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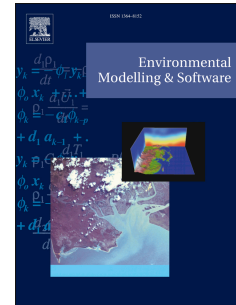
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# Multivariate overall and dependence trend tests, applied to hydrology

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## **CRedit authorship contribution statement**

**Dorsaf Goutali:** Conceptualization, Formal analysis, Methodology, Software,

Validation, Writing – original draft, Writing - Review & Editing. **Fateh Chebana:**

Conceptualization, Methodology, Supervision, Validation, Writing – review &

editing

## 24 **Abstract**

25 Given climate change, trend detection is gaining increasing attention in the context of multivariate  
26 frequency analysis. In this paper, we propose new statistical tests for multivariate trend detection.  
27 The first one, a multivariate overall trend (MOT) test, is designed to detect trend in all components  
28 of the multivariate distribution (margins and dependence structure) whereas the second test is a  
29 multivariate dependence trend (MDT) test focusing on detecting trend in the dependence structure.  
30 A simulation study is used to evaluate the performance of the proposed tests. Results show that the  
31 proposed MOT test performs well when trend is present in margins, in the dependence structure  
32 and/or in both. Likewise, results of the proposed MDT test indicate an interesting power when the  
33 trend is in the dependence structure. Moreover, an application to a real dataset is provided.  
34 Performing the proposed tests with the univariate tests provides a complete overview of trend  
35 detection.

36 **Keywords:** Trend, Hydrology, Multivariate, Non-stationarity, Copula, dependence structure.

## 37 **Highlights**

- 38 • Two multivariate trend tests for multivariate hydrological series are proposed.
- 39 • New multivariate overall trend (MOT) test dealing with trend in all the components of the  
40 whole multivariate distribution.
- 41 • New multivariate dependence trend (MDT) test focuses on trend in the dependence  
42 structure.
- 43 • Vast simulation study is considered to evaluate the performance of the tests.
- 44 • The developed tests show high performance, with increasing power observed as the trend  
45 slope and sample size increase..

## 46 **Software and/or data availability**

47 **Software:**

- 48 • The code used for developing the multivariate trend tests and the simulation study
- 49 scenarios, implemented in the R language, can be found on GitHub at the following
- 50 link: <https://github.com/GOUD05/Multivariate-Trend-Tests.git>
- 51 • Repository creator: Dorsaf Goutali.
- 52 • Creation date: 2024.
- 53 • Contact Information: [Dorsaf.goutali@inrs.ca](mailto:Dorsaf.goutali@inrs.ca).
- 54 • Program Language: R version 4.1.3 (64bit).
- 55 • Required Software: R (Download from <https://cran.r-project.org/>), RStudio
- 56 (Download from <https://www.rstudio.com/>).
- 57 • Cost: free.
- 58 • Required R Packages:
  - 59 - copula <https://cran.r-project.org/web/packages/copula/index.html>,
  - 60 - Kendall , <https://cran.r-project.org/web/packages/Kendall/index.html>,
  - 61 - resample , <https://cran.r-project.org/web/packages/resample/index.html>,
  - 62 - VGAM: <https://cran.r-project.org/web/packages/VGAM/index.html>,
  - 63 - openxlsx: <https://cran.r-project.org/web/packages/openxlsx/index.html>,
  - 64 - gtools: <https://cran.r-project.org/web/packages/gtools/index.html>.
- 65 • Used Hardware: Computer with Windows 10, Intel i5 8th Gen processor 8 GB RAM,
- 66 256 GB storage.

67 **Data:**

- 68 • This study relies on the generation of synthetic data as a requisite part of the
- 69 methodology to conduct simulations study. The provided code on GitHub generates
- 70 data and simultaneously calculates the performance of the tests. The data used in
- 71 the illustrative applications will be available on request.

72

## 73 1. Introduction

74 Hydrological frequency analysis (HFA) is widely used for modeling extreme hydro-  
75 meteorological events like floods, droughts, and storms (e.g. Hamed & Rao, 1998). Such events  
76 are often identified by correlated features, such as peak, volume, and duration for floods (e.g.  
77 Chebana & Ouarda, 2021; Grimaldi & Serinaldi, 2006). These dependent features highlight the  
78 need for a multivariate HFA approach, supported by various studies (e.g. Genest & Chebana, 2017;  
79 Li *et al.*, 2019; Requena *et al.*, 2013). Univariate HFA can provide only limited assessment of  
80 extreme events and their probability of occurrence (e.g. Chebana & Ouarda, 2011; Joyce *et al.*,  
81 2018).

82 Commonly, HFA is based on the assumptions of stationarity, homogeneity, and serial  
83 independence. In the multivariate context, checking these assumptions, particularly stationarity,  
84 attracted less attention compared to modeling (e.g. Chebana & Ouarda, 2021; Gu *et al.*, 2018).  
85 Ignoring the testing step of these assumptions can lead to inaccurate results and potentially wrong  
86 decisions (e.g. Chebana *et al.*, 2013). Indeed, this step contributes to the choice of the appropriate  
87 model, which should integrate possible trends in some or all components of the multivariate  
88 distribution (margins and dependence structure). The stationarity assumption has long been  
89 compromised by climate change and human activities such as deforestation, and overuse of  
90 extraction from surface water and ground water (e.g. Milly *et al.*, 2008; Tan & Gan, 2015; Vidrio-  
91 Sahagún *et al.*, 2024). Related to theoretical considerations, it is no longer valid to believe that the  
92 design flood is always stationary (e.g. Aissia *et al.*, 2014; Kang *et al.*, 2019; Milly *et al.*, 2008).  
93 Therefore, in recent years, increasing attention has been paid to hydrological designs under non-  
94 stationarity (NS) conditions and particularly in the multivariate setting (e.g. Chebana & Ouarda,  
95 2021; Li *et al.*, 2016; Zhang *et al.*, 2022).

96 A wide variety of parametric and non-parametric tests has been employed for trend detection (e.g.  
97 De Luca & Napolitano, 2023). The Mann-Kendall (MK) and the Spearman rank order correlation

**Table 1: Overview of existing tests for trend in univariate and multivariate framework**

98 (SR) tests are among the most non-parametric considered univariate trend tests (e.g. Chong *et al.*,  
99 2022; Conover, 1980; Kendall, 1975; Ouarda *et al.*, 2018). In addition, Chebana *et al.* (2013)  
100 presented an overview of the available multivariate extensions of the univariate MK and SR tests.  
101 Being non-parametric and powerful is the main advantage of these multivariate tests. However, the  
102 latter were initially developed and designed for water quality analysis even though they have been  
103 directly employed later in HFA. Moreover, these multivariate tests are essentially based on their  
104 univariate counterparts (component-wise tests), do not take into account the dependence between  
105 the variables, and cannot identify the affected component . On the other hand, it seems that testing  
106 for trends in the dependence structure has not been explored yet.. Furthermore, upon reviewing the  
107 literature, it appears that there are no recently developed trend tests and recent studies (e.g. Chebana  
108 & Ouarda, 2021; Chebana *et al.*, 2013; Jalili Pirani & Najafi, 2020; Kang *et al.*, 2019; Karahacane  
109 *et al.*, 2020; Modarres, 2018; Xu *et al.*, 2023) consider multivariate tests reviewed by Chebana *et*  
110 *al.* (2013). In Table 1 the univariate and multivariate trend tests are summarised including their  
111 advantages and drawbacks.

112 In order to overcome the drawbacks of the multivariate trend tests, the objective of the present  
113 paper is to propose two multivariate trend tests. The first proposed test is a multivariate overall  
114 trend (MOT) test dealing with trend in all the components of the whole multivariate distribution  
115 (margins and the dependence structure). The second proposed one, a multivariate dependence trend  
116 (MDT) test, focuses on trend in the dependence structure. Therefore, the proposed tests, along with  
117 the existing univariate trend tests, allow dealing with the multivariate distribution as whole as well  
118 as its components.

Tests		Advantages	Drawbacks	Some references
Univariate tests	Mann-Kendall (MK)	<ul style="list-style-type: none"> <li>Both tests have been recommended by the World Meteorological Organization as standard nonparametric procedures</li> <li>Powerful</li> <li>Robustness against missing values and outliers</li> <li>Making very few assumptions</li> <li>Detect increasing decreasing trend</li> <li>Simple to apply</li> </ul>	<ul style="list-style-type: none"> <li>The existence of positive autocorrelation in the data increases the probability of detecting trends when actually none exist, and vice versa</li> <li>Inability to detect non-monotonic trend structures</li> </ul>	Mann (1945); Yue <i>et al.</i> (2002) Bihrat and Bayazit (2003) Yue and Pilon (2004) Rutkowska (2015) Wang <i>et al.</i> (2020) Hamed and Rao (1998)
	Spearman's rho (SR)			
Multivariate component wise tests	Covariance-Inversion test (CIT)	<ul style="list-style-type: none"> <li>Non-parametric tests do not make any assumption or precondition about the models</li> <li>Detect increasing/decreasing trends</li> <li>Simple to apply</li> </ul>	<ul style="list-style-type: none"> <li>Designed for water quality analysis and not for hydrological fields, existing comparisons and evaluations of these tests are often based on scenarios that do not align with the hydrological context (e.g. sample size, distributions)</li> <li>Essentially based on their univariate counterparts (component-wise tests)</li> <li>Simple combinations of univariate tests and do not take into account the dependence between the variables</li> <li>Cannot identify the affected components</li> </ul>	Dietz and Killeen (1981) Hirsch and Slack (1984) Lettenmaier (1988) Loftis <i>et al.</i> (1991) Smith <i>et al.</i> (1993) Thas <i>et al.</i> (1998) Chebana <i>et al.</i> (2013)
	Covariance-Eigenvalue test (CET)			
	Covariance Sum test (CST)			

119  
120 The developed tests, MOT and MDT, are based on multivariate extension of Kendall's  $\tau$  and not  
121 on combinations of univariate statistic tests. A simulation study to evaluate and compare the  
122 performances of the proposed tests is presented. The proposed tests are general and can be  
123 considered in other contexts and applications dealing with trends.

124 The paper is organized as follows. A brief theoretical background, related to the developed tests,  
125 is presented in Section 2. The proposed statistical tests for trend are described in Section 3. The  
126 simulation study to evaluate the performance of the tests is given in Section 4. The conclusions are  
127 reported to Section 5.

## 128 2. Available multivariate trend tests

129 In this section, we briefly present the available univariate and multivariate tests for trend. In  
130 statistical hydrology, mainly two non-parametric rank-based statistical tests are considered, namely

131 the MK and SR tests. Even though, they have the same aim and similar performance, the univariate  
 132 SR tests are less employed than MK ones (e.g. Sneyers, 1990; Yue *et al.*, 2002). More details about  
 133 multivariate SR tests are provided in Chebana *et al.* (2013). In the following, we focus on  
 134 presenting MK tests. Either in the univariate or multivariate settings, the null hypothesis of no trend  
 135  $TR$  is  $H_0 : TR = 0$  against the general alternative hypothesis of a monotonic trend  $H_1 : TR \neq 0$  and  
 136 there exists at least a component  $u$  such that  $TR^{(u)}$  is monotonic (e.g. Chebana *et al.*, 2013).  
 137 The univariate MK test is the most used test to detect monotonic univariate trends. Given a data  
 138 series  $(x_1, x_2, \dots, x_n)$  of length  $n$ , the MK test statistic is given by

$$139 \quad M = \sum_{i=1}^{n-1} \sum_{j=i+1}^n \operatorname{sgn}(x_j - x_i) \quad (1)$$

140 where  $x_j$  and  $x_i$  are both values in the series, and  $\operatorname{sgn}(\cdot)$  is a sign function:

$$141 \quad \operatorname{sgn}(x) = -1 \text{ if } x < 0, \quad = 0 \text{ if } x = 0, \quad = 1 \text{ if } x > 0 \quad (2)$$

142 Under  $H_0$ , the test statistic  $M$  has asymptotically normal distribution with mean  $E(M) = 0$  and

$$143 \quad \operatorname{Var}(M) = \frac{n(n-1)(2n+5)}{18} \quad (3)$$

144 Multivariate extensions of the univariate MK tests have been established to analyze multivariate  
 145 trends in the hydrological context. Table 2 gives an overview of the main properties of those  
 146 existing multivariate MK tests covering their expression and the asymptotic distributions.

