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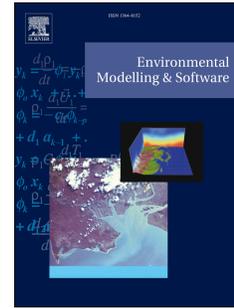
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Multivariate overall and dependence trend tests, applied to hydrology

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Dorsaf Goutali: Conceptualization, Formal analysis, Methodology, Software,

Validation, Writing – original draft, Writing - Review & Editing. **Fateh Chebana:**

Conceptualization, Methodology, Supervision, Validation, Writing – review &

editing

24 **Abstract**

25 Given climate change, trend detection is gaining increasing attention in the context of multivariate
26 frequency analysis. In this paper, we propose new statistical tests for multivariate trend detection.
27 The first one, a multivariate overall trend (MOT) test, is designed to detect trend in all components
28 of the multivariate distribution (margins and dependence structure) whereas the second test is a
29 multivariate dependence trend (MDT) test focusing on detecting trend in the dependence structure.
30 A simulation study is used to evaluate the performance of the proposed tests. Results show that the
31 proposed MOT test performs well when trend is present in margins, in the dependence structure
32 and/or in both. Likewise, results of the proposed MDT test indicate an interesting power when the
33 trend is in the dependence structure. Moreover, an application to a real dataset is provided.
34 Performing the proposed tests with the univariate tests provides a complete overview of trend
35 detection.

36 **Keywords:** Trend, Hydrology, Multivariate, Non-stationarity, Copula, dependence structure.

37 **Highlights**

- 38 • Two multivariate trend tests for multivariate hydrological series are proposed.
- 39 • New multivariate overall trend (MOT) test dealing with trend in all the components of the
40 whole multivariate distribution.
- 41 • New multivariate dependence trend (MDT) test focuses on trend in the dependence
42 structure.
- 43 • Vast simulation study is considered to evaluate the performance of the tests.
- 44 • The developed tests show high performance, with increasing power observed as the trend
45 slope and sample size increase..

46 **Software and/or data availability**

47 **Software:**

- 48 • The code used for developing the multivariate trend tests and the simulation study
- 49 scenarios, implemented in the R language, can be found on GitHub at the following
- 50 link: <https://github.com/GOUD05/Multivariate-Trend-Tests.git>
- 51 • Repository creator: Dorsaf Goutali.
- 52 • Creation date: 2024.
- 53 • Contact Information: Dorsaf.goutali@inrs.ca.
- 54 • Program Language: R version 4.1.3 (64bit).
- 55 • Required Software: R (Download from <https://cran.r-project.org/>), RStudio
- 56 (Download from <https://www.rstudio.com/>).
- 57 • Cost: free.
- 58 • Required R Packages:
 - 59 - copula <https://cran.r-project.org/web/packages/copula/index.html>,
 - 60 - Kendall , <https://cran.r-project.org/web/packages/Kendall/index.html>,
 - 61 - resample , <https://cran.r-project.org/web/packages/resample/index.html>,
 - 62 - VGAM: <https://cran.r-project.org/web/packages/VGAM/index.html>,
 - 63 - openxlsx: <https://cran.r-project.org/web/packages/openxlsx/index.html>,
 - 64 - gtools: <https://cran.r-project.org/web/packages/gtools/index.html>.
- 65 • Used Hardware: Computer with Windows 10, Intel i5 8th Gen processor 8 GB RAM,
- 66 256 GB storage.

67 **Data:**

- 68 • This study relies on the generation of synthetic data as a requisite part of the
- 69 methodology to conduct simulations study. The provided code on GitHub generates
- 70 data and simultaneously calculates the performance of the tests. The data used in
- 71 the illustrative applications will be available on request.

72

73 1. Introduction

74 Hydrological frequency analysis (HFA) is widely used for modeling extreme hydro-
75 meteorological events like floods, droughts, and storms (e.g. Hamed & Rao, 1998). Such events
76 are often identified by correlated features, such as peak, volume, and duration for floods (e.g.
77 Chebana & Ouarda, 2021; Grimaldi & Serinaldi, 2006). These dependent features highlight the
78 need for a multivariate HFA approach, supported by various studies (e.g. Genest & Chebana, 2017;
79 Li *et al.*, 2019; Requena *et al.*, 2013). Univariate HFA can provide only limited assessment of
80 extreme events and their probability of occurrence (e.g. Chebana & Ouarda, 2011; Joyce *et al.*,
81 2018).

82 Commonly, HFA is based on the assumptions of stationarity, homogeneity, and serial
83 independence. In the multivariate context, checking these assumptions, particularly stationarity,
84 attracted less attention compared to modeling (e.g. Chebana & Ouarda, 2021; Gu *et al.*, 2018).
85 Ignoring the testing step of these assumptions can lead to inaccurate results and potentially wrong
86 decisions (e.g. Chebana *et al.*, 2013). Indeed, this step contributes to the choice of the appropriate
87 model, which should integrate possible trends in some or all components of the multivariate
88 distribution (margins and dependence structure). The stationarity assumption has long been
89 compromised by climate change and human activities such as deforestation, and overuse of
90 extraction from surface water and ground water (e.g. Milly *et al.*, 2008; Tan & Gan, 2015; Vidrio-
91 Sahagún *et al.*, 2024). Related to theoretical considerations, it is no longer valid to believe that the
92 design flood is always stationary (e.g. Aissia *et al.*, 2014; Kang *et al.*, 2019; Milly *et al.*, 2008).
93 Therefore, in recent years, increasing attention has been paid to hydrological designs under non-
94 stationarity (NS) conditions and particularly in the multivariate setting (e.g. Chebana & Ouarda,
95 2021; Li *et al.*, 2016; Zhang *et al.*, 2022).

96 A wide variety of parametric and non-parametric tests has been employed for trend detection (e.g.
97 De Luca & Napolitano, 2023). The Mann-Kendall (MK) and the Spearman rank order correlation

Table 1: Overview of existing tests for trend in univariate and multivariate framework

98 (SR) tests are among the most non-parametric considered univariate trend tests (e.g. Chong *et al.*,
99 2022; Conover, 1980; Kendall, 1975; Ouarda *et al.*, 2018). In addition, Chebana *et al.* (2013)
100 presented an overview of the available multivariate extensions of the univariate MK and SR tests.
101 Being non-parametric and powerful is the main advantage of these multivariate tests. However, the
102 latter were initially developed and designed for water quality analysis even though they have been
103 directly employed later in HFA. Moreover, these multivariate tests are essentially based on their
104 univariate counterparts (component-wise tests), do not take into account the dependence between
105 the variables, and cannot identify the affected component . On the other hand, it seems that testing
106 for trends in the dependence structure has not been explored yet.. Furthermore, upon reviewing the
107 literature, it appears that there are no recently developed trend tests and recent studies (e.g. Chebana
108 & Ouarda, 2021; Chebana *et al.*, 2013; Jalili Pirani & Najafi, 2020; Kang *et al.*, 2019; Karahacane
109 *et al.*, 2020; Modarres, 2018; Xu *et al.*, 2023) consider multivariate tests reviewed by Chebana *et*
110 *al.* (2013). In Table 1 the univariate and multivariate trend tests are summarised including their
111 advantages and drawbacks.

112 In order to overcome the drawbacks of the multivariate trend tests, the objective of the present
113 paper is to propose two multivariate trend tests. The first proposed test is a multivariate overall
114 trend (MOT) test dealing with trend in all the components of the whole multivariate distribution
115 (margins and the dependence structure). The second proposed one, a multivariate dependence trend
116 (MDT) test, focuses on trend in the dependence structure. Therefore, the proposed tests, along with
117 the existing univariate trend tests, allow dealing with the multivariate distribution as whole as well
118 as its components.

Tests		Advantages	Drawbacks	Some references
Univariate tests	Mann-Kendall (MK)	<ul style="list-style-type: none"> Both tests have been recommended by the World Meteorological Organization as standard nonparametric procedures Powerful Robustness against missing values and outliers Making very few assumptions Detect increasing decreasing trend Simple to apply 	<ul style="list-style-type: none"> The existence of positive autocorrelation in the data increases the probability of detecting trends when actually none exist, and vice versa Inability to detect non-monotonic trend structures 	Mann (1945); Yue <i>et al.</i> (2002) Bihrat and Bayazit (2003) Yue and Pilon (2004) Rutkowska (2015) Wang <i>et al.</i> (2020) Hamed and Rao (1998)
	Spearman's rho (SR)			
Multivariate component wise tests	Covariance-Inversion test (CIT)	<ul style="list-style-type: none"> Non-parametric tests do not make any assumption or precondition about the models Detect increasing/decreasing trends Simple to apply 	<ul style="list-style-type: none"> Designed for water quality analysis and not for hydrological fields, existing comparisons and evaluations of these tests are often based on scenarios that do not align with the hydrological context (e.g. sample size, distributions) Essentially based on their univariate counterparts (component-wise tests) Simple combinations of univariate tests and do not take into account the dependence between the variables Cannot identify the affected components 	Dietz and Killeen (1981) Hirsch and Slack (1984) Lettenmaier (1988) Loftis <i>et al.</i> (1991) Smith <i>et al.</i> (1993) Thas <i>et al.</i> (1998) Chebana <i>et al.</i> (2013)
	Covariance-Eigenvalue test (CET)			
	Covariance Sum test (CST)			

119
120 The developed tests, MOT and MDT, are based on multivariate extension of Kendall's τ and not
121 on combinations of univariate statistic tests. A simulation study to evaluate and compare the
122 performances of the proposed tests is presented. The proposed tests are general and can be
123 considered in other contexts and applications dealing with trends.

124 The paper is organized as follows. A brief theoretical background, related to the developed tests,
125 is presented in Section 2. The proposed statistical tests for trend are described in Section 3. The
126 simulation study to evaluate the performance of the tests is given in Section 4. The conclusions are
127 reported to Section 5.

128 2. Available multivariate trend tests

129 In this section, we briefly present the available univariate and multivariate tests for trend. In
130 statistical hydrology, mainly two non-parametric rank-based statistical tests are considered, namely

131 the MK and SR tests. Even though, they have the same aim and similar performance, the univariate
 132 SR tests are less employed than MK ones (e.g. Sneyers, 1990; Yue *et al.*, 2002). More details about
 133 multivariate SR tests are provided in Chebana *et al.* (2013). In the following, we focus on
 134 presenting MK tests. Either in the univariate or multivariate settings, the null hypothesis of no trend
 135 TR is $H_0 : TR = 0$ against the general alternative hypothesis of a monotonic trend $H_1 : TR \neq 0$ and
 136 there exists at least a component u such that $TR^{(u)}$ is monotonic (e.g. Chebana *et al.*, 2013).
 137 The univariate MK test is the most used test to detect monotonic univariate trends. Given a data
 138 series (x_1, x_2, \dots, x_n) of length n , the MK test statistic is given by

$$139 \quad M = \sum_{i=1}^{n-1} \sum_{j=i+1}^n \text{sgn}(x_j - x_i) \quad (1)$$

140 where x_j and x_i are both values in the series, and $\text{sgn}(\cdot)$ is a sign function:

$$141 \quad \text{sgn}(x) = -1 \text{ if } x < 0, \quad = 0 \text{ if } x = 0, \quad = 1 \text{ if } x > 0 \quad (2)$$

142 Under H_0 , the test statistic M has asymptotically normal distribution with mean $E(M) = 0$ and

$$143 \quad \text{Var}(M) = \frac{n(n-1)(2n+5)}{18} \quad (3)$$

144 Multivariate extensions of the univariate MK tests have been established to analyze multivariate
 145 trends in the hydrological context. Table 2 gives an overview of the main properties of those
 146 existing multivariate MK tests covering their expression and the asymptotic distributions.

