically, based on the received information, the TMC with predefined signal timings will optimize the lane

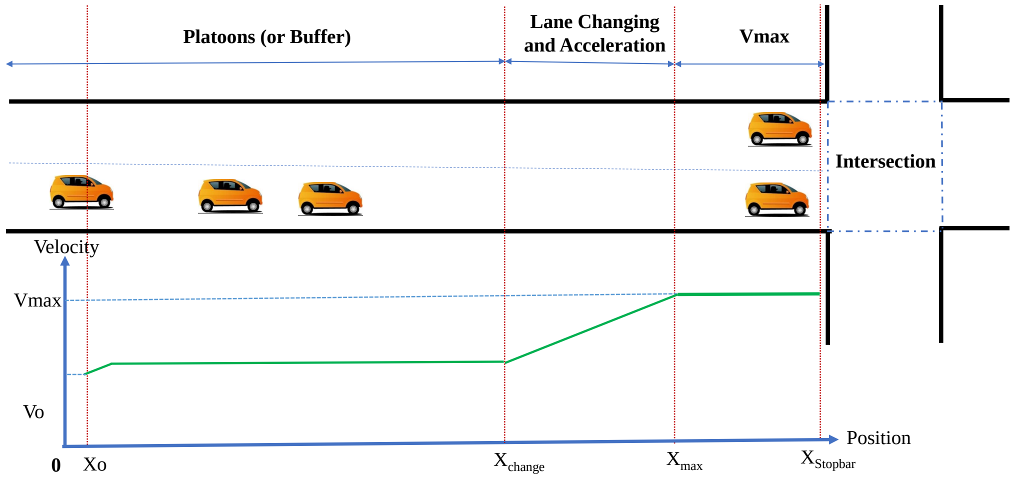


Figure 5.1: The optimal CAV's trajectory.

usage variables and the number of CAVs having permission to cross the intersection (called GAVs). Finally, each GAV will calculate its own trajectory (e.g. speed and acceleration) and position distribution (e.g. lane changes) in order to pass through the intersection with maximum speed and smallest gap.

The remainder of this paper is organized as follows. A kinematic vehicle model with safety constraints and the optimization problem are described in Section 5.2. Next, Section 5.3 introduces the SDM to be used in our bi-level controller method in order to utilize the benefits of the CAVs. The numerical results are presented in Section 5.4 to show the advantages of the proposed method. Finally, we conclude the paper in Section 5.5.

5.2 Kinematic Vehicle Model and Optimization Problem

In this paper, we will divide each approach into three segments as follows: i) platoon segment, ii) lane changing and acceleration segment, and iii) maximum speed segment (see Fig. 5.1). The first segment is dedicated to a constant speed pattern. In a platoon, each CAV communicates with the adjacent ones to obtain their trajectories and then adjusts its own speed to satisfy the safety constraints. The gap between two adjacent vehicles in the same lane and moving along the same direction is at least equal to the distance that they travel in 1 second. The second segment is used for changing lanes and accelerating to reach a maximum speed without a collision. Finally, in the last segment, the CAVs maintain the maximum speed and smallest gap before reaching the stop bar, and then passing through the intersection.

Next, we will describe the kinematic vehicle model and formulate the optimization problem.

5.2.1 Kinematic Vehicle Model

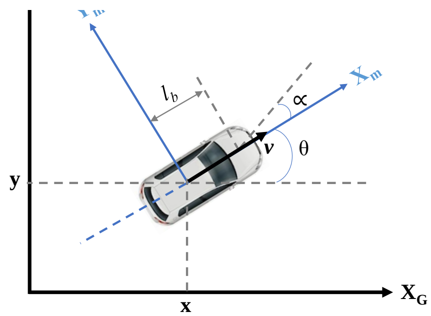


Figure 5.2: The variables in the global reference frame.

Fig. 5.2 describes the variables of CAVs in the global reference frame $X_G Y_G$. We have the following equations of the kinematic vehicle model [65]:

$$\dot{x}(t) = v(t)\cos(\theta(t)), \quad (5.1)$$

$$\dot{y}(t) = v(t)\sin(\theta(t)), \quad (5.2)$$

$$\dot{\theta}(t) = v(t)\sin(\alpha(t))/l_b, \quad (5.3)$$

where $(x(t),\, y(t),\, \theta(t))$ are the vehicle positions in the global reference frame; $v(t)$ and $\alpha(t)$ are the linear velocity and orientation of the vehicle front wheel, respectively; and l_b is the wheelbase of the vehicle. These equations are used to avoid collisions when the CAVs change lanes, merge or diverge from the platoon.

Let $\mathcal{L} = \{1, 2, ..., Lm\}$, $\mathcal{J} = \{$'west', 'east', 'north','south'$\}$, $\mathcal{P} = \{1, 2, 3, 4\}$, denotes the set of lanes, approaches, and phases, respectively. It is noted that all approaches have the same maximum number of lanes, which is equal to Lm. Let l, j and p be the elements in $\mathcal{L}$, $\mathcal{J}$, and $\mathcal{P}$, respectively. Moreover, the set of CAVs in lanes l, approaches j is named $\mathcal{I}_{lj}$ whose element is indicated as i.

If the CAVs travel in the same lane of the same approach, we can simplify these equations as follows:

$$
\begin{aligned}
\dot{x}_{ilj}(t) &= v_{ilj}(t), && (5.4)\\
\dot{v}_{ilj}(t) &= a_{ilj}(t), && (5.5)\\
x_{ilj}(t) - x_{(i-1)lj}(t) = g_{ilj}(t) &\geq v_{ilj}, && (5.6)\\
V_{\max} \geq v_{ilj}(t) &\geq V_{\min}, && (5.7)\\
a_{\max} \geq a_{ilj}(t) &\geq a_{\min}, && (5.8)
\end{aligned}
$$

where $x_{ilj}(t)$ and $x_{(i-1)lj}(t)$ are the position of vehicle i and its follower $(i-1)$ in lane l, approach j, respectively; $v_{ilj}(t)$ and $a_{ilj}(t)$ are the speed and the acceleration of vehicle i on approach j, respectively; $g_{ilj}(t)$ denotes the distance between vehicle i and $(i-1)$ in lane l, approach j; $V_{\max}$ and $V_{\min}$ are the upper and lower bound of the vehicles' speed; $a_{\max}$ and $a_{\min}$ are the maximum and minimum acceleration, respectively.

It is worth noting that inequality (5.6) is a lower limit of the gap between adjacent CAVs (i.e. the distance traveled in 1s). Inequalities (5.7) and (5.8) are the constraints for the feasible range of speed and acceleration of the CAVs, respectively.