147 For all the tests presented in Table 2, let  $M^{(u)}$  be the univariate MK test statistic for the observed  
 148 time series  $X_i^{(u)}, i = 1, \dots, n$  and component  $u = 1, \dots, d$ . For a given  $u$ ,  $M^{(u)}$  is defined as:

$$149 \quad M^{(u)} = \sum_{1 \leq i < j \leq n} \operatorname{sgn}(x_j^{(u)} - x_i^{(u)}) \quad (4)$$



152 Under the null hypothesis  $H_0$  of no trend,  $M^{(u)}$  is asymptotically  $d$ -dimensional normal with zero  
 153 mean and covariance matrix  $C_M = (C_{u,v})_{u,v=1,\dots,d}$  with  $C_{u,v} = \text{cov}(M^{(u)}, M^{(v)})$  which is estimated  
 154 by

$$155 \quad \hat{C}_{u,v} = \frac{t_{u,v} + r_{u,v}}{3} \text{ for } u \neq v \quad (5)$$

156 where

$$157 \quad t_{u,v} = \sum_{1 \leq i \leq j \leq n} \text{sgn}((x_j^{(u)} - x_i^{(u)})(x_j^{(v)} - x_i^{(v)})) \quad (6)$$

$$158 \quad r_{u,v} = \sum_{i,j,k=1}^n \text{sgn}((x_k^{(u)} - x_j^{(u)})(x_k^{(v)} - x_i^{(v)})) \quad (7)$$

**Table 2: Summary of the available multivariate MK-based trend tests**

Expression of the test statistic	Asymptotic distribution under $H_0$ and decision rule
Covariance Inversion test (CIT) $D = M' C_M^{-1} M \quad (8)$ where $C_M^{-1}$ is the inverse matrix of $C_M$	<ul style="list-style-type: none"> <li>It is asymptotically <math>\chi^2(q)</math> distributed under <math>H_0</math>, where <math>q</math> is the rank of the matrix <math>1 \leq q \leq d</math>.</li> <li>The null hypothesis is rejected: if the value of <math>D</math> exceeds the critical threshold determined according to <math>\chi^2(q)</math> distribution quantile, depending on the fixed significance level <math>\alpha</math>.</li> </ul>
Covariance Sum test (CST) $H = \sum_{u=1}^d M^{(u)} \quad (9)$	<ul style="list-style-type: none"> <li>The statistic <math>H</math> is asymptotically normal under <math>H_0</math>, with mean <math>E(H) = 0</math> and variance:               <math display="block">\text{var}(H) = \sum_{u=1}^d \text{var}(M^u) + 2 \sum_{v=1, v \neq u}^{d-1} C_{u,v} \quad (10)</math> </li> </ul>
	where $C_{u,v} = \text{cov}(M^{(u)}, M^{(v)}) \quad (11)$
Covariance Eigenvalue test (CET) $L = \sum_{u=1}^d (M^{(u)})^2 \quad (12)$	<ul style="list-style-type: none"> <li>The null hypothesis is rejected: similar to CIT</li> <li>The statistic <math>(M^{(u)})</math> for <math>u=1, \dots, d</math> are asymptotically normally distributed with zero mean and the approximate variance is <math>\sigma^2 = \text{var}(M^{(u)})</math> as in (3)</li> <li>If <math>(M^{(u)})</math> are independent, The statistic <math>L</math> would be asymptotically <math>\sigma^2 \chi^2(q)</math>- distributed under <math>H_0</math> where <math>q</math> is the rank of the covariance matrix as given in (5)</li> </ul>

160 Notations:  $n$  is the sample size and  $d$  is the dimension or the number of components. More details about multivariate  
 161 tests are provided in Chebana *et al.* (2013)

### 162 3. Proposed multivariate trend tests

163 To overcome the drawbacks mentioned above, the developed multivariate tests rely on two main  
 164 aspects. The first one is the multivariate extension of the rank correlation coefficient Kendall's  $\tau$ .  
 165 This idea draws from Kendall's  $\tau$  relationship with the univariate MK trend test statistic. The  
 166 second ingredient is the moving window technique over the dependence.

#### 167 Kendall's $\tau$ and univariate MK test

168 Kendall's  $\tau$  is defined, in the bivariate and usual case, as the difference between the probabilities  
 169 of concordance and discordance between two variables  $X$  and  $Y$  respectively with series  $x_1, x_2$   
 170  $\dots, x_n$  and  $y_1, y_2, \dots, y_n$  (e.g. Kendall & Gibbons, 1990):

$$171 \tau_{(X,Y)_n} = \frac{2}{n(n-1)} \sum_{(i<j)} \text{sgn}(x_j - x_i)(y_j - y_i) \quad (13)$$

172 Hence, the statistic of the univariate MK test statistic is a particular case of Kendall's  $\tau$  (e.g. Dietz  
 173 & Killeen, 1981; Hamed & Rao, 1998). Indeed, Kendall's  $\tau$  has also been used to test the  
 174 significance of trends in univariate data if the values in  $Y$  are replaced by  $T$  the time order of the  
 175 time series  $X$ , i.e.  $T = 1, 2, \dots, n$ . In that case, the test is called as Mann-Kendall test and the  
 176 equations in (1) and (13) become the same (e.g. Hamed & Rao, 1998; Hirsch & Slack, 1984).  
 177 Therefore, in an analogous way, the multivariate proposed test statistics are based on multivariate  
 178 extension of Kendall's  $\tau$ .

#### 179 Kendall's $\tau$ in $d$ -dimension and the proposed tests

180 In the literature, two extensions of Kendall's  $\tau$  have been proposed in higher dimensions (e.g.  
 181 Genest *et al.*, 2011). Consider a random vector  $X$  taking values in  $\mathbb{R}^d$  with cdf  $H(x) = \mathbb{P}(X \leq x)$  and  
 182 continuous marginal distribution  $F_1, \dots, F_d$ . Referring to Joe (1990), the first option of  $d$ -variate  
 183 version of Kendall's  $\tau$  for  $H$  is defined by:

$$184 \quad \tau_d(X) = \frac{2^d \mathbb{E}_H\{H(X)\} - 1}{2^{d-1} - 1} \quad (14)$$

185 where  $\mathbb{E}_H$  denotes the expectation with respect to  $H$ . Note that  $\mathbb{E}_H\{H(X)\} = \mathbb{E}_C\{C(U)\}$ , where  $C$   
 186 is the copula of  $H$  and  $U = (F_1(X_1), \dots, F_d(X_d))$ . The second option was established by Kendall and  
 187 Smith (1940). It is defined as the average value of Kendall's  $\tau$  taken over all possible pairs  $(X_r, X_s)$ ,  
 188 with  $r, s = 1, \dots, d$  and  $r \neq s$ , viz. and  $H_{r,s}$  is the bivariate cdf of  $(X_r, X_s)$ :

$$189 \quad t_d(X) = \frac{1}{d(d-1)} \sum_{r \neq s} \tau(X_r, X_s) \quad (15)$$

190 To develop the proposed tests, we used the  $d$ -variate extension given in the first option in (14).  
 191 Indeed, this extension has the advantage to be expressed in terms of copulas. The use of copula  
 192 allows to take into account the whole dependence structure instead of only dependence between  
 193 pairs as in (16) (e.g. Genest *et al.*, 2011; Li *et al.*, 2011). Further, Nelsen (1996) mentioned that  
 194 when  $d = 3$ , both extensions (14) and (15) coincide leading to:

$$195 \quad \tau_3 = t_3 = \frac{1}{3} \{ \tau(X_1, X_2) + \tau(X_1, X_3) + \tau(X_2, X_3) \} \quad (16)$$

196 In our developed tests, the moving window technique has been employed in order to take into  
 197 account the dependence evolution according to time. Indeed, contrary to the margins, the evolution  
 198 of the dependence structure cannot be directly seen (e.g. Chebana & Ouarda, 2021). Moreover, the  
 199 result of Kendall's  $\tau$  between two series is a single value that represents the strength of the  
 200 dependence and not the evolution of the dependence structure over time. Consequently, in order to  
 201 bring out the aspect of the trend in dependence, Kendall's  $\tau$  should be used in a series. This has  
 202 been achieved by employing a moving window technique. The chosen window size, denoted  $s$ ,  
 203 should be selected in a way to be neither too large nor too small, in order to perform reliable  
 204 analysis and adequate number of values for the identification of the dependence structure (e.g.  
 205 Bender *et al.*, 2014; Chebana *et al.*, 2013).

206 Based on multivariate extension of Kendall's  $\tau$   $\tau_3$  in equation (16) developed by Joe (1990), let's  
 207 substitute  $X_1$  with  $X$ ,  $X_2$  with  $Y$ , and replace  $X_3$  with the time order  $T = 1, 2, \dots, n$ . Let  $\tau_n$  denote  
 208 the empirical version of bivariate Kendall's  $\tau$ . In this context, we introduce the first proposed test  
 209 statistic,  $T_{MOT}$  is given by:

$$210 \quad T_{MOT} = \frac{1}{3} (\tau_n(X, T)^2 + \tau_n(Y, T)^2 + \tau_n(\tau_{nw}(X, Y), T')^2) \quad (17)$$

211 where  $\tau_{nw}$  is the series of the empirical Kendall's  $\tau$  obtained through moving window for  
 212 corresponding series  $X$  and  $Y$  (see Figure 1).  $T'$  is the new series of time order that has the same  
 213 length of  $\tau_{nw}$ . Note that the length  $q$  of the obtained series  $\tau_{nw}$  is related to the sample size  $n$  and  
 214 the width  $s$  of the window as  $q = n - s + 1$ . Choosing the size of  $s$  involves a trade-off. On one  
 215 hand, a small  $s$  increases the number of rolling window series  $q$  for reliable analysis. On the other  
 216 hand, a large  $s$  is necessary to have a sufficient number of values to identify the dependence  
 217 structure, but this might decrease  $q$  (e.g. Bender *et al.*, 2014; Chebana & Ouarda, 2021). In addition,  
 218 the selection of the width  $s$  of the windows is a common challenge to various tests (e.g. Bücher *et*  
 219 *al.*, 2019; Chebana, 2022). To the best of our knowledge, and considering the existing literature,  
 220 formal statistical inference procedures specifically designed to address this purpose appear to be  
 221 not clearly established (e.g. Bücher *et al.*, 2019; Kojadinovic & Yan, 2011).

222 Note that the window step in the moving windows shifts point-by-point in this study. Previous  
 223 studies by Vidrio-Sahagún and He (2022) have shown that a potential bias is introduced due to the  
 224 fact that data points located in the center of the series would be counted more times than those  
 225 located at the bounds, thereby exerting a significant influence on the estimates.