147 For all the tests presented in Table 2, let $M^{(u)}$ be the univariate MK test statistic for the observed
 148 time series $X_i^{(u)}, i = 1, \dots, n$ and component $u = 1, \dots, d$. For a given u , $M^{(u)}$ is defined as:

$$149 \quad M^{(u)} = \sum_{1 \leq i < j \leq n} \text{sgn}(x_j^{(u)} - x_i^{(u)}) \quad (4)$$

152 Under the null hypothesis H_0 of no trend, $M^{(u)}$ is asymptotically d -dimensional normal with zero
 153 mean and covariance matrix $C_M = (C_{u,v})_{u,v=1,\dots,d}$ with $C_{u,v} = cov(M^{(u)}, M^{(v)})$ which is estimated
 154 by

$$155 \quad \hat{C}_{u,v} = \frac{t_{u,v} + r_{u,v}}{3} \text{ for } u \neq v \quad (5)$$

156 where

$$157 \quad t_{u,v} = \sum_{1 \leq i \leq j \leq n} sgn((x_j^{(u)} - x_i^{(u)})(x_j^{(v)} - x_i^{(v)})) \quad (6)$$

$$158 \quad r_{u,v} = \sum_{i,j,k=1}^n sgn((x_k^{(u)} - x_j^{(u)})(x_k^{(v)} - x_i^{(v)})) \quad (7)$$

Table 2: Summary of the available multivariate MK-based trend tests

Expression of the test statistic	Asymptotic distribution under H_0 and decision rule
Covariance Inversion test (CIT) $D = M' C_M^{-1} M \quad (8)$ where C_M^{-1} is the inverse matrix of C_M	<ul style="list-style-type: none"> It is asymptotically $\chi^2(q)$ distributed under H_0, where q is the rank of the matrix $1 \leq q \leq d$. The null hypothesis is rejected: if the value of D exceeds the critical threshold determined according to $\chi^2(q)$ distribution quantile, depending on the fixed significance level α.
Covariance Sum test (CST) $H = \sum_{u=1}^d M^{(u)} \quad (9)$	<ul style="list-style-type: none"> The statistic H is asymptotically normal under H_0, with mean $E(H) = 0$ and variance: $var(H) = \sum_{u=1}^d var(M^u) + 2 \sum_{v=1, u=1}^{d,v-1} C_{u,v} \quad (10)$
	where $C_{u,v} = cov(M^{(u)}, M^{(v)}) \quad (11)$
Covariance Eigenvalue test (CET) $L = \sum_{u=1}^d (M^{(u)})^2 \quad (12)$	<ul style="list-style-type: none"> The null hypothesis is rejected: similar to CIT The statistic $(M^{(u)})$ for $u=1, \dots, d$ are asymptotically normally distributed with zero mean and the approximate variance is $\sigma^2 = var(M^{(u)})$ as in (3) If $(M^{(u)})$ are independent, The statistic L would be asymptotically $\sigma^2 \chi^2(q)$- distributed under H_0 where q is the rank of the covariance matrix as given in (5)

160 Notations: n is the sample size and d is the dimension or the number of components. More details about multivariate
 161 tests are provided in Chebana *et al.* (2013)

162 3. Proposed multivariate trend tests

163 To overcome the drawbacks mentioned above, the developed multivariate tests rely on two main
 164 aspects. The first one is the multivariate extension of the rank correlation coefficient Kendall's τ .
 165 This idea draws from Kendall's τ relationship with the univariate MK trend test statistic. The
 166 second ingredient is the moving window technique over the dependence.

167 Kendall's τ and univariate MK test

168 Kendall's τ is defined, in the bivariate and usual case, as the difference between the probabilities
 169 of concordance and discordance between two variables X and Y respectively with series x_1, x_2
 170 $\dots x_n$ and y_1, y_2, \dots, y_n (e.g. Kendall & Gibbons, 1990):

$$171 \tau_{(X,Y)_n} = \frac{2}{n(n-1)} \sum_{(i<j)} \text{sgn}(x_j - x_i)(y_j - y_i) \quad (13)$$

172 Hence, the statistic of the univariate MK test statistic is a particular case of Kendall's τ (e.g. Dietz
 173 & Killeen, 1981; Hamed & Rao, 1998). Indeed, Kendall's τ has also been used to test the
 174 significance of trends in univariate data if the values in Y are replaced by T the time order of the
 175 time series X , i.e. $T = 1, 2, \dots, n$. In that case, the test is called as Mann-Kendall test and the
 176 equations in (1) and (13) become the same (e.g. Hamed & Rao, 1998; Hirsch & Slack, 1984).
 177 Therefore, in an analogous way, the multivariate proposed test statistics are based on multivariate
 178 extension of Kendall's τ .

179 Kendall's τ in d -dimension and the proposed tests

180 In the literature, two extensions of Kendall's τ have been proposed in higher dimensions (e.g.
 181 Genest *et al.*, 2011). Consider a random vector X taking values in \mathbb{R}^d with cdf $H(x) = \mathbb{P}(X \leq x)$ and
 182 continuous marginal distribution F_1, \dots, F_d . Referring to Joe (1990), the first option of d -variate
 183 version of Kendall's τ for H is defined by:

$$184 \quad \tau_d(X) = \frac{2^d \mathbb{E}_H\{H(X)\} - 1}{2^{d-1} - 1} \quad (14)$$

185 where \mathbb{E}_H denotes the expectation with respect to H . Note that $\mathbb{E}_H\{H(X)\} = \mathbb{E}_C\{C(U)\}$, where C
 186 is the copula of H and $U = (F_1(X_1), \dots, F_d(X_d))$. The second option was established by Kendall and
 187 Smith (1940). It is defined as the average value of Kendall's τ taken over all possible pairs (X_r, X_s) ,
 188 with $r, s = 1, \dots, d$ and $r \neq s$, viz. and $H_{r,s}$ is the bivariate cdf of (X_r, X_s) :

$$189 \quad t_d(X) = \frac{1}{d(d-1)} \sum_{r \neq s} \tau(X_r, X_s) \quad (15)$$

190 To develop the proposed tests, we used the d -variate extension given in the first option in (14).
 191 Indeed, this extension has the advantage to be expressed in terms of copulas. The use of copula
 192 allows to take into account the whole dependence structure instead of only dependence between
 193 pairs as in (16) (e.g. Genest *et al.*, 2011; Li *et al.*, 2011). Further, Nelsen (1996) mentioned that
 194 when $d = 3$, both extensions (14) and (15) coincide leading to:

$$195 \quad \tau_3 = t_3 = \frac{1}{3} \{ \tau(X_1, X_2) + \tau(X_1, X_3) + \tau(X_2, X_3) \} \quad (16)$$

196 In our developed tests, the moving window technique has been employed in order to take into
 197 account the dependence evolution according to time. Indeed, contrary to the margins, the evolution
 198 of the dependence structure cannot be directly seen (e.g. Chebana & Ouarda, 2021). Moreover, the
 199 result of Kendall's τ between two series is a single value that represents the strength of the
 200 dependence and not the evolution of the dependence structure over time. Consequently, in order to
 201 bring out the aspect of the trend in dependence, Kendall's τ should be used in a series. This has
 202 been achieved by employing a moving window technique. The chosen window size, denoted s ,
 203 should be selected in a way to be neither too large nor too small, in order to perform reliable
 204 analysis and adequate number of values for the identification of the dependence structure (e.g.
 205 Bender *et al.*, 2014; Chebana *et al.*, 2013).

206 Based on multivariate extension of Kendall's τ τ_3 in equation (16) developed by Joe (1990), let's
 207 substitute X_1 with X , X_2 with Y , and replace X_3 with the time order $T = 1, 2, \dots, n$. Let τ_n denote
 208 the empirical version of bivariate Kendall's τ . In this context, we introduce the first proposed test
 209 statistic, T_{MOT} is given by:

$$210 \quad T_{MOT} = \frac{1}{3} (\tau_n(X, T)^2 + \tau_n(Y, T)^2 + \tau_n(\tau_{nw}(X, Y), T')^2) \quad (17)$$

211 where τ_{nw} is the series of the empirical Kendall's τ obtained through moving window for
 212 corresponding series X and Y (see Figure 1). T' is the new series of time order that has the same
 213 length of τ_{nw} . Note that the length q of the obtained series τ_{nw} is related to the sample size n and
 214 the width s of the window as $q = n - s + 1$. Choosing the size of s involves a trade-off. On one
 215 hand, a small s increases the number of rolling window series q for reliable analysis. On the other
 216 hand, a large s is necessary to have a sufficient number of values to identify the dependence
 217 structure, but this might decrease q (e.g. Bender *et al.*, 2014; Chebana & Ouarda, 2021). In addition,
 218 the selection of the width s of the windows is a common challenge to various tests (e.g. Bücher *et*
 219 *al.*, 2019; Chebana, 2022). To the best of our knowledge, and considering the existing literature,
 220 formal statistical inference procedures specifically designed to address this purpose appear to be
 221 not clearly established (e.g. Bücher *et al.*, 2019; Kojadinovic & Yan, 2011).

222 Note that the window step in the moving windows shifts point-by-point in this study. Previous
 223 studies by Vidrio-Sahagún and He (2022) have shown that a potential bias is introduced due to the
 224 fact that data points located in the center of the series would be counted more times than those
 225 located at the bounds, thereby exerting a significant influence on the estimates.

226 We employed the square of each term in order to avoid them cancelling each other or reduce the
 227 final value of the statistics. This is similar when passing from the test CST in (10) to the test in
 228 (12). This test is designed to test overall trend in a multivariate series. Indeed, the first two terms

229 $\tau_n(X, T)^2$ and $\tau_n(Y, T)^2$ focus on the univariate trends in the variables X and Y , respectively, with
 230 respect to the time order T . However, the last term $\tau_n(\tau_{nw}(X, Y), T')^2$ introduces a distinctive
 231 multivariate perspective by considering the Kendall's τ between X and Y with a moving window
 232 applied through T' . Unlike other available multivariate trend tests, this term allows to integrate the
 233 dependence between the variables in the proposed test. Hence, the developed overall multivariate
 234 trend test T_{MOT} considers the trend both in margins and in the dependence structure.