5.2.2 Optimization Problem

The goal of an optimization problem in this paper is to achieve the maximum throughput at a signalized multi-lane intersection, which is defined as follows:

$$
\max \int_0^{T_o} \sum_{j \in \mathcal{J}} Q_j(t)dt = \max \sum_{j \in \mathcal{J}} \int_0^{T_o} Q_j(t)dt = \max \sum_{j \in \mathcal{J}} \int_0^{T_o} \sum_{l=1}^{Lm} Q_{lj}(t)dt, \qquad (5.9)
$$

where $Q_j(t)$ is the flow rate per second (i.e. the number of CAVs crossing the intersection in a second) from approach j; $Q_{lj}(t)$ is the flow rate per second from lane l of approach j; T_o is the time horizon considered in the optimization process.

Without loss of generality, we only consider the optimization problem in one cycle of pre-determined signal timings. Let us define:

$$NA_j = \int_0^{T_c} Q_j(t)dt = \sum_{p \in \mathcal{P}} \int_0^{t_p} Q_j(t)dt = \sum_{p \in \mathcal{P}} NA_{jp}, \tag{5.10}$$

where NA_j is the total number of CAVs crossing the intersection from approach j during a cycle; NA_{jp} is the total number of CAVs crossing the intersection from approach j during phase p; T_c is the cycle length; t_p is the predefined green time of phase p. Besides, for a multi-lane intersection, NA_{jp} can be computed as

$$NA_{jp} = \int_0^{t_p} \sum_{l=1}^{\sigma_{jp}} Q_{lj}(t)dt = \sum_{l=1}^{\sigma_{jp}} \int_0^{t_p} Q_{lj}(t)dt, \tag{5.11}$$

where σ_{jp} is the number of assigned lanes on approach j during phase p.

In each phase p, there are only two approaches that have vehicles crossing the intersection (during the green time) so $Q_{lj}(t) \geq 0$, while the others are held back (during the red time) so $Q_{lj}(t) = 0$. If $Q_{lj}(t)$ is a constant for all lane l during phase p then:

$$NA_{jp} = t_p \sigma_{jp} Q_{lj}. \tag{5.12}$$

From equations (5.10) and (5.11), the optimization problem (5.9) for one cycle can be rewritten as

$$\max \int_0^{T_c} \sum_{j \in \mathcal{J}} Q_j(t)dt = \max \sum_{j \in \mathcal{J}} NA_j = \max \sum_{j \in \mathcal{J}} \sum_{p \in \mathcal{P}} NA_{jp}. \tag{5.13}$$

5.2.3 Problem Transformation

To the best of our knowledge, the relationship between the CAV flow rate and the total travel time at a signalized multi-lane intersection has not been shown. So, we will find this relationship.

First, we will deduce the total travel time as a function of the flow rate $Q(t)$, where $Q(t) = \sum_{j \in \mathcal{J}} Q_j(t)$. Indeed, this total travel time can be computed as:

$$TT = \sum_{j \in \mathcal{J}} \sum_{l \in \mathcal{L}} \sum_{i \in \mathcal{I}_{lj}} (t_{ilj}^{out} - t_{ilj}^{in}) = \sum_{j \in \mathcal{J}} \sum_{l \in \mathcal{L}} \sum_{i \in \mathcal{I}_{lj}} t_{ilj}^{out} - \underbrace{\sum_{j \in \mathcal{J}} \sum_{l \in \mathcal{L}} \sum_{i \in \mathcal{I}_{lj}} t_{ilj}^{in}}_{C} = \int_0^{T_e} Q(t) t dt - C, \tag{5.14}$$

where t_{ilj}^{out}, t_{ilj}^{in} are time when vehicle i in lane l, approach j enters the network and exits the intersection, respectively; C is a constant number which indicates the total time when vehicles enter the network; T_e is the time when the last vehicles exit the intersection.

Note that $\int_0^{T_e} Q(t)dt$ defines the total number of vehicles so $\int_0^{T_e}(Q(t)tdt)$ defines the total travel time. This is also in line with the definition of the total travel time of vehicles to exit the intersection in [76].

Therefore, we have the following *Proposition*.

Proposition 5.1. *If we could increase the flow rate $Q(t)$ as quickly as possible then the total travel time of vehicles TT in the network will be reduced.*

Now, let us consider the flow sequence $Q^m(t)$ such that it has the maximum flow rate in the first time period as follows:

$$\int_0^{T_e} Q^m(t)dt = \int_0^{T_e} Q^k(t)dt, \tag{5.15}$$

$$Q^m(t) \geq Q^k(t), \quad \text{if} \quad 0 \leq t \leq t_k, \tag{5.16}$$

$$Q^m(t) \leq Q^k(t), \quad \text{if} \quad t_k < t \leq T_e, \tag{5.17}$$

where $Q^k(t)$ is an arbitrary flow sequence.

In other words, *Proposition 1* could be stated as: *maximizing the throughput will minimize the total travel time.*

It is worth mentioning here that the capacity does not correspond to the lower speed as in the case of the steady state of the well-known (link-based) fundamental diagrams. In the case of signalized intersections, the traffic state is highly unlikely to be stable or steady because of stopping in the red phase (i.e. signal effects) and queuing effects. Therefore, the maximum throughput does occur at the maximum speed.

Let us define the CAVs that have the same direction (e.g. either turning left or going straight) in each approach as PAVs. Now, we will find a way to increase the throughput during a phase by using *Proposition 2*.

Proposition 5.2. *Assuming that the signal timings are determined so that the total number of PAVs crossing the intersection from approach j during phase p, denoted as NA_{jp}, is maximized if they travel in a platoon with the maximum speed, the smallest gap and the highest number of assigned lanes.*

Thanks to *Proposition 2*, the optimum throughput in one cycle, given in (5.13), can be calculated as follows:

$$\begin{aligned}\max_{v^A_{lj},\, g_{ilj},\, \sigma_{jp}} \sum_{j\in\mathcal{J}} NA_j &= \max_{v^A_{lj},\, g_{ilj},\, \sigma_{jp}} \sum_{j\in\mathcal{J}} \sum_{p\in\mathcal{P}} NA_{jp} \\ = \max_{v^A_{lj},\, g_{ilj},\, \sigma_{jp}} \sum_{p\in\mathcal{P}} \sum_{j\in\mathcal{J}} NA_{jp} &= \max_{v^A_{lj},\, g_{ilj},\, \sigma_{jp}} \sum_{p\in\mathcal{P}} \sum_{j\in\mathcal{J}} \sum_{l=1}^{\sigma_{jp}} \int_0^{t_p} Q_{lj}(t)dt.\end{aligned} \tag{5.18}$$

5.3 Bi-Level Controller and Space Distribution Method

As we cannot control the speed and the position of HVs in the traffic network, the lane-based optimization method in [20] is a passive control. One of the advantages of CAVs is that we are able to cleverly distribute their positions and control their speed in order to maximize the throughput via V2X capability (i.e. trajectory planning method). To this end, we propose here a bi-level optimization framework using a space distribution method (SDM) for traffic control under the connected environment to get the benefits of CAVs. The main idea of the proposed framework is that the positions nearest to the stop bar in all possible lanes are allocated for all PAVs having the ability to cross the intersection during their phase with the maximum speed and smallest gap. These CAVs are called AAVs.