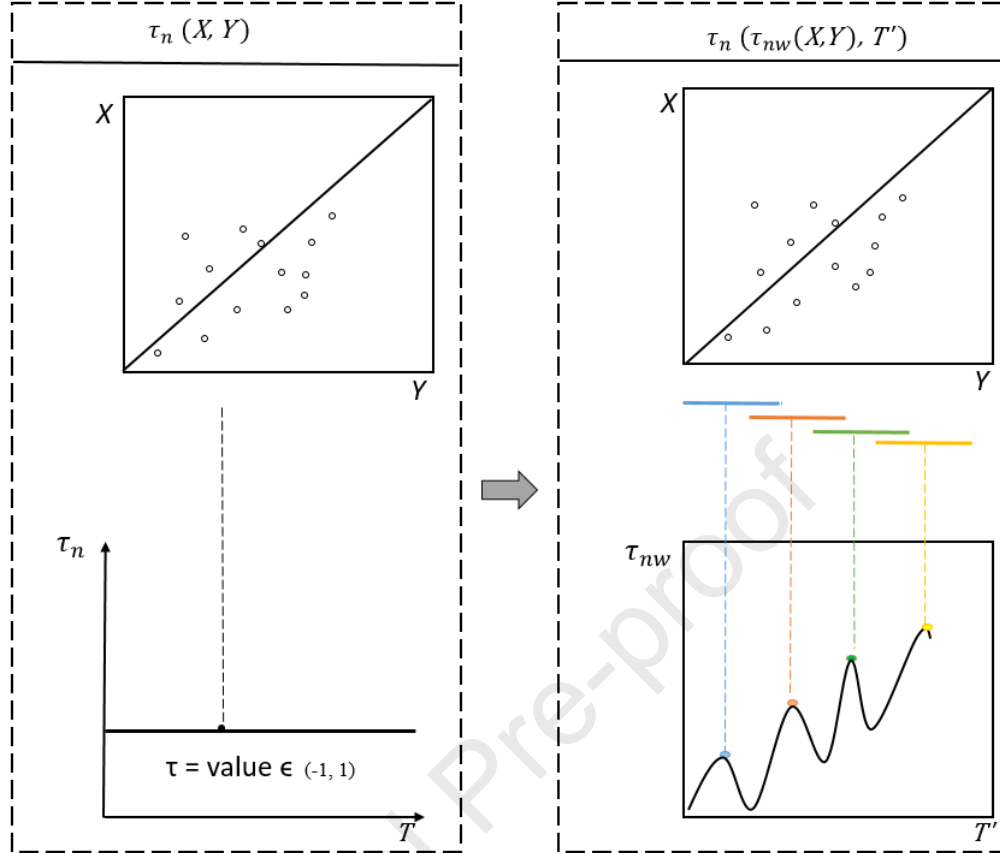
226 We employed the square of each term in order to avoid them cancelling each other or reduce the  
 227 final value of the statistics. This is similar when passing from the test CST in (10) to the test in  
 228 (12). This test is designed to test overall trend in a multivariate series. Indeed, the first two terms

229  $\tau_n(X, T)^2$  and  $\tau_n(Y, T)^2$  focus on the univariate trends in the variables  $X$  and  $Y$ , respectively, with  
 230 respect to the time order  $T$ . However, the last term  $\tau_n(\tau_{nw}(X, Y), T')^2$  introduces a distinctive  
 231 multivariate perspective by considering the Kendall's  $\tau$  between  $X$  and  $Y$  with a moving window  
 232 applied through  $T'$ . Unlike other available multivariate trend tests, this term allows to integrate the  
 233 dependence between the variables in the proposed test. Hence, the developed overall multivariate  
 234 trend test  $T_{MOT}$  considers the trend both in margins and in the dependence structure.

235 In order to focus on the trend in the dependence structure, the following multivariate dependence  
 236 trend ( $T_{MDT}$ ) test is proposed:

$$237 \quad T_{MDT} = \tau_n(\tau_{nw}(X, Y), T') \quad (18)$$

238 It represents the last term in (17) dealing only with dependence.



**Figure 1:** Illustration of evolution of the dependence structure obtained through moving windows for corresponding series X and Y

239

240 To evaluate the *p-values* corresponding to the proposed tests, the bootstrap procedure is considered  
 241 (e.g. Good, 2005). The asymptotic distribution of the proposed statistics is beyond the framework  
 242 of this paper since the distributions of these statistics  $T_{MOT}$  and  $T_{MDT}$  under the null hypothesis  
 243 depend on the unknown copula. Moreover, asymptotic results could be inappropriate in the context  
 244 of HFA, and other fields dealing with extreme values, since the series are usually very short (e.g.  
 245 Nasr & Chebana, 2019; Rutkowska, 2015).

246 The methodology of the proposed tests is based on two well-known notions in statistics and  
 247 applications, i.e. Kendall's  $\tau$  extension and the moving window technique. Regarding the moving  
 248 window technique, used to integrate the evolution of the dependence structure, it has been

249 considered in other studies for different reasons, such as in econometrics (e.g. Selvin *et al.*, 2017),  
250 in finance (e.g. Siami-Namini & Namin, 2018), in medicine (e.g. Dinh *et al.*, 1999) and in statistic  
251 (e.g. Genest & Rémillard, 2004).

252 The proposed tests have several conceptual advantages (along with their performance presented  
253 below). Indeed, they allow overcoming some drawbacks of the existing multivariate tests (Table  
254 1). In fact, the proposed overall statistic test  $T_{MOT}$  is designed to detect the trend in different  
255 components (both margins and dependence structure) and it is not componentwise. The second  
256 proposed test  $T_{MDT}$  is constructed to focus on detecting trend in dependence structure. Then, the  
257 use of the proposed tests  $T_{MOT}$  and  $T_{MDT}$ , along with the univariate testing for each margin,  
258 provides an attractive and complete procedure for testing trend in the multivariate framework. Even  
259 though the proposed tests are introduced and evaluated as part of multivariate HFA, they can be  
260 considered in other fields and application dealing with multivariate trends such as economics,  
261 finance, medicine, and climatology. It is important to recall that these tests are designed to test  
262 *monotonic* trends only.

#### 263 **4. Simulation study**

264 A Monte Carlo simulation study is conducted to evaluate the performance of the proposed  
265 multivariate trend tests (e.g. Hirsch *et al.*, 2015; Hirsch & Slack, 1984) and compare them with the  
266 existing multivariate tests. Since the test CST has already lower performance compared than those  
267 of CIT and CET (e.g. Modarres, 2018), then CST test is not considered. In addition, CET test is  
268 the one recommended among the available multivariate ones (e.g. Chebana & Ouarda, 2021;  
269 Chebana *et al.*, 2013; Lettenmaier, 1988; Modarres, 2018).

#### 270 **4.1 Simulation design**

271 Given that a multivariate distribution can be composed of margins and dependence structure, a  
272 trend can affect these components in different ways. Therefore, we considered the following  
273 scenarios for the bivariate case:

- 274 a) Trend only in the dependence structure
- 275 b) Trend only in one margin
- 276 c) Trend in both margins with the same direction (increasing)
- 277 d) Trend in both margins with different directions
- 278 e) Trend in both margins and also in the dependence structure.

279 The above scenarios were considered in different levels and values in order to evaluate the possible  
280 effects on the performances of the considered trend tests with different factors (direction and  
281 magnitudes of the trend, degrees of dependence and sample size).

282 Data are generated from representative margins and copulas in hydrometeorology analyses to  
283 evaluate the performance of the considered tests (e.g. Nasr & Chebana, 2019; Salvadori & De  
284 Michele, 2010; Zhang & Singh, 2006). The employed copulas are in two groups: Archimedean  
285 (Clayton, Frank, Joe and Gumbel), and Extreme-Value (Galambos and Husler-Reiss).

286 .Even though, a large number of univariate distributions are available, the generalized extreme  
287 value (GEV), lognormal (LN2) and three-parameter lognormal (LN3) have been those developed  
288 in non-stationarity hydrological framework (e.g. Chebana & Ouarda, 2021). In this study, we have  
289 opted for the GEV as the marginal distribution as by previous studies (e.g. El Adlouni *et al.*, 2007;  
290 Gado, 2016).

291 The GEV distribution is parameterized with location ( $\mu$ ), scale ( $\sigma$ ), and shape ( $\xi$ ) parameters. As  
292 in previous studies, the non-stationary aspect is introduced by allowing the location parameter to



293 be linear function of time ( $\mu_t$ ), while keeping the scale and shape parameters constant ( $\mu_0 + \mu_1 t$ ,  $\sigma$ ,  
294  $\xi$ ) where  $t$  is the time order.

295 In this study, the location parameter that characterize the non-stationarity are selected to have weak  
296 trends, a condition frequently observed in hydrometeorological series (e.g. El Adlouni *et al.*, 2007;  
297 Gado, 2016). As in Gado (2016), the location parameter was chosen in the range of  $-0.3 \leq \mu_1 \leq$   
298  $+0.5$  and  $\mu_0 = 0$  in order to test the sensitivity of the proposed tests to the values of a variety of  
299 trends. The scale and shape parameters were fixed at  $\sigma=1$  and  $\xi=-0.1$  respectively (e.g. El Adlouni  
300 *et al.*, 2007). Other values of the shape parameter, such as  $-0.3$  as considered by El Adlouni *et al.*  
301 (2007), have been checked. The obtained results showed no significant changes leading to similar  
302 conclusions (for space limitations, those results are not presented). Note that, given the main  
303 contribution of the present study is in the multivariate framework, the focus is not on univariate  
304 aspects (e.g. selection of marginal distributions and their parameters).

305 In order to consider trend in the dependence structure, we generated random samples from time-  
306 dependent copula  $C_t$  where the corresponding parameter  $\theta_t$  in terms of Kendall's  $\tau$   $\tau_t$  is assumed  
307 to be linear with respect to time, similarly to Nasri *et al.* (2019). Each copula has a specific  
308 parameter range and related to Kendall's  $\tau$  (e.g. Chebana, 2022). On the other hand, in the majority  
309 of flood events, the Kendall's  $\tau$  is between 0.3 and 0.8 (e.g. Nasr & Chebana, 2019; Requena *et al.*,  
310 2013; Zhang & Singh, 2007). Hence, we considered three values of  $\tau = 0.2, 0.6, 0.8$ , representing  
311 weak, moderate, and strong dependence respectively.

312 Different factors could affect the performance of a trend test, either univariate or multivariate,  
313 specifically the sample size  $n$  and magnitudes of the trend (e.g. Bihrat & Bayazit, 2003;  
314 Lettenmaier, 1988; Rutkowska, 2015; Yue *et al.*, 2002). Moreover, the proposed tests could be  
315 affected by dependence strength and copula type (e.g. Quessy *et al.*, 2013). Hydrologic series are

316 usually characterized by small sample sizes. Hence, we considered sample sizes of  $n = 30, 50$  and  
317  $100$  as in other studies (e.g. Barth *et al.*, 2017; Nasr & Chebana, 2019; Santhosh & Srinivas, 2013).  
318 Since the size  $s$  of the rolling window series is related to the sample size  $n$  (e.g. Chebana & Ouarda,  
319 2021), the window size  $s$  is selected respectively as  $s = 10, 15, 20$  for  $n = 30, 50, 100$  similarly to  
320 Nasr and Chebana (2019). This is short enough to have large rolling window series and lengthy  
321 enough to have an adequate number of values for identifying the dependence structure (e.g. Bender  
322 *et al.*, 2014). While the challenge of choosing the size  $s$  of moving windows is common to other  
323 tests in the literature, formal statistical inference procedures for this purpose are lacking in the  
324 literature (e.g. Kojadinovic & Yan, 2011). However, Bücher *et al.* (2019) discussed this matter and  
325 suggest, considering values such as  $s = 2, 3, \text{ or } 4$  and depending on the ultimate interest, one might  
326 also consider choosing  $s$  differently. Chebana and Ouarda (2021) consider  $s = 12$  for 27-sample  
327 size. In the case of Bender *et al.* (2014), the time window length is set to  $s = 50$  years for 191 years.  
328 It is important to note that, across all scenarios,  $N_{sim} = 1000$  samples are generated to ensure stable  
329 results. Preliminary trials were conducted to assess convergence, and stability in results was achieved  
330 with  $N_{sim} = 1000$  samples. Note that, no significant differences have been observed with other values  
331 greater than  $N_{sim} = 1000$  such as  $N_{sim} = 5000$ . The first kind error, or nominal level,  $\alpha$  is set to the  
332 usual value  $\alpha = 5\%$ . To compare the considered tests, we evaluate their ability to estimate  $\alpha$  as  
333 well as to quantify their power  $(1-\beta)$ . Figure 2 summaries the conducted simulation study.

334

#### 335 **4.2 Simulation results**

336 This section presents the obtained results of simulation study by estimating the nominal level and  
337 evaluation the power of the considered tests.

**Figure 2: Diagram of the simulation study**

### 338 *4.2.1. Nominal level estimation*

339 This section reports the estimates  $\hat{\alpha}$  of  $\alpha$  for the proposed tests  $T_{MOT}$ ,  $T_{MDT}$  along with the

**Table 3: First type error estimates (%) by the proposed multivariate tests ( $T_{MOT}, T_{MDT}$ ) and the existing ones (CIT, CET)**

340 multivariate existing tests CIT and CET for different factors: sample sizes, dependence strengths  
 341 and different copula types as presented in Table 3. From Table 3, we observe that the proposed  
 342 tests  $T_{MOT}$  and  $T_{MDT}$  provide estimates  $\hat{\alpha}$  close to the selected significance level  $\alpha = 5\%$  for  
 343 different factors. The first type error should be close to the chosen significance level  $\alpha$  to be exact  
 344 which is the main advantage of proposed tests.