235 In order to focus on the trend in the dependence structure, the following multivariate dependence
 236 trend (T_{MDT}) test is proposed:

$$237 \quad T_{MDT} = \tau_n(\tau_{nw}(X, Y), T') \quad (18)$$

238 It represents the last term in (17) dealing only with dependence.

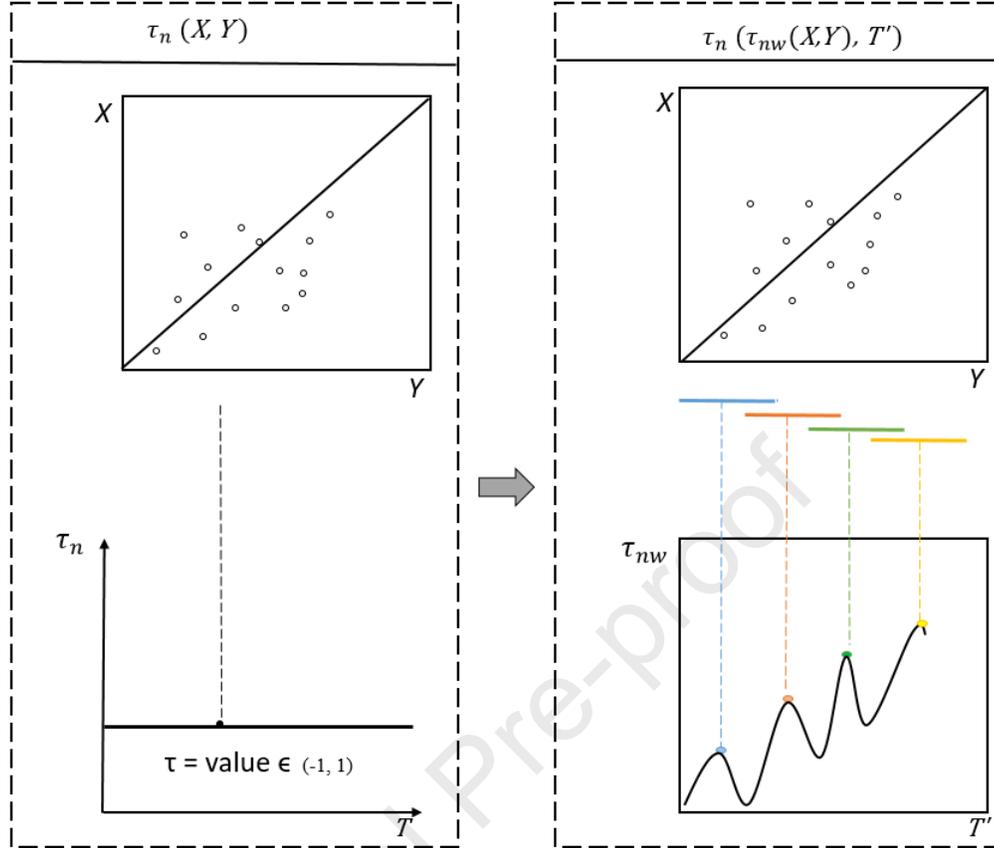


Figure 1: Illustration of evolution of the dependence structure obtained through moving windows for corresponding series X and Y

239

240 To evaluate the *p-values* corresponding to the proposed tests, the bootstrap procedure is considered
 241 (e.g. Good, 2005). The asymptotic distribution of the proposed statistics is beyond the framework
 242 of this paper since the distributions of these statistics T_{MOT} and T_{MDT} under the null hypothesis
 243 depend on the unknown copula. Moreover, asymptotic results could be inappropriate in the context
 244 of HFA, and other fields dealing with extreme values, since the series are usually very short (e.g.
 245 Nasr & Chebana, 2019; Rutkowska, 2015).

246 The methodology of the proposed tests is based on two well-known notions in statistics and
 247 applications, i.e. Kendall's τ extension and the moving window technique. Regarding the moving
 248 window technique, used to integrate the evolution of the dependence structure, it has been

249 considered in other studies for different reasons, such as in econometrics (e.g. Selvin *et al.*, 2017),
250 in finance (e.g. Siami-Namini & Namin, 2018), in medicine (e.g. Dinh *et al.*, 1999) and in statistic
251 (e.g. Genest & Rémillard, 2004).

252 The proposed tests have several conceptual advantages (along with their performance presented
253 below). Indeed, they allow overcoming some drawbacks of the existing multivariate tests (Table
254 1). In fact, the proposed overall statistic test T_{MOT} is designed to detect the trend in different
255 components (both margins and dependence structure) and it is not componentwise. The second
256 proposed test T_{MDT} is constructed to focus on detecting trend in dependence structure. Then, the
257 use of the proposed tests T_{MOT} and T_{MDT} , along with the univariate testing for each margin,
258 provides an attractive and complete procedure for testing trend in the multivariate framework. Even
259 though the proposed tests are introduced and evaluated as part of multivariate HFA, they can be
260 considered in other fields and application dealing with multivariate trends such as economics,
261 finance, medicine, and climatology. It is important to recall that these tests are designed to test
262 *monotonic* trends only.

263 **4. Simulation study**

264 A Monte Carlo simulation study is conducted to evaluate the performance of the proposed
265 multivariate trend tests (e.g. Hirsch *et al.*, 2015; Hirsch & Slack, 1984) and compare them with the
266 existing multivariate tests. Since the test CST has already lower performance compared than those
267 of CIT and CET (e.g. Modarres, 2018), then CST test is not considered. In addition, CET test is
268 the one recommended among the available multivariate ones (e.g. Chebana & Ouarda, 2021;
269 Chebana *et al.*, 2013; Lettenmaier, 1988; Modarres, 2018).

270 **4.1 Simulation design**

271 Given that a multivariate distribution can be composed of margins and dependence structure, a
272 trend can affect these components in different ways. Therefore, we considered the following
273 scenarios for the bivariate case:

- 274 a) Trend only in the dependence structure
- 275 b) Trend only in one margin
- 276 c) Trend in both margins with the same direction (increasing)
- 277 d) Trend in both margins with different directions
- 278 e) Trend in both margins and also in the dependence structure.

279 The above scenarios were considered in different levels and values in order to evaluate the possible
280 effects on the performances of the considered trend tests with different factors (direction and
281 magnitudes of the trend, degrees of dependence and sample size).

282 Data are generated from representative margins and copulas in hydrometeorology analyses to
283 evaluate the performance of the considered tests (e.g. Nasr & Chebana, 2019; Salvadori & De
284 Michele, 2010; Zhang & Singh, 2006). The employed copulas are in two groups: Archimedean
285 (Clayton, Frank, Joe and Gumbel), and Extreme-Value (Galambos and Husler-Reiss).

286 .Even though, a large number of univariate distributions are available, the generalized extreme
287 value (GEV), lognormal (LN2) and three-parameter lognormal (LN3) have been those developed
288 in non-stationarity hydrological framework (e.g. Chebana & Ouarda, 2021). In this study, we have
289 opted for the GEV as the marginal distribution as by previous studies (e.g. El Adlouni *et al.*, 2007;
290 Gado, 2016).

291 The GEV distribution is parameterized with location (μ), scale (σ), and shape (ξ) parameters. As
292 in previous studies, the non-stationary aspect is introduced by allowing the location parameter to

293 be linear function of time (μ_t), while keeping the scale and shape parameters constant ($\mu_0 + \mu_1 t$, σ ,
294 ξ) where t is the time order.

295 In this study, the location parameter that characterize the non-stationarity are selected to have weak
296 trends, a condition frequently observed in hydrometeorological series (e.g. El Adlouni *et al.*, 2007;
297 Gado, 2016). As in Gado (2016), the location parameter was chosen in the range of $-0.3 \leq \mu_1 \leq$
298 $+0.5$ and $\mu_0 = 0$ in order to test the sensitivity of the proposed tests to the values of a variety of
299 trends. The scale and shape parameters were fixed at $\sigma=1$ and $\xi=-0.1$ respectively (e.g. El Adlouni
300 *et al.*, 2007). Other values of the shape parameter, such as -0.3 as considered by El Adlouni *et al.*
301 (2007), have been checked. The obtained results showed no significant changes leading to similar
302 conclusions (for space limitations, those results are not presented). Note that, given the main
303 contribution of the present study is in the multivariate framework, the focus is not on univariate
304 aspects (e.g. selection of marginal distributions and their parameters).

305 In order to consider trend in the dependence structure, we generated random samples from time-
306 dependent copula C_t where the corresponding parameter θ_t in terms of Kendall's τ τ_t is assumed
307 to be linear with respect to time, similarly to Nasri *et al.* (2019). Each copula has a specific
308 parameter range and related to Kendall's τ (e.g. Chebana, 2022). On the other hand, in the majority
309 of flood events, the Kendall's τ is between 0.3 and 0.8 (e.g. Nasr & Chebana, 2019; Requena *et al.*,
310 2013; Zhang & Singh, 2007). Hence, we considered three values of $\tau = 0.2, 0.6, 0.8$, representing
311 weak, moderate, and strong dependence respectively.

312 Different factors could affect the performance of a trend test, either univariate or multivariate,
313 specifically the sample size n and magnitudes of the trend (e.g. Bihrat & Bayazit, 2003;
314 Lettenmaier, 1988; Rutkowska, 2015; Yue *et al.*, 2002). Moreover, the proposed tests could be
315 affected by dependence strength and copula type (e.g. Quessy *et al.*, 2013). Hydrologic series are

316 usually characterized by small sample sizes. Hence, we considered sample sizes of $n = 30, 50$ and
317 100 as in other studies (e.g. Barth *et al.*, 2017; Nasr & Chebana, 2019; Santhosh & Srinivas, 2013).
318 Since the size s of the rolling window series is related to the sample size n (e.g. Chebana & Ouarda,
319 2021), the window size s is selected respectively as $s = 10, 15, 20$ for $n = 30, 50, 100$ similarly to
320 Nasr and Chebana (2019). This is short enough to have large rolling window series and lengthy
321 enough to have an adequate number of values for identifying the dependence structure (e.g. Bender
322 *et al.*, 2014). While the challenge of choosing the size s of moving windows is common to other
323 tests in the literature, formal statistical inference procedures for this purpose are lacking in the
324 literature (e.g. Kojadinovic & Yan, 2011). However, Bücher *et al.* (2019) discussed this matter and
325 suggest, considering values such as $s = 2, 3, \text{ or } 4$ and depending on the ultimate interest, one might
326 also consider choosing s differently. Chebana and Ouarda (2021) consider $s = 12$ for 27-sample
327 size. In the case of Bender *et al.* (2014), the time window length is set to $s = 50$ years for 191 years.
328 It is important to note that, across all scenarios, $N_{sim} = 1000$ samples are generated to ensure stable
329 results. Preliminary trials were conducted to assess convergence, and stability in results was achieved
330 with $N_{sim} = 1000$ samples. Note that, no significant differences have been observed with other values
331 greater than $N_{sim} = 1000$ such as $N_{sim} = 5000$. The first kind error, or nominal level, α is set to the
332 usual value $\alpha = 5\%$. To compare the considered tests, we evaluate their ability to estimate α as
333 well as to quantify their power $(1-\beta)$. Figure 2 summaries the conducted simulation study.

334

335 **4.2 Simulation results**

336 This section presents the obtained results of simulation study by estimating the nominal level and
337 evaluation the power of the considered tests.

Figure 2: Diagram of the simulation study

338 **4.2.1. Nominal level estimation**

339 This section reports the estimates $\hat{\alpha}$ of α for the proposed tests T_{MOT} , T_{MDT} along with the

Table 3: First type error estimates (%) by the proposed multivariate tests (T_{MOT}, T_{MDT}) and the existing ones (CIT, CET)

340 multivariate existing tests CIT and CET for different factors: sample sizes, dependence strengths
 341 and different copula types as presented in Table 3. From Table 3, we observe that the proposed
 342 tests T_{MOT} and T_{MDT} provide estimates $\hat{\alpha}$ close to the selected significance level $\alpha = 5\%$ for
 343 different factors. The first type error should be close to the chosen significance level α to be exact
 344 which is the main advantage of proposed tests.