In more detail, *Propososiotion 2* states that the throughput from each approach j during phase p, $\left(NA_{jp}\right)$ depends on how the positions (i.e. gap and lane) are allocated and the speed of the

AAVs is controlled. This SDM allows us to increase the necessary number of assigned lanes for each approach, maximize the speed, and minimize the gap of the AAVs.

5.3.1 Architecture of the Proposed Method

Firstly, let us explain how to optimize the number of assigned lanes for each approach. Consider a 4-lane-4-approach intersection with 4 phases: left turning in phases 1 and 3, straight in phases 2 and 4 as Fig. 5.3. For the sake of simplicity, we ignore the right-turning phase in this example. In the case of HVs, during the straight phase, only the HVs in lane 1 (or 4) could pass through the intersection because vehicles in lane 2 (or 3) are prevented by the preceding ones. Consequently, the throughput might be reduced. In the case of CAVs, however, with the CAVs' capability, we can allocate all PAVs that could cross the intersection during the green time of this phase to go nearest to the stop bar as Fig. 5.4b. Especially, if the number of AAVs in approach A (upstream) is much higher than that in approach B (downstream), we could assign three lanes for approach A and only one lane for approach B. As a result, the throughput during the straight phase could increase significantly (i.e. nearly three times in this example). Note that the number of lanes assigned during the left turning phase for HVs' case is only one lane or two lanes [21]. However, by applying the proposed SDM and a suitable phase sequence for the CAVs' case, we can allow the AAVs to cross the intersection in three lanes as Fig. 5.4a.

Secondly, as the information (i.e. the speed, position, etc.) of the CAVs is known and the traffic signals are predefined, a PAV is considered an AAV if it satisfies the following condition:

$$t_{rp} \geq \frac{d^s_{ilj}}{V_{\max}}, \tag{5.19}$$

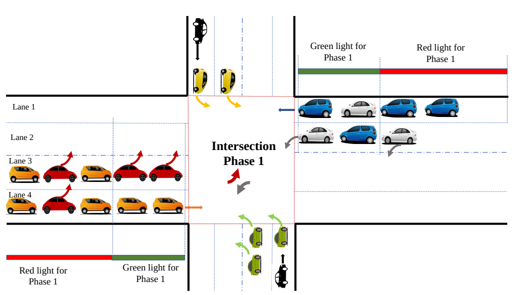
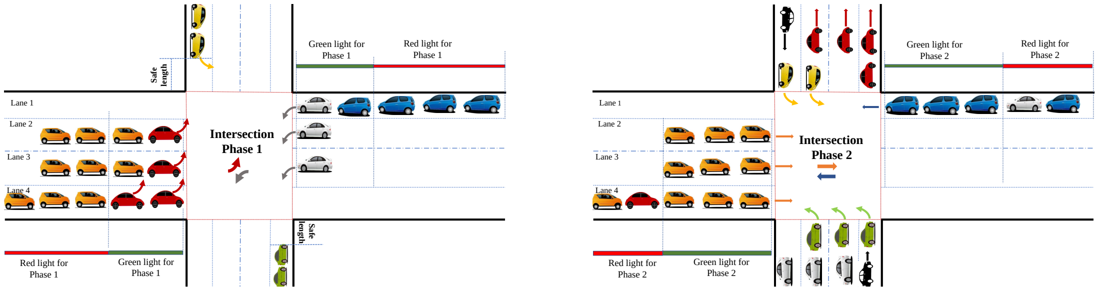


Figure 5.3: The 4-lane-4-approach intersection and 4 phases with predefined signal times for human-driven vehicles. The red and white cars will turn left, orange and blue cars will go straight, etc.



(a) If Phase 1 is active (b) If Phase 2 is active

Figure 5.4: Space Distribution Method for CAVs.

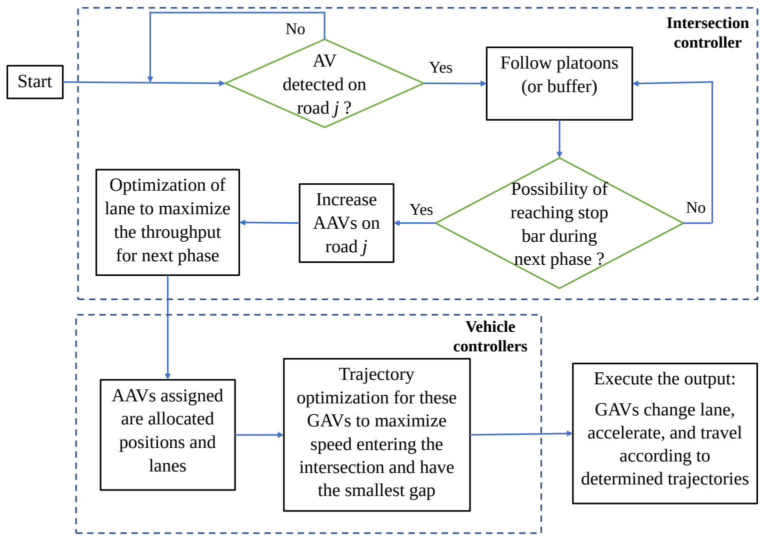


Figure 5.5: The layout of the bi-level controller framework.

where t_{rp} is the remaining time to a red signal of phase p, d^s_{ijl} is the distance of vehicle i on lane l of approach j to the stop bar. Thus, we can estimate the maximum total number of AAVs and consider them a virtual queue waiting for crossing in the next phase.

Fig. 5.5 displays the layout of the proposed bi-level controller framework and describes how to maximize the speed and minimize the gap of the AAVs. It is composed of two levels: the intersection controller (i.e. the upper level) and the vehicle controllers (i.e. the lower level). When the intersection controller (TMC) receives information that there are CAVs entering the intersection on approach j, it commands these CAVs to join a platoon. Next, the TMC checks the CAVs' possibility of reaching the stop bar during the next phase. If they could, the number of AAVs on approach j would be increased. Then, the number of AAVs for the two approaches in this phase will be used in the lane optimization stage to maximize the throughput. After the optimization problem is solved, the TMC will send the position and lane information to each AAV waiting for permission to get through the intersection.