345 Results from Table 3 show that the proposed tests  $T_{MOT}$  and  $T_{MDT}$  are generally not sensitive to  
 346 different factors. First, it can be seen that the proposed tests are almost insensitive to the copula  
 347 type regarding the estimation of  $\alpha$ . Indeed, as an example, for  $T_{MOT}$  and  $T_{MDT}$  tests,  $\hat{\alpha}$  is  
 348 respectively in the range 3.8-5.9%, and 3.8-5.5%, for  $\tau = 0.6$  and  $n = 50$  for different copula type.

349 Regarding the dependence strength, for a given sample size, it has almost no effect on the  
 350 estimation of  $\alpha$  by  $T_{MOT}$  and  $T_{MDT}$  tests. As an example, when considering the Frank copula with  
 351  $n = 50$ , the estimation ranges between 3.5-4.5% for  $T_{MOT}$  and 3.9-5.3% for  $T_{MDT}$ , for different  
 352 values of  $\tau$ . We have similar results regarding the effect of the sample size  $n$ . For example,  
 353 considering a dependence strength  $\tau$  of 0.6 and employing the Clayton copula, estimated  $\hat{\alpha}$  for  
 354  $T_{MOT}$  ranges from 5.1-5.9%, and for  $T_{MDT}$ , it varies between 3.8-5.0% across different sample sizes.

355 As we can see under  $H_0$ , the results presented in Table 3 indicate that existing multivariate CIT  
 356 and CET tests lead to estimates  $\hat{\alpha}$  close to the selected significance level  $\alpha = 5\%$  for different  
 357 factors (in the range 3.1-6.6%). Not that, in the context of water quality, Lettenmaier (1988) found  
 358 that CIT and CET tests provide under-estimations of the nominal values.

359

360 **4.2.2. Power evaluation**

361 In this section, we examine the power of the proposed tests in detecting the trend in the margins  
 362 and dependence structure (combined or separately).

Test	Copula under $H_0$	$\tau = 0.2$			$\tau = 0.6$			$\tau = 0.8$		
		$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$
$T_{MOT}$	Clayton	4.0	4.9	5.6	5.9	5.2	5.1	3.2	5.6	3.9
	Frank	4.9	4.5	4.6	4.5	4.5	6.1	4.6	3.5	4.5
	Joe	4.7	5.2	4.9	6.1	5.9	4.8	4.3	4.3	4.4
	Gumbel	5.8	3.9	5.4	3.8	4.9	4.7	3.8	4.7	5.0
	Galambos	4.6	4.7	6.3	4.9	4.4	4.5	4.9	4.9	4.1
	Husler-Reiss	4.2	5.1	4.9	3.3	3.8	5.5	2.9	4.4	5.4
	Clayton	5.3	4.7	4.8	5.0	3.8	3.9	3.9	3.5	6.0
$T_{MDT}$	Frank	4.8	5.3	4.3	3.4	4.1	4.0	3.6	3.9	6.5
	Joe	4.2	5.2	5.4	3.6	5.4	5.4	3.6	4.5	4.8
	Gumbel	3.9	4.9	3.9	4.4	4.4	4.2	3.6	4.0	5.3
	Galambos	4.2	5.3	4.5	4.4	5.5	5.2	4.9	4.2	3.7
	Husler-Reiss	4.3	5.2	4.7	4.5	4.9	4.6	4.3	5.6	4.4
	Clayton	4.6	6.1	6.1	4.2	5.2	5.3	4.7	4.8	4.5
	Frank	3.1	3.7	5.9	5.5	5.7	3.9	3.5	4.4	6.0
CIT	Joe	5.1	4.7	5.5	4.9	4.8	5.3	4.6	4.4	5.8
	Gumbel	4.6	5.1	4.9	4.1	5.3	5.2	5.6	4.4	5.7
	Galambos	4.7	4.6	4.6	6.1	5.4	5.4	4.9	5.3	4.7
	Husler-Reiss	5.7	5.3	6.0	3.8	4.5	5.4	3.9	4.4	5.9
	Clayton	5.1	6.3	5.7	4.6	4.9	5.5	5.8	5.3	3.7
	Frank	3.1	4.6	6.1	5.5	6.6	3.9	4.7	5.3	4.5
	Joe	5.5	4.2	4.9	5.1	5.2	6.1	5.5	4.1	5.0
CET	Gumbel	5.3	5.6	4.9	4.2	5.7	5.0	4.5	4.8	5.4
	Galambos	5.4	5.2	4.3	6.0	5.0	6.5	4.4	5.9	4.8
	Husler-Reiss	6.0	6.0	5.9	5.2	5.9	3.8	4.4	5.9	5.8

363 **a) Trend in the dependence only**

364 The power of the proposed tests in detecting the trend in the dependence structure is studied.

365 Results for different sample sizes, different copulas and dependence strengths, are displayed in

366 Table 4. One can see overall from Table 4 that the proposed tests  $T_{MOT}$  and  $T_{MDT}$  stand out with

367 high power, in contrast to the CIT and CET tests.

368 From Table 4, one first notes that the type of the copulas and dependence strength seems to have  
 369 little influence on the power of the  $T_{MOT}$  and  $T_{MDT}$  tests; it is rather the sample size that have a  
 370 significant impact. Indeed, we can see that the power of the tests  $T_{MOT}$  and  $T_{MDT}$  is increasing with  
 371 the sample size. For instance, for  $T_{MOT}$  test with Clayton copula, the power increases from 37.0%  
 372 when  $n = 30$  to 94.7% when  $n = 100$  (similarly for  $T_{MDT}$  from 36.7 % to 95.5%). These results  
 373 align with the results from other tests, which also observed that power increase with the sample  
 374 size (e.g. Hirsch *et al.*, 1982).

375 It is also of interest to note that the power of  $T_{MOT}$  and  $T_{MDT}$  has less variability regarding copula  
 376 type. For instance, for  $T_{MOT}$  test with  $n = 50$ , powers are 66.6% and 64.4% when considering  
 377 Clayton copula and Frank copula respectively. As another example, in the  $T_{MDT}$  test with  $n = 100$ ,  
 378 the powers are 95.5% for the Clayton copula and 94.8% for the Galambos copula. These values are  
 379 considerably high. Moreover, the power of  $T_{MOT}$  and  $T_{MDT}$ , remains well regardless of the trend  
 380 direction in the dependence structure. No significant differences were observed in the powers when  
 381 considering various trend directions. For the sake of simplicity and brevity, the results pertaining  
 382 to the decreasing trend direction are not presented.

383 Overall, with some exceptions, both proposed tests lead to similar powers. According to the sample  
 384 size, the power is low values (roughly 30 to 45%), moderate (55 to 67%) to very high (88 to 95%).  
 385 In trend or non-stationarity studies, it is important and appropriate to have  $n$  as high as possible.  
 386 Hence, the proposed tests are adapted to this context.

387 In the comparison of performance between the proposed tests ( $T_{MOT}$ ,  $T_{MDT}$ ) and the existing tests  
 388 CIT and CET, a notable observation is that the latter exhibit inability to detect any trend in the  
 389 dependence structure across all examined scenarios. For instance, for CET test, the power estimate

390 consistently ranges between 3.8%-8.8%, close to those in Table 3. These very low values are  
 391 expected since these tests ignore the dependence structure explicitly in their construction.

Sample size	Test	Archimedean				ExtremeValue	
		Clayton	Frank	Joe	Gumbel*	Galambos	Husler-Reiss
$n=30$	$T_{MOT}$	<b>37.0</b>	<b>38.3</b>	<b>41.9</b>	<b>36.3</b>	<b>43.2</b>	<b>32.2</b>
	$T_{MDT}$	<b>36.7</b>	<b>35.0</b>	<b>43.9</b>	<b>37.6</b>	<b>45.6</b>	<b>31.5</b>
	$CIT$	8.8	4.0	6.5	5.2	6.1	5.1
	$CET$	6.8	4.7	8.0	4.7	4.9	4.7
$n=50$	$T_{MOT}$	<b>66.6</b>	<b>64.4</b>	<b>64.1</b>	<b>59.6</b>	<b>67.2</b>	<b>55.6</b>
	$T_{MDT}$	<b>67.2</b>	<b>59.1</b>	<b>65.5</b>	<b>59.7</b>	<b>67.5</b>	<b>56.1</b>
	$CIT$	6.9	5.5	7.3	4.9	4.8	4.9
	$CET$	6.0	5.1	6.6	4.6	3.8	3.9
$n=100$	$T_{MOT}$	<b>94.7</b>	<b>92.3</b>	<b>92.3</b>	<b>90.1</b>	<b>95.3</b>	<b>87.5</b>
	$T_{MDT}$	<b>95.5</b>	<b>92.1</b>	<b>94.7</b>	<b>88.3</b>	<b>94.8</b>	<b>88.3</b>
	$CIT$	5.4	5.2	6.3	5.5	5.5	4.3
	$CET$	5.0	4.3	6.2	5.3	4.2	3.8

392 This table presents the power of the proposed test at significance level  $\alpha=5\%$ , for different scenarios. The Gumbel\* copula belongs  
 393 both to the class of Archimedean and extreme value copulas

#### 394 ***b) Power evaluation: trend in one margin only***

395 The results corresponding to this scenario are presented in Table 5. Since in this section we are  
 396 only interested in the marginal distributions, we consider only two families of copula (Clayton and  
 397 Galambos) with fixed Kendall's  $\tau$ ,  $\tau = 0.6$ .

398 Table 5 show higher powers of the statistical test  $T_{MOT}$  as both the sample size and the trend slope  
 399 increase, eventually reaching 100%. These high power values demonstrate the efficacy of  $T_{MOT}$  in  
 400 detecting trends in one margin. From Table 5, we can see that the impact of sample size  $n$  on the  
 401 power. For a given slope of location parameter  $\mu_t$  and a copula, the power increases with  $n$ . For  
 402 instance, we consider the case with a location parameter slope  $\mu_1 = 0.1$ , generated from a Clayton  
 403 copula. In this case, the power of the  $T_{MOT}$  test rises notably from 17.5% at  $n = 30$  to 100% at  $n =$   
 404 100. These high powers, highlighting the effectiveness of the  $T_{MOT}$  test in detecting trend within

405 one margin. In addition, no differences in growth are observed between different copulas. For  
 406 example, for  $n = 100$  and  $\tau = 0.6$ , powers of  $T_{MOT}$  tests are 100% for Clayton and Galambos  
 407 copulas.

408 Table 5 shows also high powers of the  $T_{MOT}$  test, particularly when the trend slope increases.  
 409 Indeed, for sample size  $n = 50$  and Galambos copula, power estimates of the  $T_{MOT}$  test increase  
 410 from 42.2% to 100% when slope  $\mu_1$  passes from 0.1 to 0.3. This finding is expected and it is in  
 411 agreement with the literature dealing with univariate trend (e.g. Yue *et al.*, 2002). Moreover, from  
 412 Table 5, when considering a Galambos copula with  $n = 100$ , the  $T_{MOT}$  test demonstrates very high  
 413 powers, reaching 100% for both increasing trend ( $\mu_1 = 0.1$ ) and decreasing trend ( $\mu_1 = -0.1$ ). This  
 414 result demonstrates the effectiveness of the  $T_{MOT}$  test, highlighting its ability to capture trends in  
 415 margins irrespective of their direction.

416 Results from Table 5 show that the proposed test  $T_{MDT}$  is not able to detect any trend in margins.  
 417 In all cases, the power estimates are less than 3.7%. This is not surprising since this test is intended  
 418 to capture trend only in dependence structure.