345 Results from Table 3 show that the proposed tests T_{MOT} and T_{MDT} are generally not sensitive to
 346 different factors. First, it can be seen that the proposed tests are almost insensitive to the copula
 347 type regarding the estimation of α . Indeed, as an example, for T_{MOT} and T_{MDT} tests, $\hat{\alpha}$ is
 348 respectively in the range 3.8-5.9%, and 3.8-5.5%, for $\tau = 0.6$ and $n = 50$ for different copula type.

349 Regarding the dependence strength, for a given sample size, it has almost no effect on the
 350 estimation of α by T_{MOT} and T_{MDT} tests. As an example, when considering the Frank copula with
 351 $n = 50$, the estimation ranges between 3.5-4.5% for T_{MOT} and 3.9-5.3% for T_{MDT} , for different
 352 values of τ . We have similar results regarding the effect of the sample size n . For example,
 353 considering a dependence strength τ of 0.6 and employing the Clayton copula, estimated $\hat{\alpha}$ for
 354 T_{MOT} ranges from 5.1-5.9%, and for T_{MDT} , it varies between 3.8-5.0% across different sample sizes.

355 As we can see under H_0 , the results presented in Table 3 indicate that existing multivariate CIT
 356 and CET tests lead to estimates $\hat{\alpha}$ close to the selected significance level $\alpha = 5\%$ for different
 357 factors (in the range 3.1-6.6%). Not that, in the context of water quality, Lettenmaier (1988) found
 358 that CIT and CET tests provide under-estimations of the nominal values.

359

360 **4.2.2. Power evaluation**

361 In this section, we examine the power of the proposed tests in detecting the trend in the margins
 362 and dependence structure (combined or separately).

Test	Copula under H_0	$\tau = 0.2$			$\tau = 0.6$			$\tau = 0.8$		
		$n = 30$	$n = 50$	$n = 100$	$n = 30$	$n = 50$	$n = 100$	$n = 30$	$n = 50$	$n = 100$
T_{MOT}	Clayton	4.0	4.9	5.6	5.9	5.2	5.1	3.2	5.6	3.9
	Frank	4.9	4.5	4.6	4.5	4.5	6.1	4.6	3.5	4.5
	Joe	4.7	5.2	4.9	6.1	5.9	4.8	4.3	4.3	4.4
	Gumbel	5.8	3.9	5.4	3.8	4.9	4.7	3.8	4.7	5.0
	Galambos	4.6	4.7	6.3	4.9	4.4	4.5	4.9	4.9	4.1
	Husler-Reiss	4.2	5.1	4.9	3.3	3.8	5.5	2.9	4.4	5.4
	Clayton	5.3	4.7	4.8	5.0	3.8	3.9	3.9	3.5	6.0
T_{MDT}	Frank	4.8	5.3	4.3	3.4	4.1	4.0	3.6	3.9	6.5
	Joe	4.2	5.2	5.4	3.6	5.4	5.4	3.6	4.5	4.8
	Gumbel	3.9	4.9	3.9	4.4	4.4	4.2	3.6	4.0	5.3
	Galambos	4.2	5.3	4.5	4.4	5.5	5.2	4.9	4.2	3.7
	Husler-Reiss	4.3	5.2	4.7	4.5	4.9	4.6	4.3	5.6	4.4
	Clayton	4.6	6.1	6.1	4.2	5.2	5.3	4.7	4.8	4.5
	Frank	3.1	3.7	5.9	5.5	5.7	3.9	3.5	4.4	6.0
CIT	Joe	5.1	4.7	5.5	4.9	4.8	5.3	4.6	4.4	5.8
	Gumbel	4.6	5.1	4.9	4.1	5.3	5.2	5.6	4.4	5.7
	Galambos	4.7	4.6	4.6	6.1	5.4	5.4	4.9	5.3	4.7
	Husler-Reiss	5.7	5.3	6.0	3.8	4.5	5.4	3.9	4.4	5.9
	Clayton	5.1	6.3	5.7	4.6	4.9	5.5	5.8	5.3	3.7
	Frank	3.1	4.6	6.1	5.5	6.6	3.9	4.7	5.3	4.5
	Joe	5.5	4.2	4.9	5.1	5.2	6.1	5.5	4.1	5.0
CET	Gumbel	5.3	5.6	4.9	4.2	5.7	5.0	4.5	4.8	5.4
	Galambos	5.4	5.2	4.3	6.0	5.0	6.5	4.4	5.9	4.8
	Husler-Reiss	6.0	6.0	5.9	5.2	5.9	3.8	4.4	5.9	5.8

363 **a) Trend in the dependence only**

364 The power of the proposed tests in detecting the trend in the dependence structure is studied.

365 Results for different sample sizes, different copulas and dependence strengths, are displayed in

366 Table 4. One can see overall from Table 4 that the proposed tests T_{MOT} and T_{MDT} stand out with

367 high power, in contrast to the CIT and CET tests.

368 From Table 4, one first notes that the type of the copulas and dependence strength seems to have
 369 little influence on the power of the T_{MOT} and T_{MDT} tests; it is rather the sample size that have a
 370 significant impact. Indeed, we can see that the power of the tests T_{MOT} and T_{MDT} is increasing with
 371 the sample size. For instance, for T_{MOT} test with Clayton copula, the power increases from 37.0%
 372 when $n = 30$ to 94.7% when $n = 100$ (similarly for T_{MDT} from 36.7 % to 95.5%). These results
 373 align with the results from other tests, which also observed that power increase with the sample
 374 size (e.g. Hirsch *et al.*, 1982).

375 It is also of interest to note that the power of T_{MOT} and T_{MDT} has less variability regarding copula
 376 type. For instance, for T_{MOT} test with $n = 50$, powers are 66.6% and 64.4% when considering
 377 Clayton copula and Frank copula respectively. As another example, in the T_{MDT} test with $n = 100$,
 378 the powers are 95.5% for the Clayton copula and 94.8% for the Galambos copula. These values are
 379 considerably high. Moreover, the power of T_{MOT} and T_{MDT} , remains well regardless of the trend
 380 direction in the dependence structure. No significant differences were observed in the powers when
 381 considering various trend directions. For the sake of simplicity and brevity, the results pertaining
 382 to the decreasing trend direction are not presented.

383 Overall, with some exceptions, both proposed tests lead to similar powers. According to the sample
 384 size, the power is low values (roughly 30 to 45%), moderate (55 to 67%) to very high (88 to 95%).
 385 In trend or non-stationarity studies, it is important and appropriate to have n as high as possible.
 386 Hence, the proposed tests are adapted to this context.

387 In the comparison of performance between the proposed tests (T_{MOT} , T_{MDT}) and the existing tests
 388 CIT and CET, a notable observation is that the latter exhibit inability to detect any trend in the
 389 dependence structure across all examined scenarios. For instance, for CET test, the power estimate

390 consistently ranges between 3.8%-8.8%, close to those in Table 3. These very low values are
 391 expected since these tests ignore the dependence structure explicitly in their construction.

Sample size	Test	Archimedean				ExtremeValue	
		Clayton	Frank	Joe	Gumbel*	Galambos	Husler-Reiss
$n=30$	T_{MOT}	37.0	38.3	41.9	36.3	43.2	32.2
	T_{MDT}	36.7	35.0	43.9	37.6	45.6	31.5
	CIT	8.8	4.0	6.5	5.2	6.1	5.1
	CET	6.8	4.7	8.0	4.7	4.9	4.7
$n=50$	T_{MOT}	66.6	64.4	64.1	59.6	67.2	55.6
	T_{MDT}	67.2	59.1	65.5	59.7	67.5	56.1
	CIT	6.9	5.5	7.3	4.9	4.8	4.9
	CET	6.0	5.1	6.6	4.6	3.8	3.9
$n=100$	T_{MOT}	94.7	92.3	92.3	90.1	95.3	87.5
	T_{MDT}	95.5	92.1	94.7	88.3	94.8	88.3
	CIT	5.4	5.2	6.3	5.5	5.5	4.3
	CET	5.0	4.3	6.2	5.3	4.2	3.8

392 This table presents the power of the proposed test at significance level $\alpha=5\%$, for different scenarios. The Gumbel* copula belongs
 393 both to the class of Archimedean and extreme value copulas

394 ***b) Power evaluation: trend in one margin only***

395 The results corresponding to this scenario are presented in Table 5. Since in this section we are
 396 only interested in the marginal distributions, we consider only two families of copula (Clayton and
 397 Galambos) with fixed Kendall's τ , $\tau = 0.6$.

398 Table 5 show higher powers of the statistical test T_{MOT} as both the sample size and the trend slope
 399 increase, eventually reaching 100%. These high power values demonstrate the efficacy of T_{MOT} in
 400 detecting trends in one margin. From Table 5, we can see that the impact of sample size n on the
 401 power. For a given slope of location parameter μ_t and a copula, the power increases with n . For
 402 instance, we consider the case with a location parameter slope $\mu_1 = 0.1$, generated from a Clayton
 403 copula. In this case, the power of the T_{MOT} test rises notably from 17.5% at $n = 30$ to 100% at $n =$
 404 100. These high powers, highlighting the effectiveness of the T_{MOT} test in detecting trend within

405 one margin. In addition, no differences in growth are observed between different copulas. For
 406 example, for $n = 100$ and $\tau = 0.6$, powers of T_{MOT} tests are 100% for Clayton and Galambos
 407 copulas.

408 Table 5 shows also high powers of the T_{MOT} test, particularly when the trend slope increases.
 409 Indeed, for sample size $n = 50$ and Galambos copula, power estimates of the T_{MOT} test increase
 410 from 42.2% to 100% when slope μ_1 passes from 0.1 to 0.3. This finding is expected and it is in
 411 agreement with the literature dealing with univariate trend (e.g. Yue *et al.*, 2002). Moreover, from
 412 Table 5, when considering a Galambos copula with $n = 100$, the T_{MOT} test demonstrates very high
 413 powers, reaching 100% for both increasing trend ($\mu_1 = 0.1$) and decreasing trend ($\mu_1 = -0.1$). This
 414 result demonstrates the effectiveness of the T_{MOT} test, highlighting its ability to capture trends in
 415 margins irrespective of their direction.

416 Results from Table 5 show that the proposed test T_{MDT} is not able to detect any trend in margins.
 417 In all cases, the power estimates are less than 3.7%. This is not surprising since this test is intended
 418 to capture trend only in dependence structure.