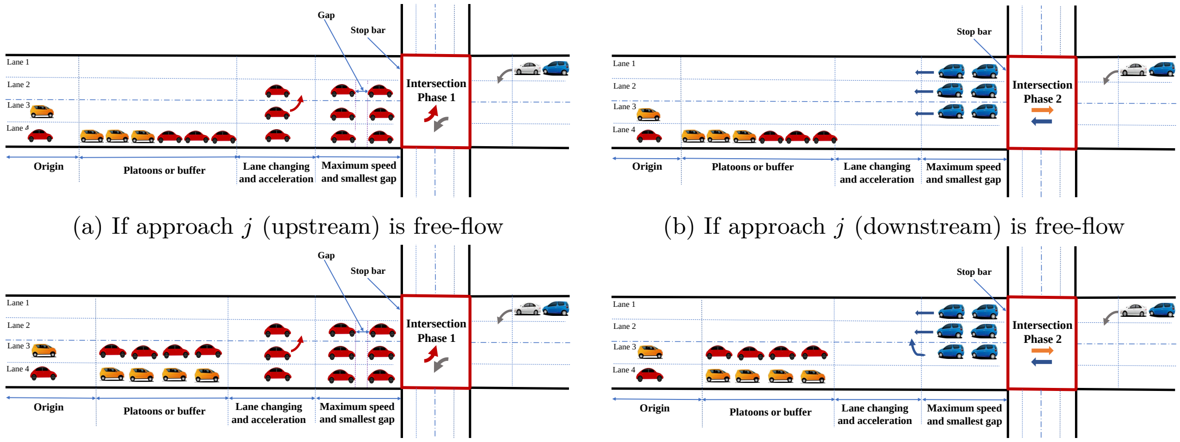
In the lower level, the vehicle controllers (VCs) of these GAVs will calculate their trajectories by themselves in order to reach an advisory speed. Besides, they communicate and cooperate with each other in order to have the smallest gap. These trajectories will satisfy safety constraints (5.4-5.8). Finally, the GAVs execute this stage by the following steps: change lanes, accelerate, and maintain the maximum speed and the minimum gap at the stop bar with safety constraints. On the other hand, if the AAVs do not receive permission, they must keep traveling in the platoon and waiting for the next cycle.

The advantage of using such a bi-level controller framework is that it alleviates the TMC's computational demand for CAVs' trajectories. In particular, if the number of CAVs grows (e.g. more than hundreds) or there are some disturbances (e.g. obstacles or interrupted communication), then the computational demand of a centralized controller may rise exponentially to guarantee the safety constraints. Accordingly, the centralized TMC no longer gets resources to do other work in the future. For instance, it could optimize the traffic signals or cooperate with the neighbor intersection controllers in a multi-intersection traffic network.

5.3.2 Distributing CAVs' Positions

In this section, we describe how to allocate the vehicle's positions on the upstream approach so that they could easily change lanes and accelerate without blocking the vehicles in the other phase. To this end, approach j is divided into 4 segments: Origin, Platoon (or buffer), Lane Changing, and Maximum Speed (see Fig. 5.6a).

The Origin is the first segment when vehicles enter the network randomly. The next one is a platoon formulation segment where vehicles are arranged in a platoon and travel with the same speed under safety constraints. If traffic on approach j is free-flow or much less than that on the other approaches, the CAVs should be allocated in one lane as Fig. 5.6a. The AAVs allowed to cross the intersection in the next phase will be nearest to the stop bar. This allocation will permit other congested approaches to get more lanes in order to boost their throughput. In contrast, if approach j is congested, then the CAVs could be distributed over two lanes as in Fig. 5.6c. In both



(a) If approach j (upstream) is free-flow

(b) If approach j (downstream) is free-flow

(c) If approach j (upstream) is congested

(d) If approach j (downstream) is congested

Figure 5.6: Distributing CAVs' Positions in the approach j.

cases, the CAVs (i.e. the red vehicles) are not blocked by the others (e.g. the yellow vehicles) and can move to the third segment easily. Having reached the last segment, the speed of the GAVs is nearly maximum, and they attempt to minimize their gaps. Finally, they reach the stop bar in time with the maximum speed and minimum gap.

Moreover, we present here ways to avoid a collision in the downstream approach j when the upstream approach is assigned many lanes (e.g. 3 out of 4 lanes). If there are few CAVs on approach j, only one lane will be used as shown in Fig. 5.6b. However, it should be careful when approaching j is congested because it could use more than two lanes. Avoiding a collision in Lane 3 can be done by merging the GAVs (coming from Lane 3) as soon as possible, as shown in Fig. 5.6d. The first method to merge GAVs is that the coming platoon in Lane 3 slows down, and then joins in the tail of Lane 2. Another way is slowing down the platoon in Lane 2 and increasing the gap between the vehicles so that each GAV in the third platoon can leave it. Therefore, the length of the two segments (i.e. the Lane changing and maximum speed) must be long enough to prevent a crash but not too long, because it will alleviate the number of AAVs to approach the stop bar during the green time of the next phase.

It is noted that the order of consecutive phases is very important because it helps to avoid conflicts at the stop bar between the GAVs crossing the intersection to approach j and the AAVs approaching from approach j. Take the situation in Fig. 5.6d as an example, if the next green time is for the red vehicles, the red AAVs will reach the stop bar in three lanes and collide with the blue GAVs (crossing) at the stop bar. The red AAVs (in the westbound) are only permitted to travel to

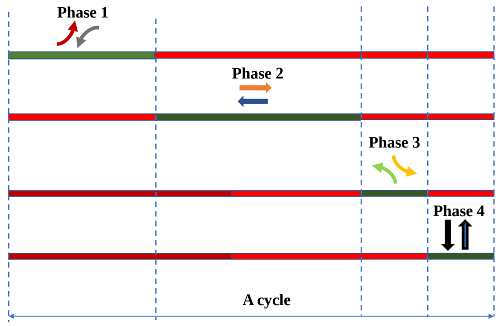


Figure 5.7: A predetermined cycle for traffic signals.

the stop bar when the prior green time is designated for other approaches (i.e. the northbound and southbound approaches). The phase sequence defined as Fig. 5.7 will guarantee this constraint.

5.3.3 Optimization Sub-Problems

So far, we have represented how the proposed method (i.e. the SDM in the upper level) could control the GAVs to obtain the maximum speed and smallest gap on each approach. Consequently, *Proposiotion 2* specifies that the maximum throughput of each approach NA_{jp} only depends on the number of lanes assigned. Thus, the optimization problem for maximum throughput (5.18) transforms to how the lane usages for each approach during each phase are assigned when the maximum number of AAVs (or the length of the virtual queues) is known. Since the traffic signals and the phase sequences are predefined, the lane variables in each phase are independent of each other. As a result, the problem becomes simpler and could be divided into 4 (optimization) sub-problems for 4 phases as follows.

$$\max_{\sigma_{jp}} \sum_{j \in \mathcal{J}} NA_{jp}, \tag{5.20}$$

where σ_{jp} is the number of the assigned lanes for approach j on phase p.

In each phase, there are only two approaches with switching green times, so let σ_{ap} and σ_{bp} denote their corresponding number of assigned lanes. In the straight phases (i.e. phase 2 or 4), to avoid a collision from two opposite directions (i.e. the north-southbound or the west-eastbound) we need to impose the following constraint:

$$\sigma_{ap} + \sigma_{bp} = Lm. \tag{5.21}$$

In addition, the constraints in the left turning phases (i.e. phase 1 or 3) are the following:

$$\sigma_{ap} \leq Lm, \text{ and } \sigma_{bp} \leq Lm. \tag{5.22}$$

It shall be noted that σ_{ap} and σ_{bp} could be equal to Lm when there is no vehicle ($AAVs = 0$) downstream during phase p. Having crossed the intersection, they must merge/change lanes as soon as possible to avoid a collision.