**Table 5: Power estimates (%) of the proposed tests ( $T_{MOT}$ ,  $T_{MDT}$ ) and existing tests (CIT, CET)-  
 trend in one margin only**

419

Test	Copula	$\tau = 0.6$											
		$\mu_1 = 0.1t$			$\mu_1 = 0.3t$			$\mu_1 = 0.5t$			$\mu_1 = -0.1t$		
		$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$
$T_{MOT}$	Clayton	17.5	40.8	100	96.6	100	100	100	100	100	17.0	39.4	100
	Galambos	15.0	42.2	100	96.0	100	100	100	100	100	17.3	42.5	100
$T_{MDT}$	Clayton	2.9	2.8	3.1	1.1	0.8	0.6	0.4	0.4	0.0	3.7	1.9	1.6
	Galambos	2.9	2.1	1.3	0.6	0.6	0.0	0.5	0.3	0.1	2.6	2.2	2.1
CIT	Clayton	99.9	100	100	100	100	100	100	100	100	99.7	100	100
	Galambos	100	100	100	100	100	100	100	100	100	100	100	100

CET	Clayton	94.3	100	100	100	100	100	100	100	100	94.4	100	100
	Galambos	94.0	100	100	100	100	100	100	100	100	93.7	100	100

420 This table presents the power of the proposed test at significance level  $\alpha=5\%$

421 As part of comparison, Table 5 reports also the performance for the existing CIT and CET tests.

422 Through Table 5, those tests have high power values almost always 100%. These high power values

423 of CIT and CET tests are different from the literature (e.g. Hirsch & Slack, 1984; Lettenmaier,

424 1988). This can be attributed to distinct simulation conditions. Initially developed for monthly

425 water quality data, the CET and CIT tests are not specifically adapted for hydrological data, as in

426 our case. The primary purpose of these tests is the detection of trends in time series, not the

427 frequency analysis of hydrological data. This dissimilarity in objectives and context introduces

428 several differences in simulation conditions. For instance, Hirsch and Slack (1984) and Lettenmaier

429 (1988) based their analyses on sample sizes ranging from 5 to 20, with a slope value from 0.0065

430 to 0.05, specifically chosen to match the features of water quality time series. When we applied

431 some of these features ( $n = 20$ , slope = 0.05), the power of the CET test decreased significantly to

432 11%.

### 433 *c) Power evaluation: trend in both margins*

434 We present the power values here when trend is present in both margins. Table 6 shows that

435 except  $T_{MDT}$ , the power of all tests is very high and can reach 100%. The high power of the

436 developed multivariate  $T_{MOT}$  test clearly emphasizes its effectiveness in detecting trends in both

437 margins. Moreover,  $T_{MOT}$  power significantly increases with  $n$ . For instance, with a Galambos

438 copula and slopes  $\mu_1 = -0.1$  and  $\mu_2 = -0.1$ , the power values for the  $T_{MOT}$  test increase from 56.4%

439 at  $n = 30$  to a 100% at  $n = 100$ .

440 Table 6 demonstrates also the impact of the trend direction between both margins on the power of

441  $T_{MOT}$ . Regardless of whether the trend is increasing or decreasing, the  $T_{MOT}$  test exhibits high



442 power. As an example, when examining different trend directions for both margins with location  
 443 parameters set at  $\mu_1 = 0.3$ ,  $\mu_2 = -0.3$ , and considering Clayton copulas, the power remains at a high  
 444 level of 100% across all sample sizes  $n$ . This high power demonstrates the effectiveness of the  
 445  $T_{MOT}$  test in detecting trends across both margins under diverse directions.

**Table 6: Power estimates (%) of the proposed tests ( $T_{MOT}$ ,  $T_{MDT}$ ) and existing tests (CIT, CET)-trend in both margins**

446 Table 6 provides also insights about the effect of the trend slope on the power on the power of the  
 447 proposed tests. In fact, the proposed test  $T_{MOT}$  performs clearly better when the slope of trend  
 448 increases. As an example, considering  $n = 30$  and Clayton copula, the test power increases from  
 449 53.2% if slope are  $\mu_1 = -0.1$ ,  $\mu_2 = -0.1$  to 100 % when the slopes are  $\mu_1 = 0.3$ ,  $\mu_2 = 0.3$ .

450 The powers here increased (except for the  $T_{MDT}$ ) compared to Table 5 specifically for  $T_{MOT}$ . The  
 451 reason is that we have additional component with trend in the margins. Moreover,  $T_{MDT}$  as designed  
 452 and expected, is not detecting such a trend. CET and CIT continue to slowly increasing to reach  
 453 100% in all cases which similar to the previous case (Table 5). However, importantly,  $T_{MOT}$  is  
 454 adapted to the situation with an increase that is realistic. The reason that tests, except  $T_{MDT}$ , reach  
 455 100% is that the trend in the margins is dominating the non-trend in the dependence (e.g. Bender  
 456 *et al.*, 2014).

457 As part of comparison, the two classical tests CIT and CET are able to detect the trend in both  
 458 marginal distributions with a high performance =100% in all cases. These obtained results are  
 459 different from the literature and the reasons are explained in the previous case (trend in one margin  
 460 only). As anticipated, Table 6 affirms that the proposed statistic  $T_{MDT}$  is not able to detect any  
 461 trend in both margins, aligning with its specific design focused on capturing trend solely within the  
 462 dependence structure. In all instances, the powers remain below 7.8%.

Test	Copula	$\tau = 0.6$											
		$\mu_1 = -0.1t$ $\mu_2 = -0.1t$			$\mu_1 = 0.3t$ $\mu_2 = -0.3t$			$\mu_1 = 0.3t$ $\mu_2 = -0.1t$			$\mu_1 = 0.3t$ $\mu_2 = 0.3t$		
		$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$
$T_{MOT}$	Clayton	53.2	99.8	100	100	100	100	100	100	100	100	100	100
	Galambos	56.4	100	100	97.0	100	100	99.9	100	100	100	100	100
$T_{MDT}$	Clayton	3.2	5.1	5.4	3.6	0.1	6,8	2.4	1.3	0.5	3.4	4.8	5.5
	Galambos	4.9	4.6	5.2	1.5	0.3	7.8	1.7	1.4	1.7	5.0	4.2	5.2
CIT	Clayton	100	100	100	100	100	100	100	100	100	100	100	100
	Galambos	100	100	100	100	100	100	100	100	100	100	100	100
CET	Clayton	100	100	100	100	100	100	100	100	100	100	100	100
	Galambos	100	100	100	100	100	100	100	100	100	100	100	100

463 This table presents the power of the proposed test at significance level  $\alpha=5\%$

464 **d) Power evaluation: trend in both margins and dependence**

465 In this case, we considered a large number of possibilities since all the components of the bivariate  
466 distribution have trends. Note that similar results are obtained when examining either an increasing  
467 or a decreasing trend in the dependence structure, as well as for the direction of the trend  
468 (increasing/decreasing) in both margins. For the sake of brevity, we do not present all the results.

469 Form, results highlight the high power of the  $T_{MOT}$  tests across various copulas and sample sizes.  
470 Notably, the  $T_{MOT}$  test exhibits high performance, reaching 100% power even with weak slopes  
471 and short sample sizes.

472 Notably, as shown in Table 7, similar to the preceding scenario, the performance of trend  $T_{MOT}$   
473 tests is influenced by the sample size. Specifically, a larger sample size correlates with higher  
474 power. This observation is most apparent for a very weak slope  $\mu_t = -0.1$  for both margins. As an  
475 example, considering Gumbel copula, the test power increases from 83.9 % for a sample size  $n =$   
476 30 to 100% when  $n = 100$ . These powers are considerably high. In this instance, the presence of  
477 trends in all components (margins and dependence structure) leads to a rapid increase in  
478 performance compared to the previous Tables (Table 5 and Table 6).

479 Moreover, the power of  $T_{MOT}$  increases with the slope of the trend. For instance, considering  
480 Husler-Reiss copula with  $n = 30$ , the power is 88.6% when  $\mu_1 = -0.1$ ,  $\mu_2 = -0.1$  and increases to  
481 100% when  $\mu_1 = 0.3$ ,  $\mu_2 = 0.3$ . However, no significant differences were found between powers  
482 when considering different copulas. Indeed, as an example, for  $n = 30$  and  $\mu_1 = -0.1$ ,  $\mu_2 = -0.1$ ,  
483 the test power is between 83.9% and 94.0% for all different copulas.

484 From Table 7, we can see also that power of  $T_{MOT}$  is very high when considering the same trend  
485 direction in margins and dependence. As an example, from Table 7 when considering time-varying  
486 location parameters for  $\mu_1 = 0.3$ ,  $\mu_2 = 0.3t$ , and increasing trend in dependence structure, the  
487 power is always 100%. Further, it can be seen from the same Table 7 that power estimates of the  
488 proposed test  $T_{MOT}$  is not sensitive to the different direction of trend between both margins and  
489 dependence. For instance, when both margins exhibit a decreasing trend ( $\mu_1 = -0.1$ ,  $\mu_2 = -0.1$ ) and  
490 the dependence shows an increasing trend, the power ranges from 83.9% to 100% across different  
491 sample sizes  $n$  and copula types. Moreover, it is important to emphasise that the proposed statistic  
492  $T_{MOT}$  performs well in detecting trend even when considering different directions between both  
493 margins. For instance, for time-varying location parameters of  $\mu_1 = 0.3$ ,  $\mu_2 = -0.3$ , and increasing  
494 trend in dependence structure, the power estimates are 100% for all different sample size and  
495 copulas. This is because of the terms in the test  $T_{MOT}$  are squared to avoid cancelling the trend  
496 with different signs. Through Table 7, considering overall test  $T_{MOT}$  it is very important to note that  
497 the powers are around 100% in the majority of simulation cases. The first column is the only one  
498 that does not reach 100% for the different copula types. Indeed, the power range between 83.9%  
499 and 94% when  $n = 30$ . This is because this column represents the lowest slope (-0.1). This is a high  
500 results despite the weak slopes of marginal distributions were chosen according to the hydrological  
501 flood context and only the location parameter ( $\mu_t$ ) is assumed to be a linear function of time.

**Table 7: Power estimates (%) of the proposed tests  $T_{MOT}$  and  $T_{MDT}$  – trend in both margins and the dependence structure**

Test	Copula	$\mu_1 = -0.1t$ $\mu_2 = -0.1t$			$\mu_1 = 0.3t$ $\mu_2 = 0.3t$			$\mu_1 = 0.5t$ $\mu_2 = 0.3t$			$\mu_1 = 0.3t$ $\mu_2 = -0.3t$		
		$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$
$T_{MOT}$	Clayton	93.1	99.8	100	100	100	100	100	100	100	100	100	100
	Frank	94.0	100	100	100	100	100	100	100	100	100	100	100
	Joe	92.0	100	100	100	100	100	100	100	100	100	100	100
	Gumbel	83.9	100	100	100	100	100	100	100	100	100	100	100
	Galambos	92.4	100	100	100	100	100	100	100	100	100	100	100
	Husler-Reiss	88.6	100	100	100	100	100	100	100	100	100	100	100
$T_{MDT}$	Clayton	25.4	37.6	70.5	26.5	34.9	78.4	11.0	20.2	50.4	7.2	9.6	15.4
	Frank	31.4	59.3	91.8	22.7	36.3	79.8	14.2	22.4	54.1	8.4	10.3	19.6
	Joe	30.4	58.6	93.7	26.5	47.8	84.5	18.7	28.4	61.7	7.2	11.5	11.5
	Gumbel	34.9	55.4	92.1	29.5	49.4	89.1	16.4	27.8	59.5	7.8	12.1	17.8
	Galambos	29.8	63.3	95.2	37.6	60.8	95.0	14.0	25.3	62.6	6.5	11.3	14.4
	Husler-Reiss	21.7	47.8	89.0	28.3	47.7	90.3	13.5	23.9	54.3	5.7	10.1	15.7

502 This table presents the power of the proposed test at significance level  $\alpha=5\%$

503 Regarding the second proposed test,  $T_{MDT}$ , as observed in Table 7, we can see that the power  
504 substantially increases with a higher sample size, specifically reaching up to 95.2% when the  
505 sample size is elevated to  $n = 100$ . For example, generated data from a Gumbel copula and a slope  
506  $\mu_1 = 0.3$ ,  $\mu_2 = 0.3$  and trend in dependence structure, the test power increase from 29.5% for a  
507 series of length  $n = 30$  to 89.1% when  $n = 100$ . It is also important to note that exceptions are  
508 observed concerning  $T_{MDT}$ . Table 7 reveals that the  $T_{MDT}$  test's power increases as the slope of both  
509 margins decreases. For instance, with the Clayton copula and  $n = 50$ , the test power increases from  
510 50.4% when the location parameters are set to  $\mu_1 = 0.5$  and  $\mu_2 = 0.3$  to 78.4% when  $\mu_1 = 0.3$  and  
511  $\mu_2 = 0.3$ . Moreover,  $T_{MDT}$  test's performance is better when the directions of the margins are the same  
512 compared to case where they differ. For example, considering the Frank copula and  $n = 50$ , the