**Table 5: Power estimates (%) of the proposed tests (T_{MOT} , T_{MDT}) and existing tests (CIT, CET)-
 trend in one margin only**

419

Test	Copula	$\tau = 0.6$											
		$\mu_1 = 0.1t$			$\mu_1 = 0.3t$			$\mu_1 = 0.5t$			$\mu_1 = -0.1t$		
		$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$
T_{MOT}	Clayton	17.5	40.8	100	96.6	100	100	100	100	100	17.0	39.4	100
	Galambos	15.0	42.2	100	96.0	100	100	100	100	100	17.3	42.5	100
T_{MDT}	Clayton	2.9	2.8	3.1	1.1	0.8	0.6	0.4	0.4	0.0	3.7	1.9	1.6
	Galambos	2.9	2.1	1.3	0.6	0.6	0.0	0.5	0.3	0.1	2.6	2.2	2.1
CIT	Clayton	99.9	100	100	100	100	100	100	100	100	99.7	100	100
	Galambos	100	100	100	100	100	100	100	100	100	100	100	100

CET	Clayton	94.3	100	100	100	100	100	100	100	100	94.4	100	100
	Galambos	94.0	100	100	100	100	100	100	100	100	93.7	100	100

420 This table presents the power of the proposed test at significance level $\alpha=5\%$

421 As part of comparison, Table 5 reports also the performance for the existing CIT and CET tests.

422 Through Table 5, those tests have high power values almost always 100%. These high power values

423 of CIT and CET tests are different from the literature (e.g. Hirsch & Slack, 1984; Lettenmaier,

424 1988). This can be attributed to distinct simulation conditions. Initially developed for monthly

425 water quality data, the CET and CIT tests are not specifically adapted for hydrological data, as in

426 our case. The primary purpose of these tests is the detection of trends in time series, not the

427 frequency analysis of hydrological data. This dissimilarity in objectives and context introduces

428 several differences in simulation conditions. For instance, Hirsch and Slack (1984) and Lettenmaier

429 (1988) based their analyses on sample sizes ranging from 5 to 20, with a slope value from 0.0065

430 to 0.05, specifically chosen to match the features of water quality time series. When we applied

431 some of these features ($n = 20$, slope = 0.05), the power of the CET test decreased significantly to

432 11%.

433 *c) Power evaluation: trend in both margins*

434 We present the power values here when trend is present in both margins. Table 6 shows that

435 except T_{MDT} , the power of all tests is very high and can reach 100%. The high power of the

436 developed multivariate T_{MOT} test clearly emphasizes its effectiveness in detecting trends in both

437 margins. Moreover, T_{MOT} power significantly increases with n . For instance, with a Galambos

438 copula and slopes $\mu_1 = -0.1$ and $\mu_2 = -0.1$, the power values for the T_{MOT} test increase from 56.4%

439 at $n = 30$ to a 100% at $n = 100$.

440 Table 6 demonstrates also the impact of the trend direction between both margins on the power of

441 T_{MOT} . Regardless of whether the trend is increasing or decreasing, the T_{MOT} test exhibits high

442 power. As an example, when examining different trend directions for both margins with location
 443 parameters set at $\mu_1 = 0.3$, $\mu_2 = -0.3$, and considering Clayton copulas, the power remains at a high
 444 level of 100% across all sample sizes n . This high power demonstrates the effectiveness of the
 445 T_{MOT} test in detecting trends across both margins under diverse directions.

Table 6: Power estimates (%) of the proposed tests (T_{MOT} , T_{MDT}) and existing tests (CIT, CET)-trend in both margins

446 Table 6 provides also insights about the effect of the trend slope on the power on the power of the
 447 proposed tests. In fact, the proposed test T_{MOT} performs clearly better when the slope of trend
 448 increases. As an example, considering $n = 30$ and Clayton copula, the test power increases from
 449 53.2% if slope are $\mu_1 = -0.1$, $\mu_2 = -0.1$ to 100 % when the slopes are $\mu_1 = 0.3$, $\mu_2 = 0.3$.

450 The powers here increased (except for the T_{MDT}) compared to Table 5 specifically for T_{MOT} . The
 451 reason is that we have additional component with trend in the margins. Moreover, T_{MDT} as designed
 452 and expected, is not detecting such a trend. CET and CIT continue to slowly increasing to reach
 453 100% in all cases which similar to the previous case (Table 5). However, importantly, T_{MOT} is
 454 adapted to the situation with an increase that is realistic. The reason that tests, except T_{MDT} , reach
 455 100% is that the trend in the margins is dominating the non-trend in the dependence (e.g. Bender
 456 *et al.*, 2014).

457 As part of comparison, the two classical tests CIT and CET are able to detect the trend in both
 458 marginal distributions with a high performance =100% in all cases. These obtained results are
 459 different from the literature and the reasons are explained in the previous case (trend in one margin
 460 only). As anticipated, Table 6 affirms that the proposed statistic T_{MDT} is not able to detect any
 461 trend in both margins, aligning with its specific design focused on capturing trend solely within the
 462 dependence structure. In all instances, the powers remain below 7.8%.

Test	Copula	$\tau = 0.6$											
		$\mu_1 = -0.1t$ $\mu_2 = -0.1t$			$\mu_1 = 0.3t$ $\mu_2 = -0.3t$			$\mu_1 = 0.3t$ $\mu_2 = -0.1t$			$\mu_1 = 0.3t$ $\mu_2 = 0.3t$		
		$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$
T_{MOT}	Clayton	53.2	99.8	100	100	100	100	100	100	100	100	100	100
	Galambos	56.4	100	100	97.0	100	100	99.9	100	100	100	100	100
T_{MDT}	Clayton	3.2	5.1	5.4	3.6	0.1	6.8	2.4	1.3	0.5	3.4	4.8	5.5
	Galambos	4.9	4.6	5.2	1.5	0.3	7.8	1.7	1.4	1.7	5.0	4.2	5.2
CIT	Clayton	100	100	100	100	100	100	100	100	100	100	100	100
	Galambos	100	100	100	100	100	100	100	100	100	100	100	100
CET	Clayton	100	100	100	100	100	100	100	100	100	100	100	100
	Galambos	100	100	100	100	100	100	100	100	100	100	100	100

463 This table presents the power of the proposed test at significance level $\alpha=5\%$

464 **d) Power evaluation: trend in both margins and dependence**

465 In this case, we considered a large number of possibilities since all the components of the bivariate
466 distribution have trends. Note that similar results are obtained when examining either an increasing
467 or a decreasing trend in the dependence structure, as well as for the direction of the trend
468 (increasing/decreasing) in both margins. For the sake of brevity, we do not present all the results.

469 Form, results highlight the high power of the T_{MOT} tests across various copulas and sample sizes.
470 Notably, the T_{MOT} test exhibits high performance, reaching 100% power even with weak slopes
471 and short sample sizes.

472 Notably, as shown in Table 7, similar to the preceding scenario, the performance of trend T_{MOT}
473 tests is influenced by the sample size. Specifically, a larger sample size correlates with higher
474 power. This observation is most apparent for a very weak slope $\mu_t = -0.1$ for both margins. As an
475 example, considering Gumbel copula, the test power increases from 83.9 % for a sample size $n =$
476 30 to 100% when $n = 100$. These powers are considerably high. In this instance, the presence of
477 trends in all components (margins and dependence structure) leads to a rapid increase in
478 performance compared to the previous Tables (Table 5 and Table 6).

479 Moreover, the power of T_{MOT} increases with the slope of the trend. For instance, considering
480 Husler-Reiss copula with $n = 30$, the power is 88.6% when $\mu_1 = -0.1$, $\mu_2 = -0.1$ and increases to
481 100% when $\mu_1 = 0.3$, $\mu_2 = 0.3$. However, no significant differences were found between powers
482 when considering different copulas. Indeed, as an example, for $n = 30$ and $\mu_1 = -0.1$, $\mu_2 = -0.1$,
483 the test power is between 83.9% and 94.0% for all different copulas.

484 From Table 7, we can see also that power of T_{MOT} is very high when considering the same trend
485 direction in margins and dependence. As an example, from Table 7 when considering time-varying
486 location parameters for $\mu_1 = 0.3$, $\mu_2 = 0.3t$, and increasing trend in dependence structure, the
487 power is always 100%. Further, it can be seen from the same Table 7 that power estimates of the
488 proposed test T_{MOT} is not sensitive to the different direction of trend between both margins and
489 dependence. For instance, when both margins exhibit a decreasing trend ($\mu_1 = -0.1$, $\mu_2 = -0.1$) and
490 the dependence shows an increasing trend, the power ranges from 83.9% to 100% across different
491 sample sizes n and copula types. Moreover, it is important to emphasise that the proposed statistic
492 T_{MOT} performs well in detecting trend even when considering different directions between both
493 margins. For instance, for time-varying location parameters of $\mu_1 = 0.3$, $\mu_2 = -0.3$, and increasing
494 trend in dependence structure, the power estimates are 100% for all different sample size and
495 copulas. This is because of the terms in the test T_{MOT} are squared to avoid cancelling the trend
496 with different signs. Through Table 7, considering overall test T_{MOT} it is very important to note that
497 the powers are around 100% in the majority of simulation cases. The first column is the only one
498 that does not reach 100% for the different copula types. Indeed, the power range between 83.9%
499 and 94% when $n = 30$. This is because this column represents the lowest slope (-0.1). This is a high
500 results despite the weak slopes of marginal distributions were chosen according to the hydrological
501 flood context and only the location parameter (μ_t) is assumed to be a linear function of time.

Table 7: Power estimates (%) of the proposed tests T_{MOT} and T_{MDT} – trend in both margins and the dependence structure

Test	Copula	$\mu_1 = -0.1t$ $\mu_2 = -0.1t$			$\mu_1 = 0.3t$ $\mu_2 = 0.3t$			$\mu_1 = 0.5t$ $\mu_2 = 0.3t$			$\mu_1 = 0.3t$ $\mu_2 = -0.3t$		
		$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$	$n=30$	$n=50$	$n=100$
T_{MOT}	Clayton	93.1	99.8	100	100	100	100	100	100	100	100	100	100
	Frank	94.0	100	100	100	100	100	100	100	100	100	100	100
	Joe	92.0	100	100	100	100	100	100	100	100	100	100	100
	Gumbel	83.9	100	100	100	100	100	100	100	100	100	100	100
	Galambos	92.4	100	100	100	100	100	100	100	100	100	100	100
	Husler-Reiss	88.6	100	100	100	100	100	100	100	100	100	100	100
T_{MDT}	Clayton	25.4	37.6	70.5	26.5	34.9	78.4	11.0	20.2	50.4	7.2	9.6	15.4
	Frank	31.4	59.3	91.8	22.7	36.3	79.8	14.2	22.4	54.1	8.4	10.3	19.6
	Joe	30.4	58.6	93.7	26.5	47.8	84.5	18.7	28.4	61.7	7.2	11.5	11.5
	Gumbel	34.9	55.4	92.1	29.5	49.4	89.1	16.4	27.8	59.5	7.8	12.1	17.8
	Galambos	29.8	63.3	95.2	37.6	60.8	95.0	14.0	25.3	62.6	6.5	11.3	14.4
	Husler-Reiss	21.7	47.8	89.0	28.3	47.7	90.3	13.5	23.9	54.3	5.7	10.1	15.7

502 This table presents the power of the proposed test at significance level $\alpha=5\%$

503 Regarding the second proposed test, T_{MDT} , as observed in Table 7, we can see that the power
504 substantially increases with a higher sample size, specifically reaching up to 95.2% when the
505 sample size is elevated to $n = 100$. For example, generated data from a Gumbel copula and a slope
506 $\mu_1 = 0.3$, $\mu_2 = 0.3$ and trend in dependence structure, the test power increase from 29.5% for a
507 series of length $n = 30$ to 89.1% when $n = 100$. It is also important to note that exceptions are
508 observed concerning T_{MDT} . Table 7 reveals that the T_{MDT} test's power increases as the slope of both
509 margins decreases. For instance, with the Clayton copula and $n = 50$, the test power increases from
510 50.4% when the location parameters are set to $\mu_1 = 0.5$ and $\mu_2 = 0.3$ to 78.4% when $\mu_1 = 0.3$ and
511 $\mu_2 = 0.3$. Moreover, T_{MDT} test's performance is better when the directions of the margins are the same
512 compared to case where they differ. For example, considering the Frank copula and $n = 50$, the