Moreover, when solving the optimization problem for cases in which the number of AAVs on approach A is much higher than that on approach B during phase p, it could lead to $\sigma_{ap} = Lm$. It means that the AAVs on approach B may have to wait for a long time to cross the intersection. To balance this, we should have the following constraint in case the number of AAVs on approach B is non-zero:

$$\sigma_{ap} < Lm. \tag{5.23}$$

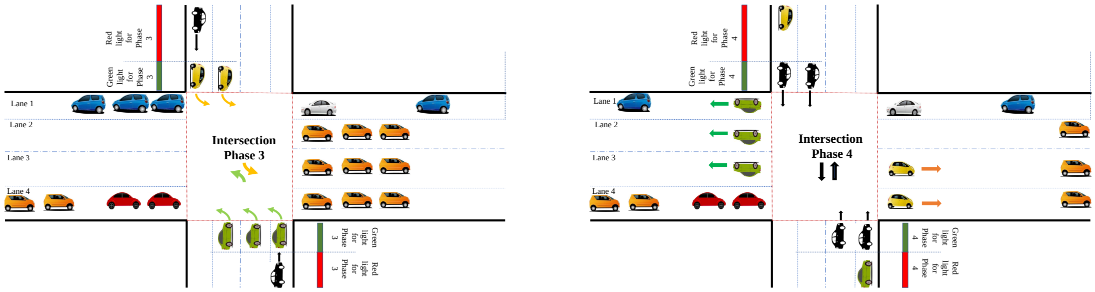
In summary, the optimization sub-problems for each phase become as follows:

$$\begin{aligned} & \max_{\sigma_{ap}, \sigma_{bp}} && (NA_{ap} + NA_{bp}) \\ \text{s.t} \quad & (5.21), && \text{if } p \in \{2, 4\}, \\ & (5.22), && \text{if } p \in \{1, 3\}, \\ & (5.23), && \text{if there are some AAVs in the downstream.} \end{aligned} \tag{5.24}$$

5.4 Numerical Results

5.4.1 Simulation Setup

Let us consider again the 4-lane-4-approach intersection as in Fig. 5.3 and the traffic signals for 4 phases are predefined as in Fig. 5.7. The order of the 4 phases sequence is left turning from the west to the north, straight from the west to the east, left turning from the north to the east, and



(a) If Phase 3 is active (b) If Phase 4 is active

Figure 5.8: Space Distribution Method for CAVs.

straight from the north to the south. It is important to obey this order because it helps to avoid collisions and gives time for the AAVs in the next phase to move to the assigned positions.

Vehicles travel with maximum velocity, which is 20 m/s, and minimum velocity, which is 3 m/s. The maximum and minimum accelerations of vehicles are set at 4.0 m/s^2 and -3.0 m/s^2, respectively. There are a total of 36 vehicles considered in this test, including 17 vehicles from the west, 8 vehicles from the east, 5 vehicles from the north, and 6 vehicles from the south. The red and white CAVs turn left in Phase 1, the orange and blue ones go straight in Phase 2, the yellow and green ones turn left in Phase 3, and the black ones go straight in Phase 4.

To compare with the state-of-art results, four methods are studied to optimize the throughput as follows: human-driven vehicles method (HVM), non-lane base method (NLM), lane base method (LM), and space distribution method (SDM). In more detail, the HVM is used for only HVs without communication between them, so they only attempt to reach the stop bar quickly. Whereas the NLM and the LM are adopted for CAVs and attempt to avoid stop-and-go waves by calculating their trajectories, so that the CAVs could reach the stop bar with maximum speed under safety constraints. However, the NLM does not optimize the number of lanes, but the LM does. The SDM proposed in this paper does not only calculate the maximum number of lanes but also estimates and allocates the AAVs' positions as well as minimizes their gaps.

5.4.2 Simulation Results

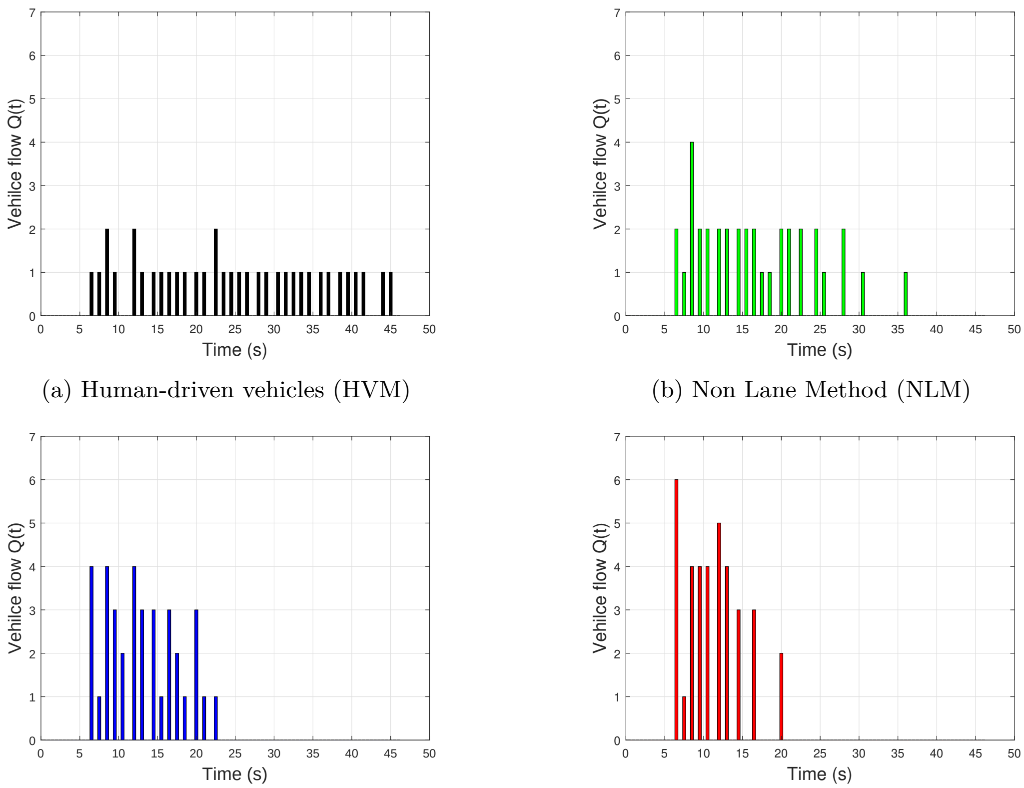
Let us look at Fig. 5.3 again, in which we will optimize the throughput for each phase.