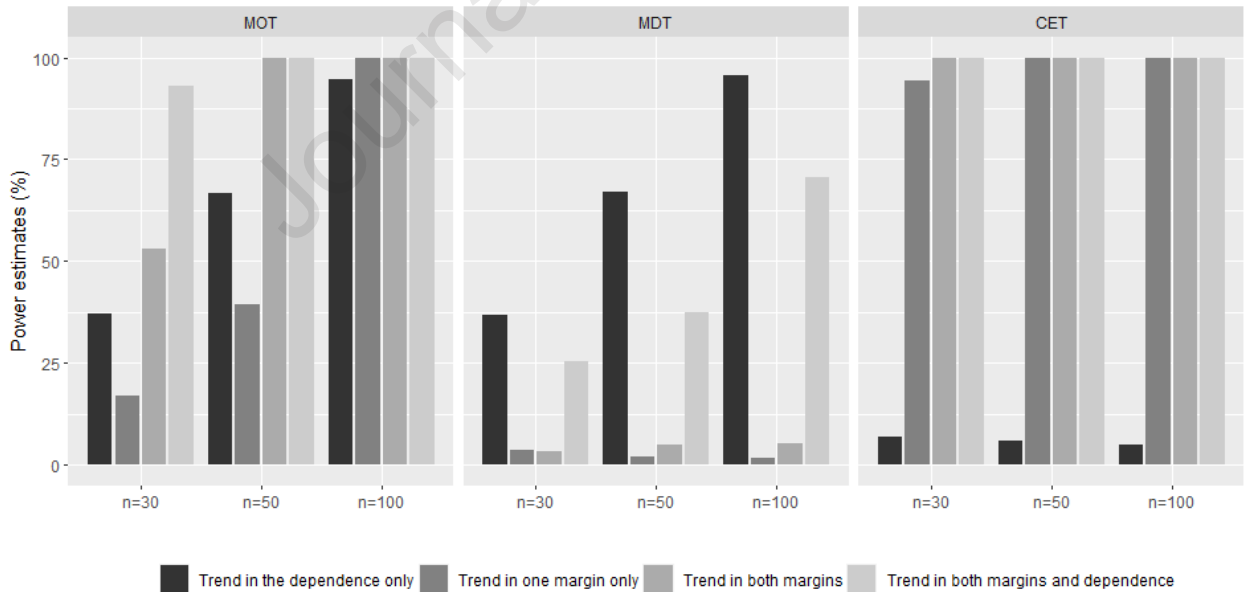
513 power estimates increase from 10.3% when location parameters are set to  $\mu_1 = 0.3$ ,  $\mu_2 = -0.3$  to  
514 36.3% when  $\mu_1 = 0.3$ ,  $\mu_2 = 0.3$ . This exceptions on results can be explained by the fact that  
515 incorporating trends in the margins can dilute or mask the trend in the dependence, especially when  
516 the trend in the margins is stronger than the trend in the dependence (e.g. Bender *et al.*, 2014).

517 In order to have an overview of the power of the proposed tests,  $T_{MOT}$  test performs well, especially  
518 with more data and stronger trends. It is flexible, width different trend directions. However,  $T_{MDT}$ 's  
519 power varies more, improving with larger sample sizes but sometimes decreasing with stronger  
520 trends in margins. The  $T_{MDT}$  test seems to have some exception associated with varying directions  
521 in margins trends. Comparatively, existing tests (CIT and CET) have high powers when trends are  
522 only in margins but fall short in spotting trends in the dependence. This highlights the importance  
523 of  $T_{MOT}$  and  $T_{MDT}$ , which consider both margins and dependence for a more complete picture.

524 Note that the performance of the existing multivariate tests CIT and CET is not interesting in this  
525 scenario (trend in both margins and dependence). Notably, Table 4 demonstrate that both CIT and  
526 CET do not identify any trend in the dependence structure. This demonstrates that current existing  
527 multivariate tests fall short in capturing trends across the entire system, encompassing both margins  
528 and the dependence structure. In particular, they neglect to discern whether a trend is present or  
529 absent in the dependence structure.

530 It is important to extract information from different tables (4, 5, 6, and 7) in order to quantify the  
531 trend in all components. We chose Clayton copula and slope of trend equal to -0.1 for both margins.  
532 Considering  $T_{MOT}$  test, powers of the cases of trend in both margins and dependence are higher  
533 than in trend on the only the dependence or only in the margins. This can be explained by the fact  
534 that the higher the trend in terms of the number components, the higher the power will be.

535 From Figure 3 we can see for  $n = 30$  that the power of the  $T_{MOT}$  test increases from 17.5% when  
 536 trend in one margin, 53.2% when trend in both marginal distributions and to 93.1 % when trend in  
 537 all components. This test allows a quantification of the trend since it detects it in all components  
 538 unlike the multivariate existing tests. Through Figure 3, it is clear to see that  $T_{MDT}$  test performs  
 539 well when a trend in dependence structure and in both components (margins and dependence  
 540 structure). Moreover, we can see that the presence of the trend in the margins influences the  
 541 performance of  $T_{MDT}$  test. For example, for  $n = 100$ , Figure 3 show that the power of test increase  
 542 from 70.5% when trend in both margins and dependence, to 95.5% when trend only in dependence  
 543 structure. We note that CET test not able to detect the trend in the dependence structure. The high  
 544 power of the CET is misleading and once a gain it ignores that there is no trend in the dependence.  
 545 The proposed tests, although with lower power, provide realistic and representative results in  
 546 detecting trends.



547 Figure 3: Quantification of trend by statistics  $T_{MOT}$ ,  $T_{MDT}$  and CET test for Clayton copulas and  
 548 slope of margins equal to -0.1.

## 5. Applications to Hydrological Data

In this section, the purpose is to assess the appropriateness of the proposed tests for practical use. We apply the developed and existing tests to three real-world hydrological datasets in Canada. They have been chosen to cover different affected components of trends. The first data series correspond to the Southwest Margaree River. The second data series correspond to the Long Creek stations. This station exhibits regulated flow regimes and located below Boundary dam Reservoir. Note that reservoir construction is one of the primary factors contributing to changes in the characteristics of natural river flow regimes (e.g. Ekka *et al.*, 2022). The same series was considered in Tan and Gan (2015) to study the contribution of human change impacts to changes in streamflow of Canada. The third data series correspond to Athabasca River. The same station has been previously employed in analysis of hydrological univariate trends and variability by numerous studies (e.g. Bawden *et al.*, 2014; Das *et al.*, 2020). Figure 4 and Table 8 present respectively the geographical location and general information about the considered stations.

Table 8: General information about the stations

Station name	Province	Station number	Period of records (years)	Part of RHBN*
Southwest Margaree River	Nova Scotia	01FB003	1960-2021 (61)	Yes
Long Creek near Estevan	Saskatchewan	05NB001	1970-2023 (53)	No
Athabasca River below Fort McMurray	Alberta	07DA001	1970-2021 (51)	Yes

\* Reference Hydrometric Basin Network (RHBN), which consists of a set of stations with long records and minimal human impacts intended for climate change studies



Figure 4: Geographical location of stations 01FB003, 05NB001 and 07DA001

565  
 566 Based on hydrological literature, our emphasis is on the flood peak (Q) and volume (V) series (e.g.  
 567 Gaál *et al.*, 2015). All the applications considered a significance level of 5%. The results are given  
 568 in Table 9.

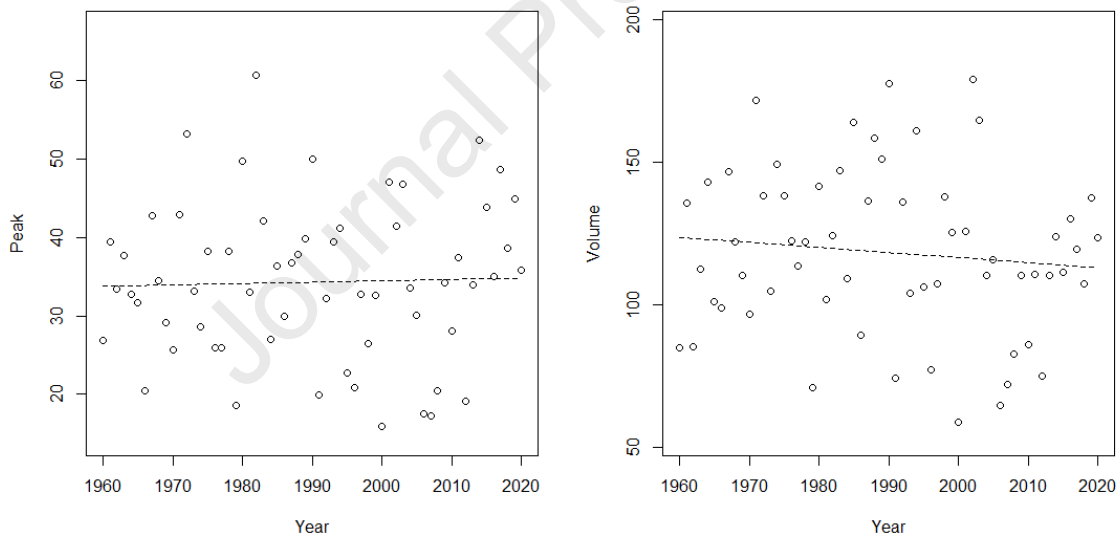
Table 9: Univariate and multivariate stationarity testing results

Station	Univariate MK test		Multivariate MK tests				
	Variable	p-value	CST	CIT	CET*	MOT	MDT
01FB003	Q	0.627	0.668	0.528	0	<b>0.008</b>	<b>0.029</b>
	V	0.713					
05NB001	Q	<b>0.000052</b>	<b>0.00523</b>	<b>0.000040</b>	<b>1</b>	<b>0.026</b>	0.574
	V	<b>0.000011</b>					
07DA001	Q	<b>0.0184</b>	0,138	0,060	<b>1</b>	<b>0.017</b>	<b>0.037</b>
	V	<b>0.026</b>					

570 The bold character indicates the rejection of corresponding null hypothesis at the 5% level.\* Note that, instead of the p-value, for the CET-test the  
 571 conclusion is presented as: 1 if there is a trend, 0 if not, since this test is based on critical thresholds.

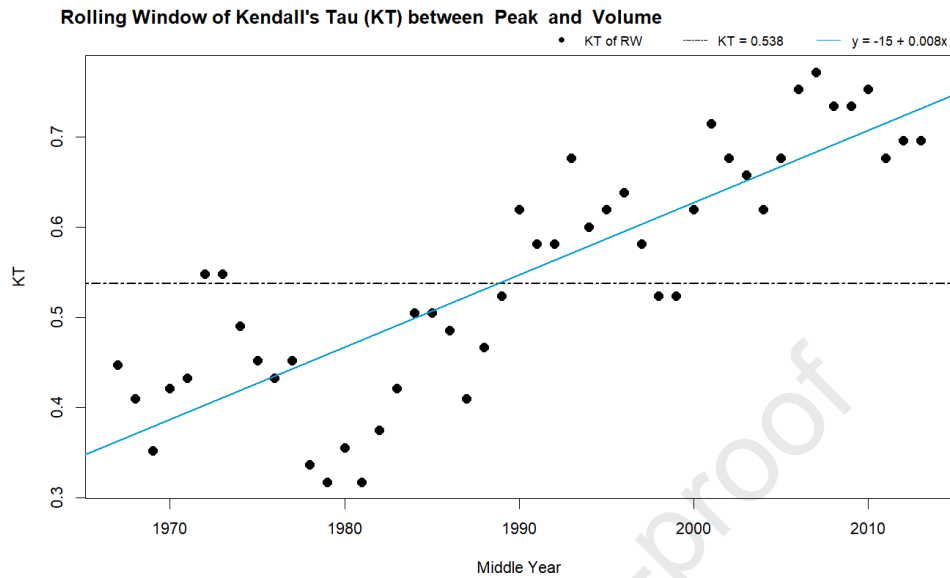


572 For station 01FB003, first, results show no trend detected in the margins (as confirmed from Figure  
573 5). In addition, a significant trend in the dependence structure detected by the proposed MDT test,  
574 which is consistent with Figure 6. In contrast, the existing multivariate tests (CST, CIT and CET)  
575 were unable to detect the trend in the dependence structure. This confirms the efficacy of MDT in  
576 detecting trend in dependence structure. Furthermore, the MOT test also indicates a significant  
577 overall trend, which confirm its ability to detect trends some components (here in the dependence  
578 structure). Given that, this station is part of the RHBN, the presence of trends in the dependence  
579 structure is driven by climate changes. It is noteworthy that Burn and Whitfield (2023) have  
580 observed changes in the nival fraction at the same station.