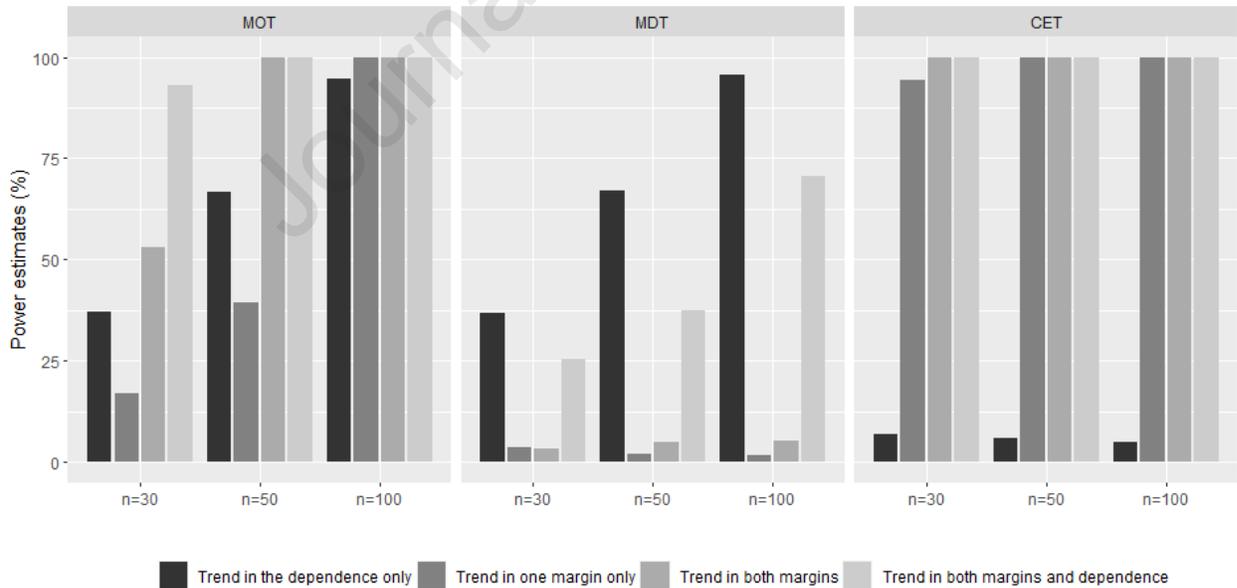
513 power estimates increase from 10.3% when location parameters are set to $\mu_1 = 0.3$, $\mu_2 = -0.3$ to
514 36.3% when $\mu_1 = 0.3$, $\mu_2 = 0.3$. This exceptions on results can be explained by the fact that
515 incorporating trends in the margins can dilute or mask the trend in the dependence, especially when
516 the trend in the margins is stronger than the trend in the dependence (e.g. Bender *et al.*, 2014).

517 In order to have an overview of the power of the proposed tests, T_{MOT} test performs well, especially
518 with more data and stronger trends. It is flexible, width different trend directions. However, T_{MDT} 's
519 power varies more, improving with larger sample sizes but sometimes decreasing with stronger
520 trends in margins. The T_{MDT} test seems to have some exception associated with varying directions
521 in margins trends. Comparatively, existing tests (CIT and CET) have high powers when trends are
522 only in margins but fall short in spotting trends in the dependence. This highlights the importance
523 of T_{MOT} and T_{MDT} , which consider both margins and dependence for a more complete picture.

524 Note that the performance of the existing multivariate tests CIT and CET is not interesting in this
525 scenario (trend in both margins and dependence). Notably, Table 4 demonstrate that both CIT and
526 CET do not identify any trend in the dependence structure. This demonstrates that current existing
527 multivariate tests fall short in capturing trends across the entire system, encompassing both margins
528 and the dependence structure. In particular, they neglect to discern whether a trend is present or
529 absent in the dependence structure.

530 It is important to extract information from different tables (4, 5, 6, and 7) in order to quantify the
531 trend in all components. We chose Clayton copula and slope of trend equal to -0.1 for both margins.
532 Considering T_{MOT} test, powers of the cases of trend in both margins and dependence are higher
533 than in trend on the only the dependence or only in the margins. This can be explained by the fact
534 that the higher the trend in terms of the number components, the higher the power will be.

535 From Figure 3 we can see for $n = 30$ that the power of the T_{MOT} test increases from 17.5% when
 536 trend in one margin, 53.2% when trend in both marginal distributions and to 93.1 % when trend in
 537 all components. This test allows a quantification of the trend since it detects it in all components
 538 unlike the multivariate existing tests. Through Figure 3, it is clear to see that T_{MDT} test performs
 539 well when a trend in dependence structure and in both components (margins and dependence
 540 structure). Moreover, we can see that the presence of the trend in the margins influences the
 541 performance of T_{MDT} test. For example, for $n = 100$, Figure 3 show that the power of test increase
 542 from 70.5% when trend in both margins and dependence, to 95.5% when trend only in dependence
 543 structure. We note that CET test not able to detect the trend in the dependence structure. The high
 544 power of the CET is misleading and once a gain it ignores that there is no trend in the dependence.
 545 The proposed tests, although with lower power, provide realistic and representative results in
 546 detecting trends.



547 Figure 3: Quantification of trend by statistics T_{MOT} , T_{MDT} and CET test for Clayton copulas and
 548 slope of margins equal to -0.1.



Figure 4: Geographical location of stations 01FB003, 05NB001 and 07DA001

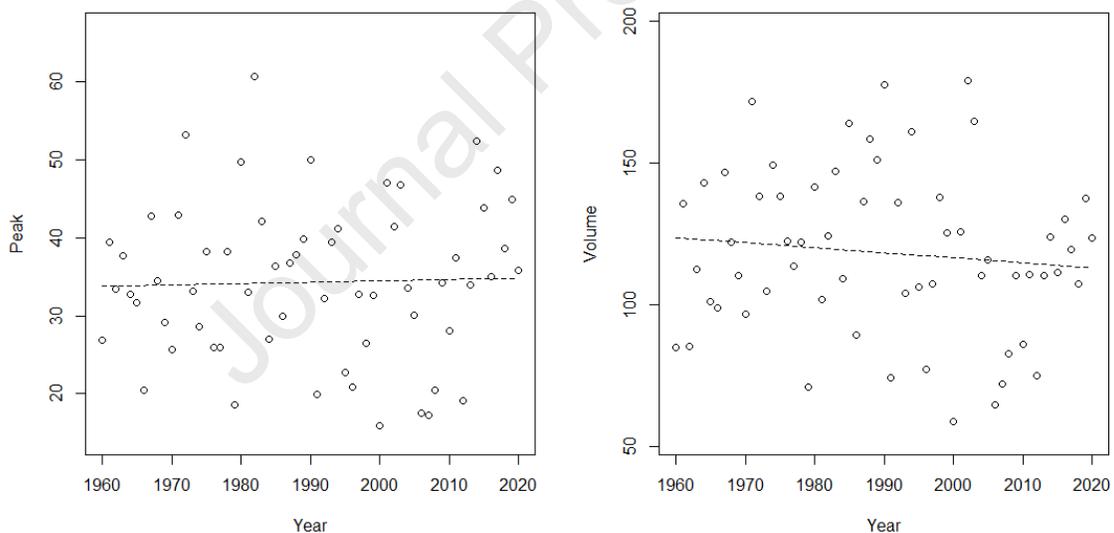
565
 566 Based on hydrological literature, our emphasis is on the flood peak (Q) and volume (V) series (e.g.
 567 Gaál *et al.*, 2015). All the applications considered a significance level of 5%. The results are given
 568 in Table 9.

569 Table 9: Univariate and multivariate stationarity testing results

Station	Univariate MK test		Multivariate MK tests				
	Variable	p-value	CST	CIT	CET*	MOT	MDT
01FB003	Q	0.627	0.668	0.528	0	0.008	0.029
	V	0.713					
05NB001	Q	0.000052	0.00523	0.000040	1	0.026	0.574
	V	0.000011					
07DA001	Q	0.0184	0,138	0,060	1	0.017	0.037
	V	0.026					

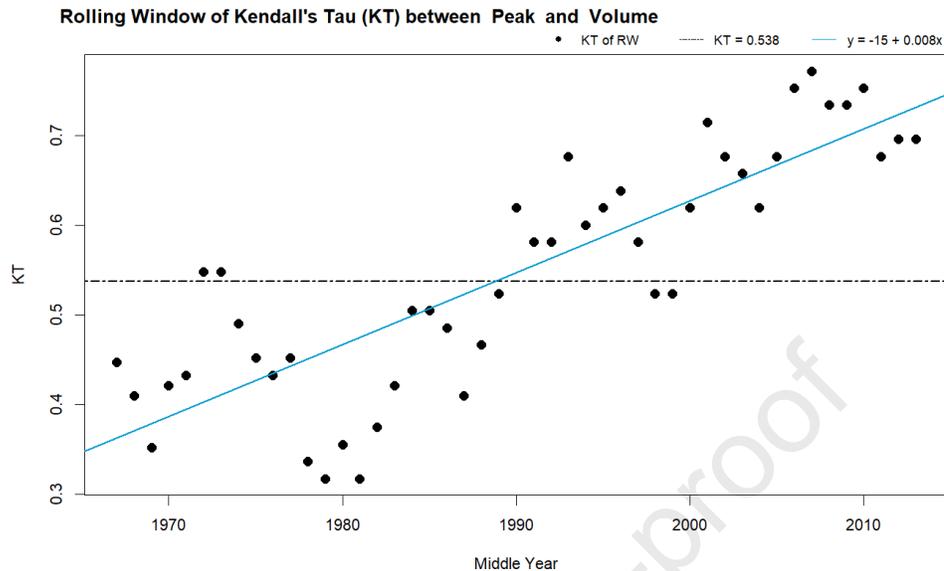
570 The bold character indicates the rejection of corresponding null hypothesis at the 5% level.* Note that, instead of the p-value, for the CET-test the
 571 conclusion is presented as: 1 if there is a trend, 0 if not, since this test is based on critical thresholds.

572 For station 01FB003, first, results show no trend detected in the margins (as confirmed from Figure
573 5). In addition, a significant trend in the dependence structure detected by the proposed MDT test,
574 which is consistent with Figure 6. In contrast, the existing multivariate tests (CST, CIT and CET)
575 were unable to detect the trend in the dependence structure. This confirms the efficacy of MDT in
576 detecting trend in dependence structure. Furthermore, the MOT test also indicates a significant
577 overall trend, which confirm its ability to detect trends some components (here in the dependence
578 structure). Given that, this station is part of the RHBN, the presence of trends in the dependence
579 structure is driven by climate changes. It is noteworthy that Burn and Whitfield (2023) have
580 observed changes in the nival fraction at the same station.