Table 5.1: The number of lanes assigned and throughput of the intersection for each phase with 4 methods: HVM, NLM, LM, and SDM

	Phase 1		Phase 2		Phase 3		Phase 4		NA
	WL	EL	WT	ET	NL	SL	NT	ST	
HVM (L)	1	1	1	1	1	1	1	1	
NA_{jp}	1	1	2	1	1	1	0	1	8
NLM (L)	1	1	2	2	1	1	1	1	
NA_{jp}	2	1	4	4	1	1	1	1	15
LM (L)	2	2	3	1	2	2	2	1	
NA_{jp}	3	2	7	2	2	2	2	1	21
SDM (L)	3	3	3	1	2	3	2	2	
NA_{jp}	4	3	9	3	2	3	2	2	28

Phase 1: Note that most methods do not allow the vehicles to change lanes or overtake when they are near the stop bar. It can be seen that for Phase 1, there are only a maximum of 3 HVs on both approaches (i.e. the west and the east one), which could pass through the intersection. It is because some vehicles (i.e. the red and the white ones) allowed to go in this phase are blocked by the others (i.e. the orange and the blue ones) in the next phase. Due to the fact that the IC, in the case of HVM is not able to control the HVs' speed and gap, so only 1 or 2 HVs could cross the intersection.

Nonetheless, we can allocate the positions of the AAVs in more lanes and should have a safe length for the other approaches to avoid a collision when turning left, as in Fig. 5.4a. In this method, the TMC gets information (i.e. the position, speed, etc.) from the CAVs, then estimates that a maximum of 7 AAVs (i.e. 4 AAVs on the west and 3 AAVs on the east) could pass the intersection during the green time of Phase 1. It is thus straightforward to solve sub-problem (5.24) which meets both constraints (5.22) and (5.23), so that the results are $\sigma_{w1} = \sigma_{e1} = 3$. In this phase, both the west and the east approaches could allow the vehicles on a maximum of three lanes to cross the intersection because of the fact that there are some AAVs in the north and the south (downstream) approach. Accordingly, 4 red cars in the west and 3 white cars in the east will speed up and change lanes in the lane-changing segment under safety constraints. Consequently, these GAVs (also AAVs in this phase) will be distributed among three lanes of the upstream approaches (i.e. the west and the east), and the CAVs in the downstream approaches (i.e. the north and the south) must be allocated in one lane. Having crossed the intersection, they should move into two



(a) Human-driven vehicles (HVM)

(b) Non Lane Method (NLM)

(c) Lane Method (LM)

(d) Space Distribution Method (SDM)

Figure 5.9: Flow rate of 4 methods: HVM, NLM, LM, SDM.

lanes to avoid a collision. In Phase 1, hence, we have:

$$\sum_{j \in \mathcal{J}} NA_{j1} = 7.$$

Phase 2: It is noted that each phase may affect the consequent ones in three methods HVM, NLM, and LM. For further information, as the red and orange vehicles share the same Lane 3, and if the red one cannot move in Phase 1, it will prevent the orange ones from moving in Phase 2. Apparently, the flow rate in the next phases is negatively affected, and the throughput of the intersection shall be alleviated. On the other hand, by using the optimal space distribution in the proposed framework, the red platoons and the orange platoons do not influence each other, hence, maximizing the throughput for each phase is independent.

Similarly, the maximum number of vehicles able to get permission to cross the intersection in the straight phase is 7 for three methods HVM, NLM, and LM. Nevertheless, by using the proposed method, we can estimate the maximum number of AAVs on the west approach and the east approach to be 9 and 4, respectively. By solving the optimization sub-problem for lane variables with constraints (5.21) and (5.23), we obtain $\sigma_{w2} = 3, \sigma_{e2} = 1$. So, three lanes are assigned for the west approach and one lane for the east approach, as in Fig. 5.4b. As a result, the GAVs' numbers of these approaches are 9 and 3 in that order. This indicates that:

$$\sum_{j \in \mathcal{J}} NA_{j2} = 12.$$

Phase 3 (Fig. 5.8a) and **Phase 4** (Fig. 5.8b) could be done in the same way as in **Phase 1** and **Phase 2**, ultimately.

Table 5.1 displays the best results of the four methods in each phase, where WL, ET, NL, etc., denote the upstream approach in each phase. That is in the WL (west-left): vehicles cross the intersection from the west in the left turning phase (Phase 1); in the NT (north-straight): vehicles cross the intersection from the north in the go straight phase (Phase 3). There are only two upstream approaches that allow vehicles to cross the intersection in this phase, such as the west-left (WL) and the east-left (EL) in Phase 1. In each method, the first row describes the number of lanes assigned for the upstream approaches at each phase. For example, the SDM permits 3 lanes for WL and 3 lanes for EL to travel at Phase 1, but it assigns 3 lanes for WT and only 1 lane for ET at Phase 2. On the other hand, the second row exhibits the throughput from the upstream approach j during phase p (NA_{jp}) as well as the total throughput (i.e. in the last column). Table 5.1 shows that the SDM always obtains the greatest throughput ($NA = 28$) along with the number of lanes assigned in four methods. In some cases, the throughput from the SDM could be nearly twice or three times in comparison with that from the NLM, especially in the left-turning phase. The LM ($NA = 21$) is better than the NLM ($NA = 15$) because it allows the CAVs to travel in more lanes. The average speed, gap, and lanes from the HVM cannot be controlled, thus it has the least throughput ($NA = 8$).

Furthermore, Fig. 5.9a- Fig. 5.9d illustrate the flow rate $Q(t)$ over time in four methods: HVM, NLM, LM, and SDM, respectively. In the first period time (from 6 s to 16 s), the SDM always has the largest flow rate and the shortest time ($T_e = 21$s) for the last vehicle to exit the intersection. The LM needs a little more time ($T_e = 23$ s) than the SDM, whereas the NLM and the HVM require much higher time, i.e. $T_e = 36$ s and $T_e = 45$ s, respectively, to do this. The reason is that the

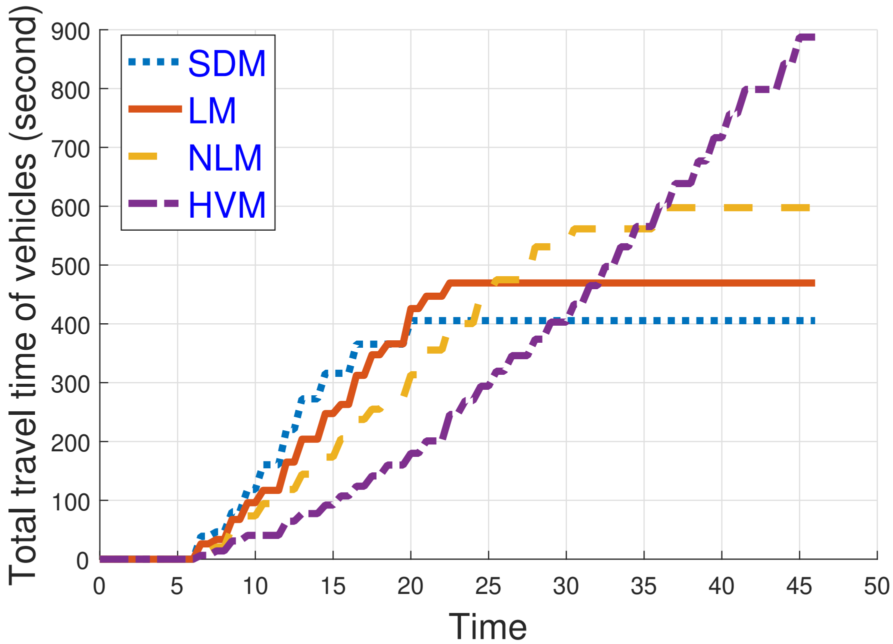


Figure 5.10: Total travel time produced from 4 methods.