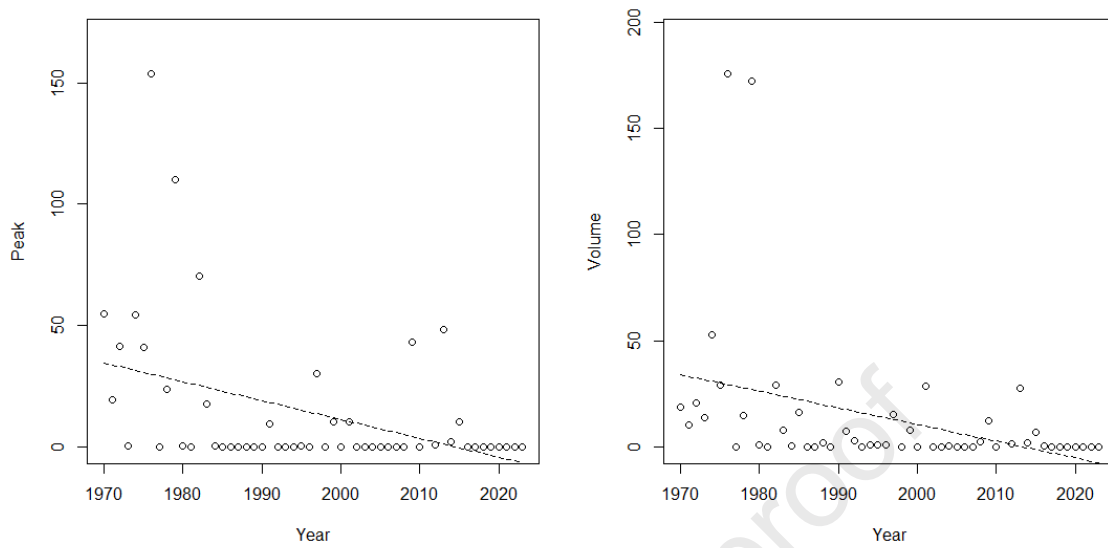
581 Figure 5: Plot of Peak, volume of the series Q (left) and V (right) with the associated regression  
582 lines (01FB003)

583

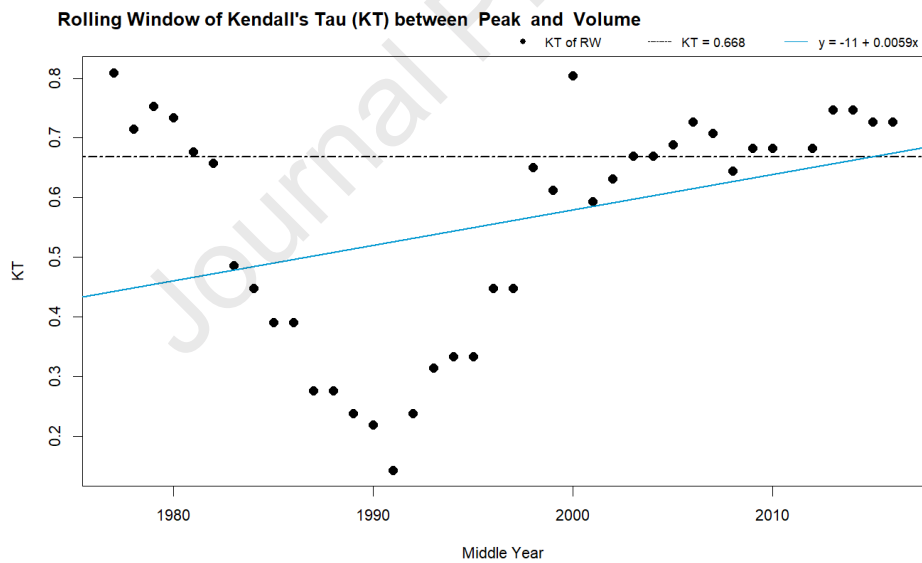


584 Figure 6: Plot of Peak, volume and rolling window of Kendall's  $\tau$  (KT) between them against year  
 585 (01FB003)

586 In station 05NB001, we observe that all multivariate existing tests are in agreement with a  
 587 significant trend. The proposed multivariate test MOT also indicated overall trend. This can also  
 588 be verified from Figure 7. Trends are also detected by MK univariate for each variable. Given that  
 589 this station is non-RHBN and located below Boundary dam Reservoir, the observed trend detection  
 590 in the margins may be attributed to anthropogenic activity and/or potential climate change impacts.  
 591 It should be noted that no visually clear monotonic trend in the dependence structure is observable,  
 592 as shown in Figure 8, as confirmed by the MDT test.

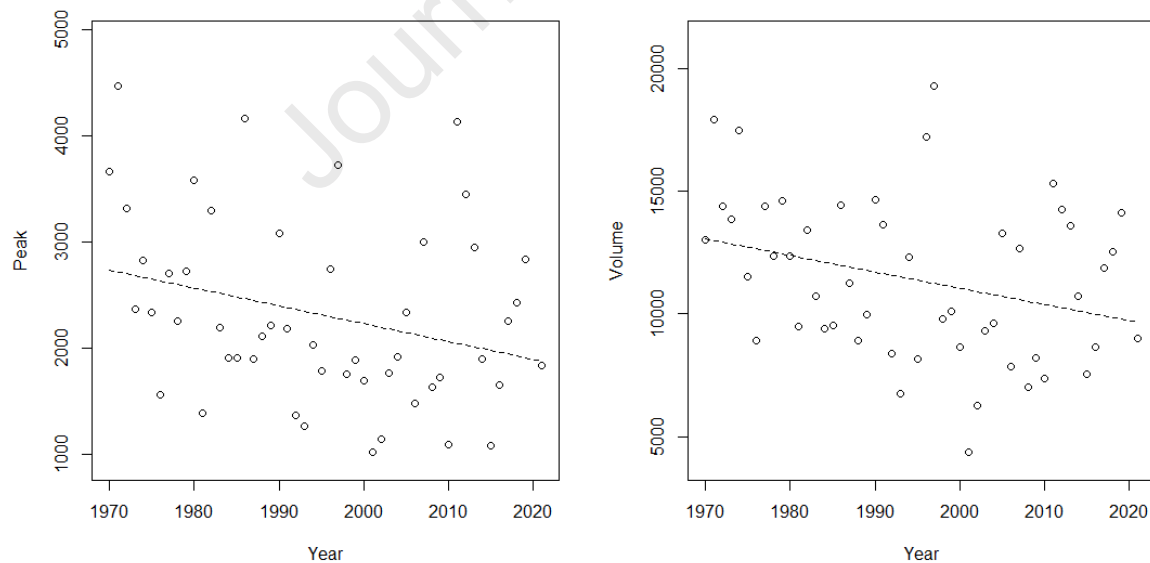


593 Figure 7: Plot of Peak, volume of the series Q (left) and V (right) with the associated regression  
 594 lines (05NB001)

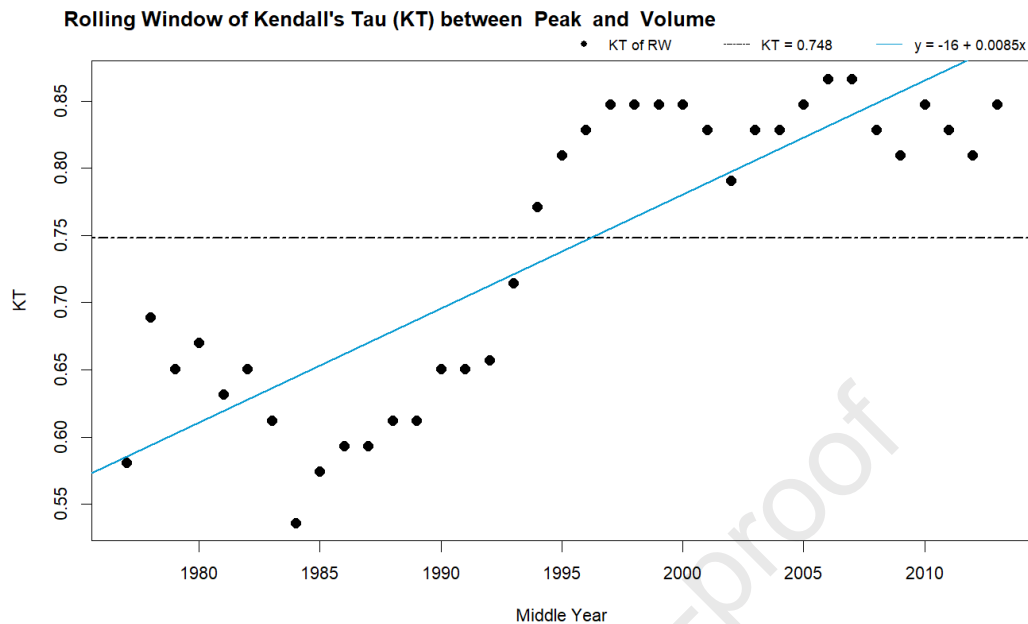


595 Figure 8: Plot of Peak, volume and rolling window of Kendall's  $\tau$  (KT) between them against year  
 596 (05NB001)

597 For 07DA001, we observe that the univariate MK test indicates the presence of trends in both  
 598 variables simultaneously (as confirmed by Figure 9). Additionally, the MDT test detects a trend in  
 599 the dependence structure, which can also be confirmed from Figure 10. The proposed multivariate  
 600 MOT test confirms an overall multivariate trend. Among the existing multivariate tests, only CET  
 601 detects the existence of a trend in the margins. The CIT statistic value is very close to the threshold.  
 602 The CST test does not detect any trend. This confirms findings in the literature that the CST test  
 603 has lower performance compared to CIT and CET (e.g. Modarres, 2018), and the CET test is  
 604 recommended among the available multivariate tests (e.g. Chebana & Ouarda, 2021). This station  
 605 is a part of the RHBN. Thus, the observed multivariate trends in the margins and dependence  
 606 structure could be driven by climate change. Note that, recent decades have seen significant  
 607 changes in the hydrological and meteorological conditions of the Athabasca River (e.g. Bawden *et*  
 608 *al.*, 2014; Beltaos & Carter, 2009).



609 Figure 9: Plot of Peak, volume of the series Q (left) and V (right) with the associated regression  
 610 lines (07DA001)



611 Figure 10: Plot of Peak, volume and rolling window of Kendall's  $\tau$  (KT) between them against year  
 612 (07DA001)

## 613 6. Conclusions and Perspectives

614 In the literature dealing with multivariate frequency analysis, in general, the stationarity assumption  
 615 is not verified. This is in part due to the absence of powerful and effective tests. However,  
 616 nowadays in climate and hydrological changing context, it is more and more important to consider  
 617 multivariate tests that can detect non-stationarity either in the margins or in the dependence  
 618 structure.

619 The aim of the present paper is to develop new tests for multivariate trend to fill a gap in the  
 620 statistical and hydrological literature. The first test  $T_{MOT}$  is designed to detect trend in the affected  
 621 component (margins and dependence), and the second test  $T_{MDT}$  is conceived to focus on trend in  
 622 the dependence structure. In comparison to existing multivariate tests, simulation results show very  
 623 promising performances in terms of first type error and power.