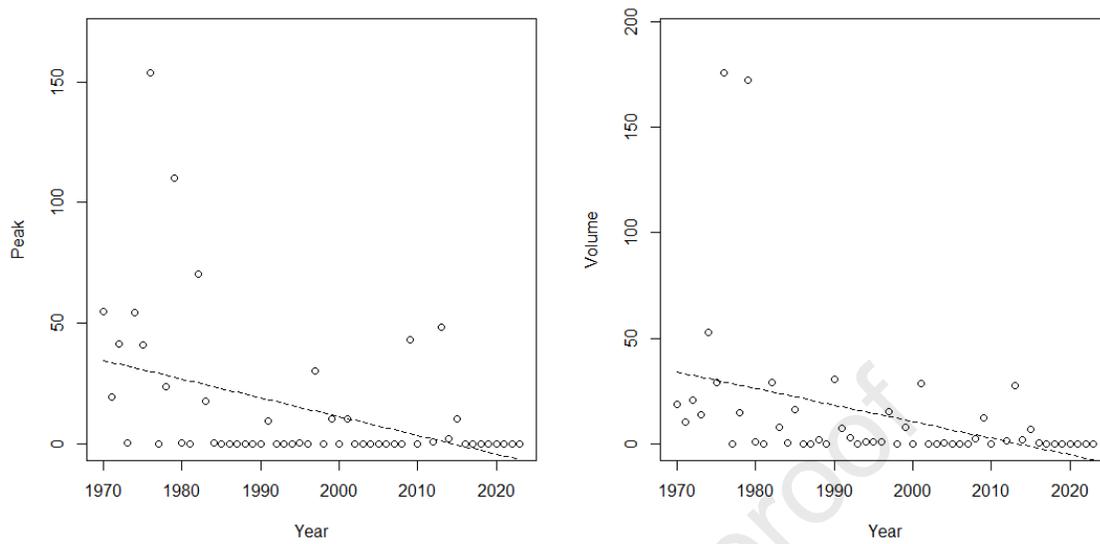
581 Figure 5: Plot of Peak, volume of the series Q (left) and V (right) with the associated regression
582 lines (01FB003)

583

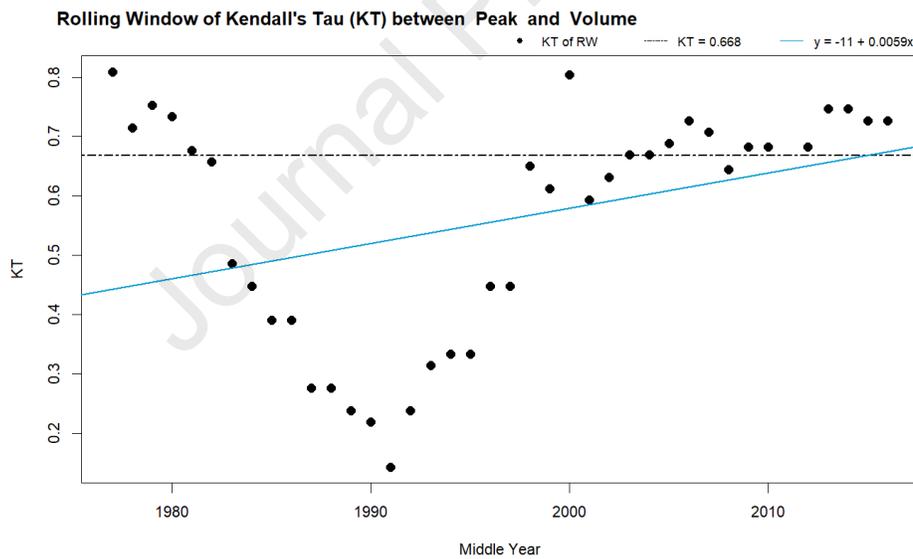


584 Figure 6: Plot of Peak, volume and rolling window of Kendall's τ (KT) between them against year
 585 (01FB003)

586 In station 05NB001, we observe that all multivariate existing tests are in agreement with a
 587 significant trend. The proposed multivariate test MOT also indicated overall trend. This can also
 588 be verified from Figure 7. Trends are also detected by MK univariate for each variable. Given that
 589 this station is non-RHBN and located below Boundary dam Reservoir, the observed trend detection
 590 in the margins may be attributed to anthropogenic activity and/or potential climate change impacts.
 591 It should be noted that no visually clear monotonic trend in the dependence structure is observable,
 592 as shown in Figure 8, as confirmed by the MDT test.

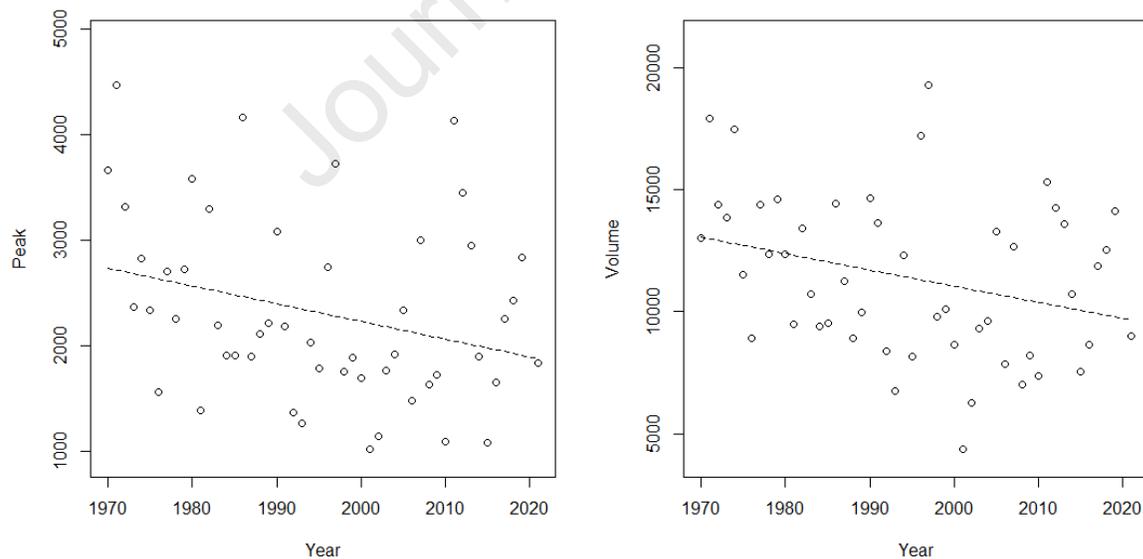


593 Figure 7: Plot of Peak, volume of the series Q (left) and V (right) with the associated regression
 594 lines (05NB001)

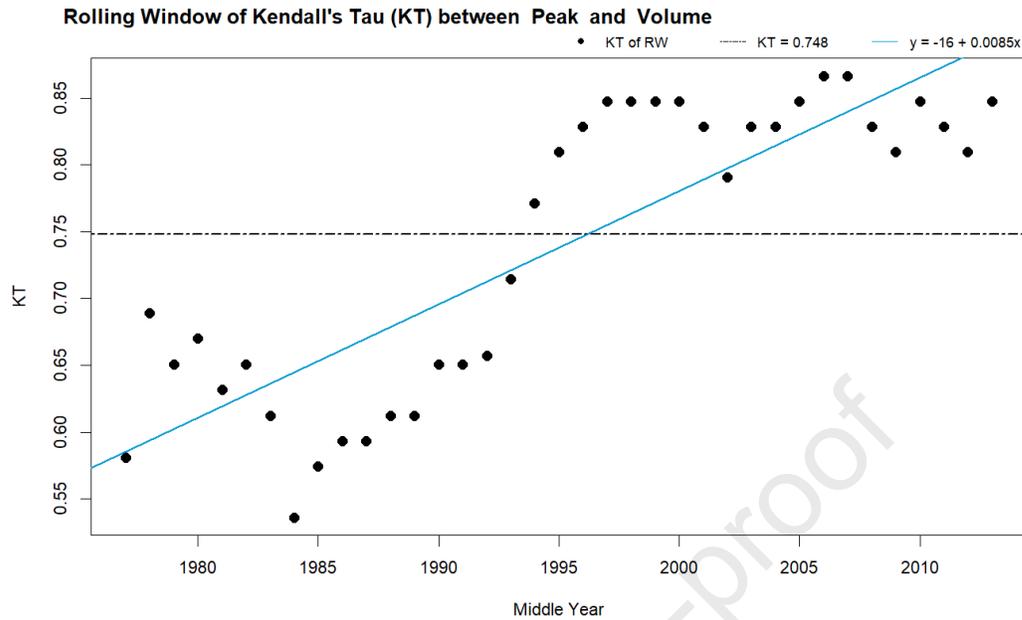


595 Figure 8: Plot of Peak, volume and rolling window of Kendall's τ (KT) between them against year
 596 (05NB001)

597 For 07DA001, we observe that the univariate MK test indicates the presence of trends in both
 598 variables simultaneously (as confirmed by Figure 9). Additionally, the MDT test detects a trend in
 599 the dependence structure, which can also be confirmed from Figure 10. The proposed multivariate
 600 MOT test confirms an overall multivariate trend. Among the existing multivariate tests, only CET
 601 detects the existence of a trend in the margins. The CIT statistic value is very close to the threshold.
 602 The CST test does not detect any trend. This confirms findings in the literature that the CST test
 603 has lower performance compared to CIT and CET (e.g. Modarres, 2018), and the CET test is
 604 recommended among the available multivariate tests (e.g. Chebana & Ouarda, 2021). This station
 605 is a part of the RHBN. Thus, the observed multivariate trends in the margins and dependence
 606 structure could be driven by climate change. Note that, recent decades have seen significant
 607 changes in the hydrological and meteorological conditions of the Athabasca River (e.g. Bawden *et*
 608 *al.*, 2014; Beltaos & Carter, 2009).



609 Figure 9: Plot of Peak, volume of the series Q (left) and V (right) with the associated regression
 610 lines (07DA001)



611 Figure 10: Plot of Peak, volume and rolling window of Kendall's τ (KT) between them against year
 612 (07DA001)

613 6. Conclusions and Perspectives

614 In the literature dealing with multivariate frequency analysis, in general, the stationarity assumption
 615 is not verified. This is in part due to the absence of powerful and effective tests. However,
 616 nowadays in climate and hydrological changing context, it is more and more important to consider
 617 multivariate tests that can detect non-stationarity either in the margins or in the dependence
 618 structure.

619 The aim of the present paper is to develop new tests for multivariate trend to fill a gap in the
 620 statistical and hydrological literature. The first test T_{MOT} is designed to detect trend in the affected
 621 component (margins and dependence), and the second test T_{MDT} is conceived to focus on trend in
 622 the dependence structure. In comparison to existing multivariate tests, simulation results show very
 623 promising performances in terms of first type error and power.

624 The proposed multivariate tests are adopted to hydrological context due to their good performance
625 when the trend is very weak and the series is short, which often happens in hydrological series. The
626 existing tests were not able to detect trend in the dependence structure alone or with the margins.
627 The mutual application of the proposed tests T_{MDT} and T_{MOT} with univariate MK test provides an
628 attractive procedure for testing multivariate trend and to discriminating its potential source.
629 In this paper, the proposed multivariate trend tests were theoretically justified and practically
630 demonstrated through both a comprehensive simulation study and practical illustrative
631 applications. However, certain limitations were observed for the developed tests. Indeed, they are
632 designed for monotonic trends whereas other forms of trends may exist. Thus, for a more flexible
633 trend detection, there is a need to advance the development of non-monotonic trend tests.
634 Moreover, the presence of autocorrelation in the data can influence the outcomes of tests to detect
635 trends. Consequently, Hamed and Rao (1998) proposed a modified univariate MK-test. Similarly,
636 in the multivariate framework, it would be of interest to develop multivariate trend tests suited for
637 autocorrelated data.