NLM and the LM do not exploit the superiority of the CAVs, while the HVM is not able to control the vehicles' speed and position.

As a result, the total travel time or the average time delay from the SDM is always the smallest, as shown in Fig. 5.10 and Table 5.2, which is consistent with *Proposition 1*. The delay from the SDM is reduced by more than a half (51.06%) compared with that from the NLM and by one-third of the delay from the HVM.

From the above results, the SDM shows that it can achieve the most efficient performance.

Table 5.2: The average travel time and delay time of vehicles from 4 methods: HVM, NLB, LM, SDM

	HVM	NLM	LM	SDM
Average Travel Time (s)	24.6528	16.5972	13.0417	11.2639
Average Delay Time (s)	18.5	10.444	6.8889	5.111
Reduction in delay (Compare with NLM)	-77.12%	0%	34.04%	51.06%

5.5 Conclusion

5.5.1 Remarks

This paper has presented a bi-level controller framework using a space distribution method for the operations of the CAVs at a signalized multi-lane intersection with predetermined signal timings. By exploiting the superiority of the CAVs and knowing the predefined signal timings as well as the phase sequence, the intersection controller (i.e. the upper-level controller) will get information from the CAVs, then optimize the lane variables and their positions to maximize the throughput. In the vehicle controllers (i.e. the lower level), each CAV having permission to cross the intersection will evaluate its own trajectory to reach the stop bar in time as well as the exact position and obtain the maximum speed and the minimum gap. This method enhances the throughput and reduces the total travel time remarkably. More specifically, our numerical results have shown that the proposed method outperformed the other methods in most cases. Particularly, the throughput from our proposed framework is increased nearly twice, and the delay is reduced by more than 51% in comparison with those from the non-lane-based method. Nevertheless, this improvement is only limited to simple cases at this proof of concept stage. More intensive experiments with realistic data should be carried out in the future to confirm our findings.

5.5.2 Recommendations

The limitation of this paper is that all vehicles are required fully connected and automated, and the TMC and the CAVs must have perfect communication without any delays or interruptions. Because, if there are some vehicles (e.g. left turning) that do not follow the instructions from the intersection controller due to some reasons (e.g. human vehicles, interrupted communication), it could prevent the others (going straight) from changing lanes to their assigned position or block them to cross the intersection at this time.

In addition, because the SDM method needs three segments on the road, and the platoon segment is used to store the vehicles without permission to move, then the road needs to be long enough to reserve these vehicles. Finally, the more the speed of CAVs increases, the more the collision risk between the CAVs in two consecutive phases at the intersection. Hence, their time and position are required to be measured exactly.

Due to these above reasons, the vehicles need to be perfectly connected and automated, then we assume that they can communicate with no delay (5G) and quickly respond (0.1 second). Therefore, the car-following model is released to simplify the problem (easier for calculation and simulation) in which the follower can simultaneously change the velocity with the leader. Future research should overcome these and consider those situations where some HVs (i.e. without communication capability) or connected vehicles (i.e. without speed control capability) are present (i.e. heterogeneous traffic flow).

The problem in this paper is solved when the signal timings are known. It is much more challenging if we do not know them in advance and thus should optimize these variables. Future research should consider determining the best traffic plans in the proposed framework to achieve better results. Besides, in each signalized intersection, the optimization problem depends on the original states (i.e. velocity and position) of vehicles, the number of lanes and the length on each road, the information (i.e. duration and sequence) of phases, and the interaction of these intersections, so the variables of the network are large and mutually related. Therefore, it is difficult to search all combinations to find the optimum solutions in a network. To solve this problem, we plan to use the decentralized methods to find the solutions in the future works. For example, we can use the Alternating Direction Method of Multiplier (ADMM) to divide the intersection network into many smaller problems in which the local optimum solutions are easier to find.

5.6 Appendices

5.6.1 Proof of Proposition 5.1

To prove this proposition, we can use the following *Lemma*:

Lemma 5.1. *If there exist two flow rate sequences $Q(t)$ and $Q^*(t)$ which have the same number of vehicles entering (or exiting) the network but have different flow rates at the same time, or satisfy* *the following conditions:*

$$\int_0^{T_e} Q(t)dt = \int_0^{T_e} Q^*(t)dt, \tag{5.25a}$$

$$Q(t) \geq Q^*(t), \quad \text{if} \quad 0 \leq t \leq t_1, \tag{5.25b}$$

$$Q(t) \leq Q^*(t), \quad \text{if} \quad t_1 < t \leq T_e. \tag{5.25c}$$

then, we have:

$$\int_0^{T_e} Q(t)tdt \leq \int_0^{T_e} Q^*(t)tdt. \tag{5.26}$$

Proof. It is straightforward to show that:

$$\begin{aligned}
&\int_0^{T_e} Q(t)tdt - \int_0^{T_e} Q^*(t)tdt = \int_0^{t_1} \Big(Q(t) - Q^*(t)\Big)tdt - \int_{t_1}^{T_e} \Big(Q^*(t) - Q(t)\Big)tdt \\
&\overset{(a)}{\leq} t_1 \int_0^{t_1} \Big(Q(t) - Q^*(t)\Big)dt - t_1 \int_{t_1}^{T_e} \Big(Q^*(t) - Q(t)\Big)dt \\
&\overset{(b)}{=} t_1 \int_0^{t_1} \Big(Q(t) - Q^*(t)\Big)dt - t_1 \int_0^{t_1} \Big(Q(t) - Q^*(t)\Big)dt = 0.
\end{aligned} \tag{5.27}$$

In equation (5.27), (a) is derived from the inequalities (5.25b) and (5.25c); whereas (b) is a result of the constraint (5.25a). Hence, equation (5.26) holds. □

From (5.25a, 5.25b, 5.25c) and (5.26), the flow sequence $Q^m(t)$ will result in the smallest total travel time.

5.6.2 Proof of Proposition 5.2

Proof.

$$Q_{lj}(t) = v_{lj}^A(t)\rho_{lj}(t) = v_{lj}^A(t)\frac{n_{lj}}{D}, \tag{5.28}$$

where v_{lj}^A, ρ_{lj}, n_{lj} are the average speed, density, and the number of vehicles crossing the intersection into a constant distant D (e.g., $D = 100$ m) from lane l, approach j, respectively.

Then, the sum of n_{lj} vehicle's length and the gaps between them is smaller than the interval D, which is expressed as follows

$$n_{lj}L + \sum_{i \in \mathcal{I}_{lj}} g_{ilj} \leq D, \tag{5.29}$$

where L is the (average) vehicle length.