624 The proposed multivariate tests are adopted to hydrological context due to their good performance  
625 when the trend is very weak and the series is short, which often happens in hydrological series. The  
626 existing tests were not able to detect trend in the dependence structure alone or with the margins.  
627 The mutual application of the proposed tests  $T_{MDT}$  and  $T_{MOT}$  with univariate MK test provides an  
628 attractive procedure for testing multivariate trend and to discriminating its potential source.  
629 In this paper, the proposed multivariate trend tests were theoretically justified and practically  
630 demonstrated through both a comprehensive simulation study and practical illustrative  
631 applications. However, certain limitations were observed for the developed tests. Indeed, they are  
632 designed for monotonic trends whereas other forms of trends may exist. Thus, for a more flexible  
633 trend detection, there is a need to advance the development of non-monotonic trend tests.  
634 Moreover, the presence of autocorrelation in the data can influence the outcomes of tests to detect  
635 trends. Consequently, Hamed and Rao (1998) proposed a modified univariate MK-test. Similarly,  
636 in the multivariate framework, it would be of interest to develop multivariate trend tests suited for  
637 autocorrelated data.

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643

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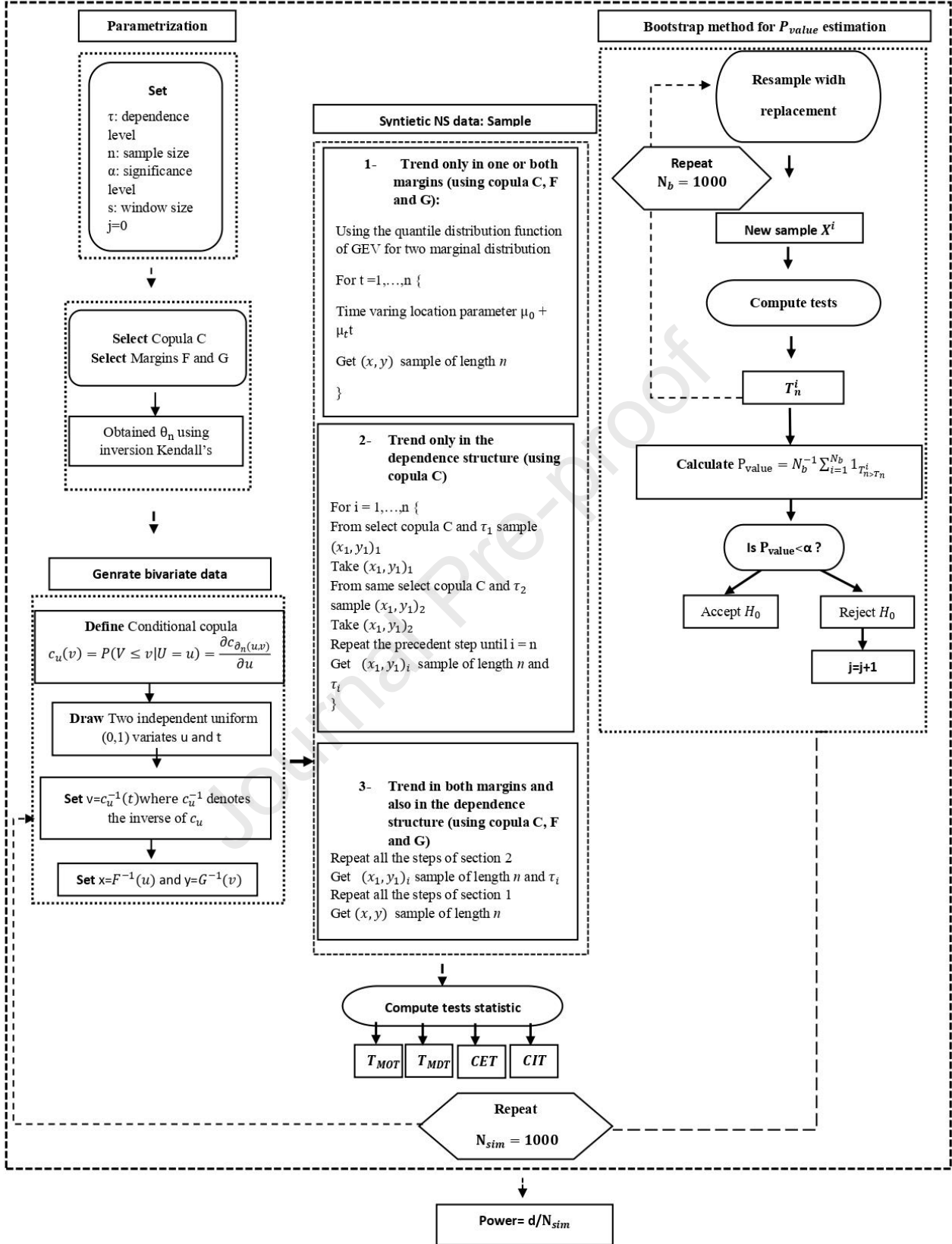


Figure 1: Diagram of the simulation study

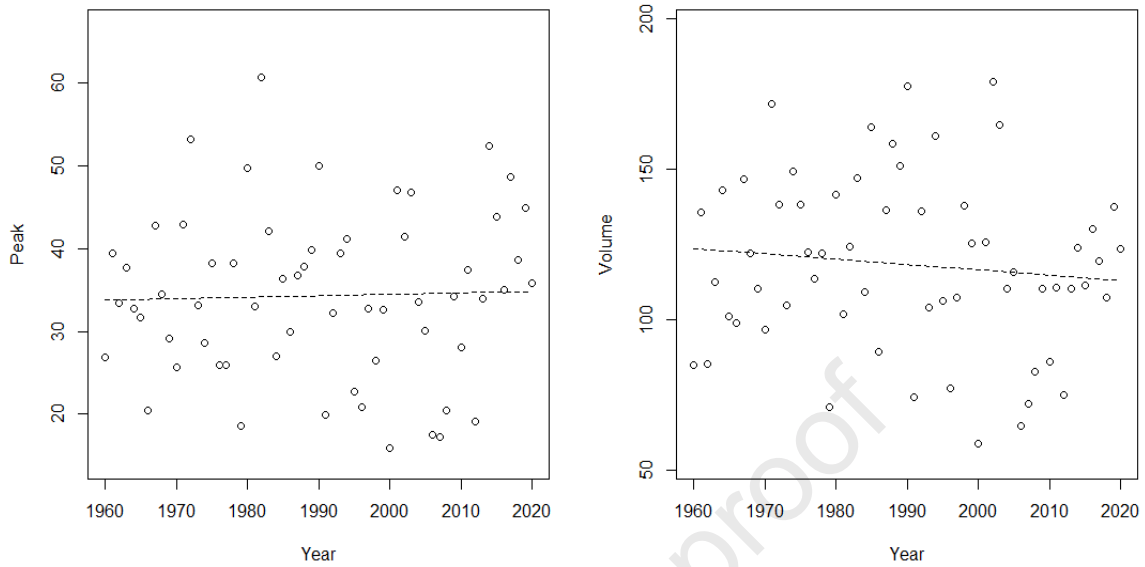


Figure 2: Plot of Peak, volume of the series Q (left) and V (right) with the associated regression lines (01FB003)

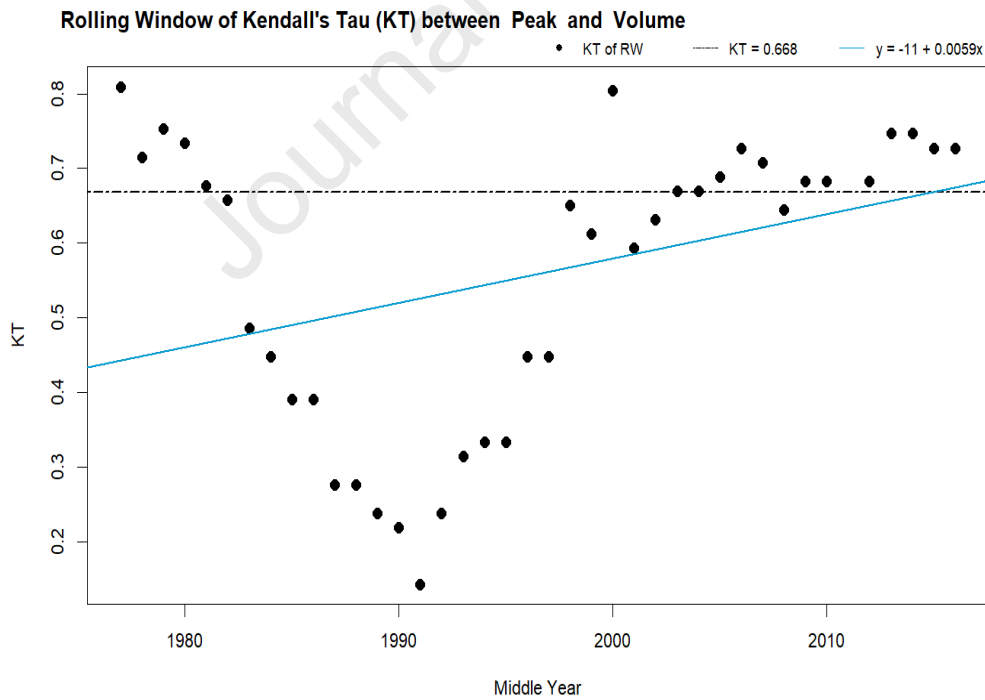


Figure 3: Plot of Peak, volume and rolling window of Kendall's  $\tau$  (KT) between them against year (05NB001)

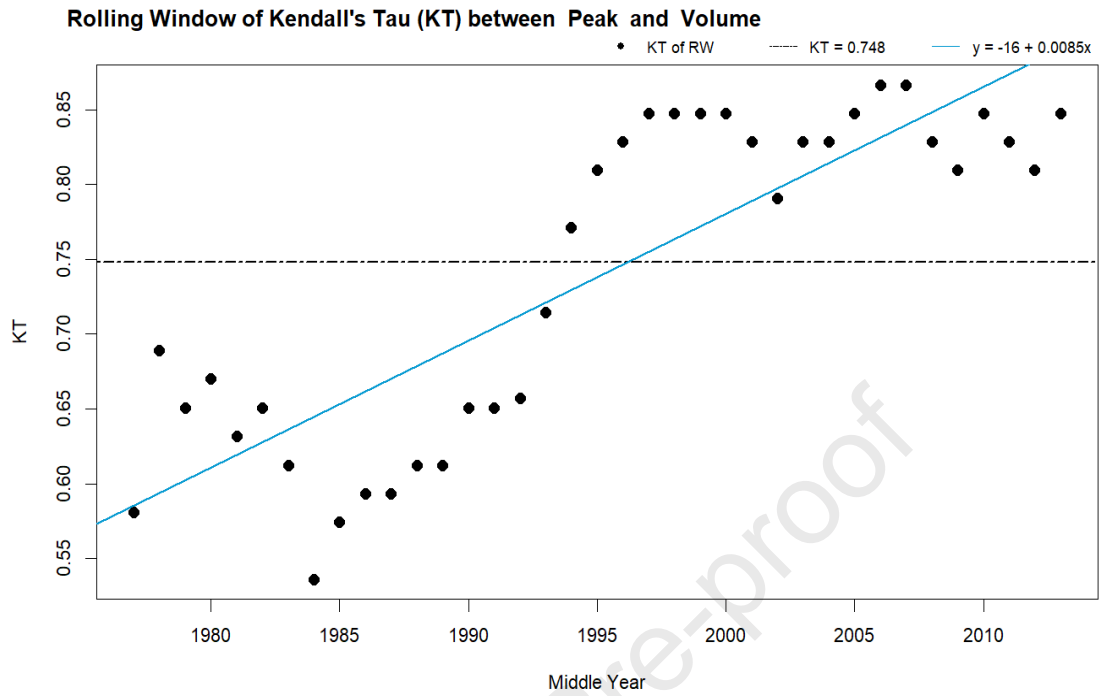


Figure 4: Plot of Peak, volume and rolling window of Kendall's  $\tau$  (KT) between them against year (07DA001)

- Two multivariate trend tests for multivariate hydrological series are proposed.
- New multivariate overall trend (MOT) test dealing with trend in all the components of the whole multivariate distribution.
- New multivariate dependence trend (MDT) test focuses on trend in the dependence structure.
- Vast simulation study is considered to evaluate the performance of the tests.
- The developed tests show high performance, with increasing power observed as the trend slope and sample size increase.

**Declaration of interests**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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