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643

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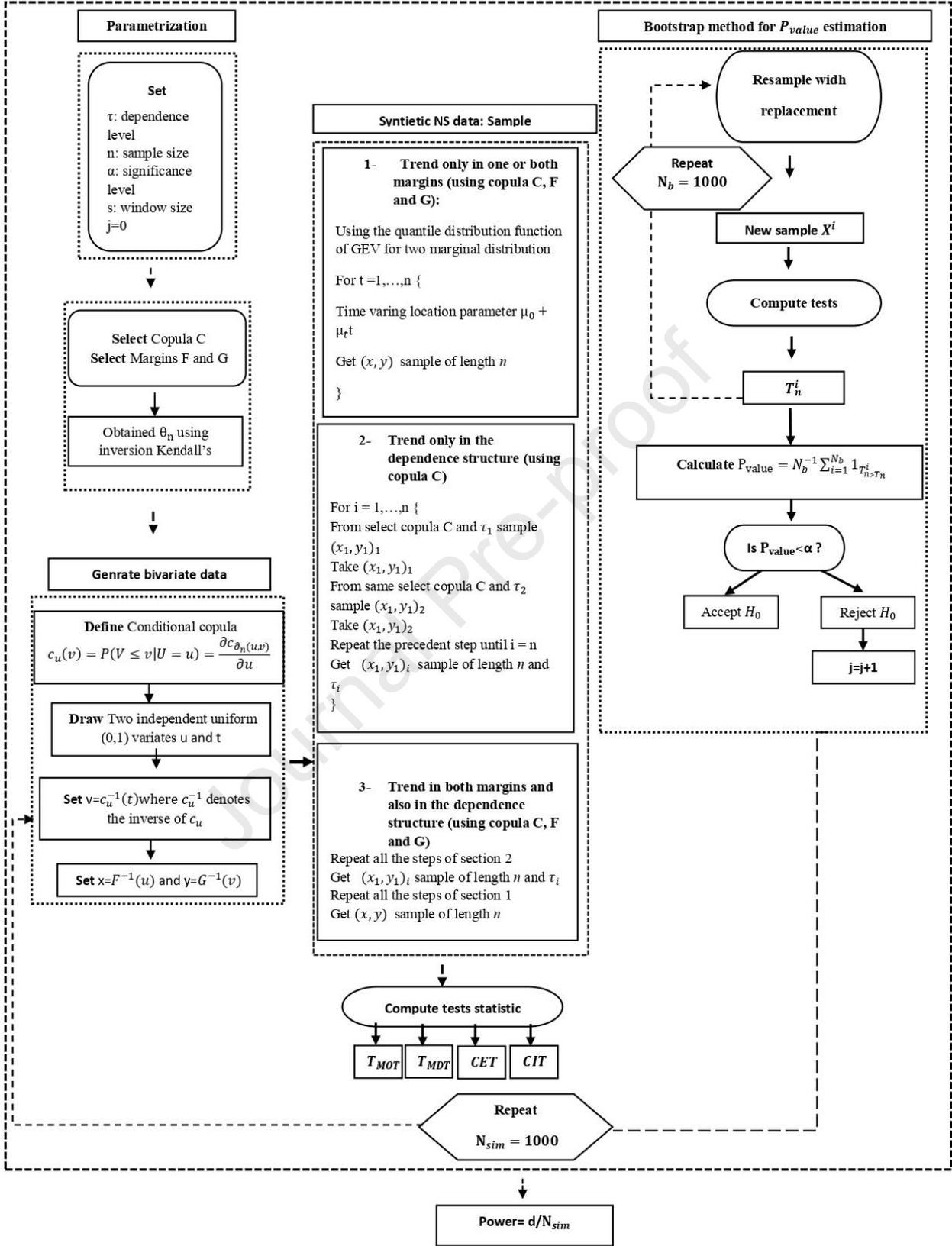


Figure 1: Diagram of the simulation study

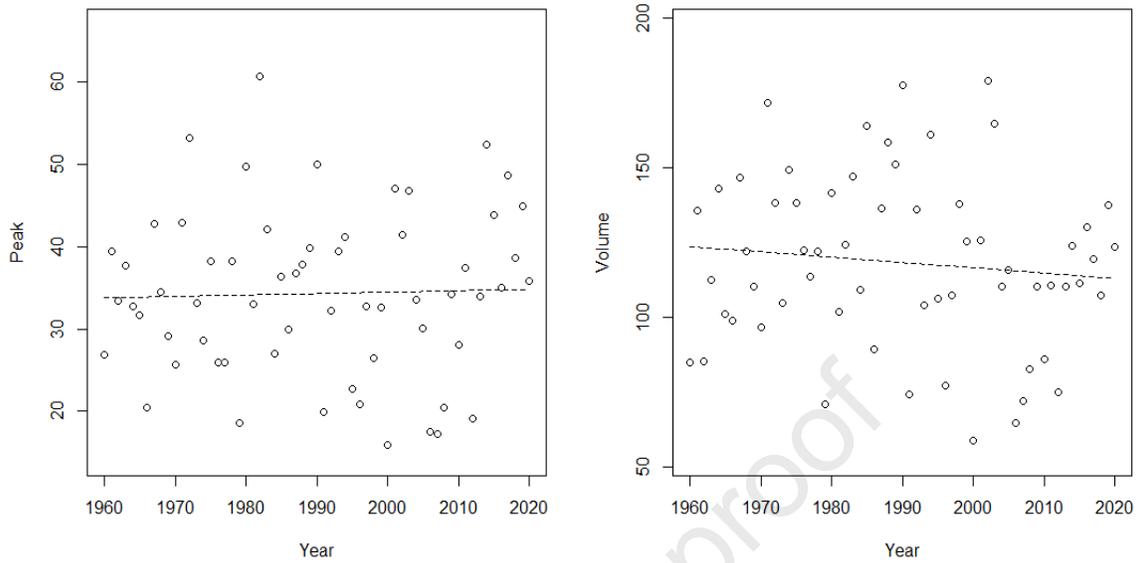


Figure 2: Plot of Peak, volume of the series Q (left) and V (right) with the associated regression lines (01FB003)

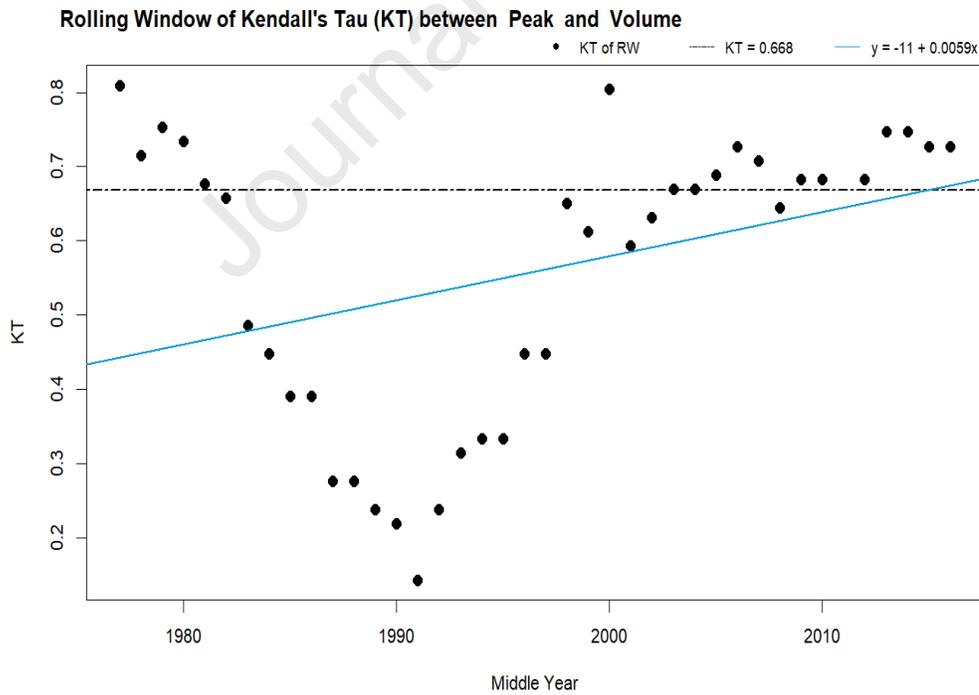


Figure 3: Plot of Peak, volume and rolling window of Kendall's τ (KT) between them against year (05NB001)

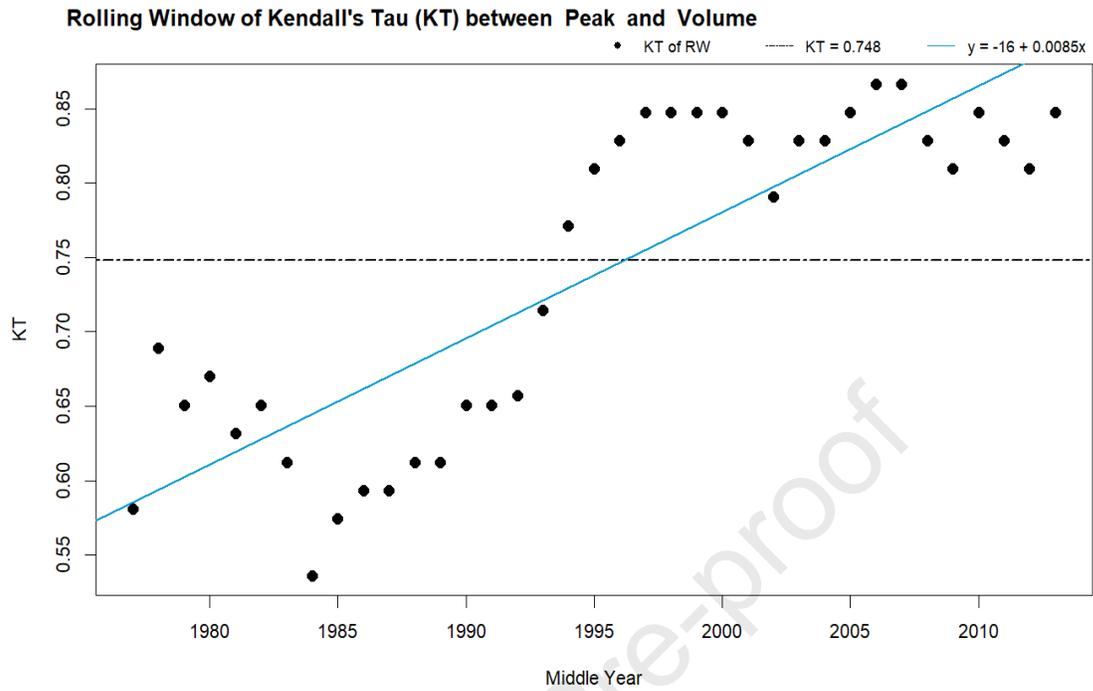


Figure 4: Plot of Peak, volume and rolling window of Kendall's τ (KT) between them against year (07DA001)

- Two multivariate trend tests for multivariate hydrological series are proposed.
- New multivariate overall trend (MOT) test dealing with trend in all the components of the whole multivariate distribution.
- New multivariate dependence trend (MDT) test focuses on trend in the dependence structure.
- Vast simulation study is considered to evaluate the performance of the tests.
- The developed tests show high performance, with increasing power observed as the trend slope and sample size increase.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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