Assuming that the CAVs travel in a platoon, so they have the same speed. From equation (5.6), we have

$$g_{lj}^{\max} \geq g_{ilj} \geq g_{lj}^{\min} \geq v_{ilj} = v_{lj}^{A}(t), \tag{5.30}$$

where $g_{lj}^{\min}$ and $g_{lj}^{\max}$ are the minimum and maximum gap between adjacent vehicles in lane l, approach j, respectively. Then constraint (5.29) can be rewritten as

$$n_{lj}L + (n_{lj} - 1)g_{lj}^{\min} \leq n_{lj}L + \sum_{i \in \mathcal{I}_{lj}} g_{ilj} \leq D. \tag{5.31}$$

Therefore, the density in the interval D is limited as

$$\rho_{lj}(t) = \frac{n_{lj}}{D} \leq \frac{D + g_{lj}^{\min}}{(L + g_{lj}^{\min})D} \leq \frac{D + v_{lj}^{A}(t)}{(L + v_{lj}^{A}(t))D}. \tag{5.32}$$

Then, the flow rate $Q_{jl}(t)$ given in equation (5.28) has the upper bound as:

$$\begin{aligned} Q_{jl}(t) &\leq v_{lj}^{A}(t) \frac{D + v_{lj}^{A}(t)}{(L + v_{lj}^{A}(t))D} \\ &\leq V_{\max} \frac{D + V_{\max}}{(L + V_{\max})D} = Q_{\max}. \end{aligned} \tag{5.33}$$

Consequently, the total number of vehicles NA_{jp} is represented as

$$\begin{aligned} NA_{jp} &= \sum_{l=1}^{\sigma_{jp}} \int_0^{t_p} Q_{lj}(t)dt \leq \sum_{l=1}^{\sigma_{jp}} \int_0^{t_p} Q_{\max}dt \\ &\leq \sigma_{jp}^{\max} Q_{\max} t_p = NA_{jp}^{\max}. \end{aligned} \tag{5.34}$$

The equality in (5.34) happens when

$$v_{lj}^{A}(t) = V_{\max}, \tag{5.35a}$$

$$g_{lj}^{\max} = g_{lj}^{\min} = v_{lj}^{A}(t), \tag{5.35b}$$

$$\sigma_{jp} = \sigma_{jp}^{\max}. \tag{5.35c}$$

As a result, *Proposition 2* holds. □

Chapter 6

Conclusions and Further Works

In this chapter, we summarize our research contributions and discuss some potential directions for further research.

6.1 Major Research Contributions

In our first set of contributions [77], we have investigated the kinematic vehicle models for the connected and autonomous vehicles (CAVs) in multi-lane freeways. Specifically, we provide an insightful exploration of CAV trajectories, which comprise two fundamental maneuvers: lane-keeping and lane-changing processes. Our primary objective in this research is to minimize the total travel time of CAVs, and we described the problem's constraints. To address these challenges, we introduce three types of cooperative space distribution methods (CSDM). The proposed methods are shown with remarkable efficiency, consistently achieving over 90% of the lower bound across various scenarios.

In our second set of contributions [72], we introduce a bi-level controller framework for the operations of CAVs at signalized multi-lane intersections, where signal timings are predetermined. We begin by considering vehicle dynamic models and proceed to formulate a minimization problem aimed at reducing the total travel time of CAVs and satisfying the safety constraints. To tackle this problem, we propose the space distribution method (SDM) to guide vehicles safely and swiftly through intersections. Our research showcases the outstanding performance of our proposed designs compared to conventional control methods that lack effective CAV-TMC cooperation.

6.2 Further Research Directions

The following research directions are of importance and deserve further investigation.

6.2.1 Mixed Traffic Systems

Our research contributions published in two IEEE journal papers only considered the scenario with fully autonomous vehicles, which are expected to become a reality in the long term. If there are some vehicles (e.g., left-turning vehicles) that do not follow the instructions from the intersection controller due to some reasons (e.g., human vehicles, interrupted communication), it could prevent the others from changing lanes to their assigned position or block them from crossing the intersection. Besides, we assume that there is no delay or interruption in communication between the CAVs and between the CAVs and the TMC. Our future work will study the practical scenario with both human-driven vehicles and autonomous vehicles in traffic flow. Moreover, we assume that there can be small disturbances that insignificantly affect CAVs' trajectories, and the vehicle controllers can quickly regulate the designed trajectories. These assumptions motivate further work that considers major disturbances (e.g., the CAVs suddenly decrease to low velocity due to accidents or loss of wireless signal).

6.2.2 Multiple/Other Design Goals

The main goal of our research works presented in this dissertation is to reduce the total travel time of vehicles. In addition to this objective, we could consider reducing the energy consumption or maximum acceleration and limiting the jerk in the acceleration segment (the second segment in our design) to improve the smooth and comfortable maneuvers of the vehicles and passengers. However, reducing the maximum acceleration and the jerk could lead to the longer length of this acceleration segment, and, therefore, a longer communication range is required between the management center and CAVs. Due to the space constraint, we do not consider this interesting comfort-related issue in our study, which is left for future work. This allows us to focus on developing CAVs' trajectory control strategy to achieve maximum flow rate or minimum total travel time for CAVs.

6.2.3 Other potential extensions

The complexity analysis was not conducted in our papers. Therefore, our future work will involve a comprehensive study to compare the computational complexity of our proposed control designs with existing baseline methods. This analysis aims to provide insights into the computational efficiency of our methods and identify areas for further optimization.

Besides, platoon formation strategies should be further studied. Another way for future investigation is the exploration of various platoon formation strategies and their impact on the achieved throughput of the proposed control mechanisms. By analyzing different formation patterns and coordination techniques, we aim to enhance the efficiency and scalability of our control strategies in real-world traffic scenarios.

Finally, it is essential to extend simulations considering various practical scenarios and traffic density. Further research will involve conducting more extensive simulations to evaluate the performance of our design under a more comprehensive range of traffic models and varying traffic density regimes. By considering diverse traffic conditions, including congested and free-flowing scenarios, we aim to assess the robustness and adaptability of our control strategies in various real-world environments.

By addressing these issues in future studies, we aim to advance our proposed control designs' understanding and practical applicability in improving traffic flow efficiency and congestion management.

6.3 List of Publications

6.3.1 Journals

[J2]. Tung Thanh Phan, Long Bao Le, and Dong Ngoduy "A cooperative space distribution method for autonomous vehicles at a lane-drop bottleneck on multi-lane freeways," *IEEE Trans. Intell. Transp. Syst,* vol. 23, no. 4, pp. 3710–3723, Apr. 2022.

[J1]. Tung Thanh Phan, Dong Ngoduy, and Long Bao Le "Space distribution method for autonomous vehicles at a signalized multi-lane intersection," *IEEE Trans. Intell. Transp. Syst,* vol. 21, no. 12, pp. 5283-5294, Dec. 2020.

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