THE PEP STANDARD
COMPUTABLE GENERAL EQUILIBRIUM MODEL
SINGLE-COUNTRY, STATIC VERSION
PEP-1-1

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PREFACE

This paper documents the PEP-1-1 model developed by four members of the PEP research network. The PEP-1-1 model is the first major output of a project that emerged spontaneously from the long-standing association between the co-authors. Our ambition is to crystallize years, even decades for some of us, of CGE modeling experience, and to share the result with the PEP MPIA Network, and with the modeling community at large. In addition, PEP-1-1 is to be our basis, from which to further deepen our understanding of CGE analysis and develop modeling techniques that will tackle new problems. But, both in sharing our experience and exploring new paths, we want to remain in the realm of operational model building. So our intentions are all at once pedagogical, experimental, and practical. And, besides, we enjoy working together: it's just plain fun !

A first, provisional edition of the PEP-1-1 documentation was issued in July 2009, in response to the interest shown by participants of the June 2009 PEP Network meeting in Santiago, Chile. So the authors have responded by hastening to make this paper available as soon as possible.

The model presented in this edition contains a few improvements relative to the provisional version. The principal changes are:

– The value added production function is now CES, rather than Cobb-Douglas.
– Investment demand distinguishes between gross fixed capital formation (GFCF) and changes in inventories.
– The aggregate output of each industry consists of several products, consistent with rectangular input-output tables which are now the standard.
– Several intermediate variables have been added to make the theoretical underpinnings of the model more explicit, and to link model variables to national accounting concepts more closely.

In addition, a new appendix contains the detailed mathematical derivations of model equations.

Of course, we have tried to eliminate as many errors and « misprints » as possible. Needless to say, we welcome comments that will help us improve either the model or its presentation. Readers are invited to send their comments to André Lemelin at the following address: andre_lemelin@ucs.inrs.ca

Bernard Decaluwé, André Lemelin, Hélène Maisonnave, and Véronique Robichaud

INTRODUCTION

This paper proposes a static computable general equilibrium (CGE) model designed for the study of a national economy. This CGE model is intended to be an operational tool for PEP Network researchers and other users, so we hope. With it, they will be able to develop a relatively standard model, and easily apply it to their country, whatever the particular structure of their social accounting matrix (SAM). The present model differs significantly from Decaluwé, Martens and Savard's (2001) EXTER model, which has been used extensively in the past by network researchers who had been trained in one of the many modeling « Schools » held over the years in many parts of the world. In many respects, the PEP-1-1 model is richer than the more pedagogical EXTER. First, the PEP-1-1 model distinguishes several categories of workers and of capital. Also, PEP-1-1 is capable of taking into account a broader set of tax instruments, and it models all possible transfers between institutions (agents). Finally, the GAMS code, presented in a separate document, has been written in a general form, thanks to the use of sets. This will facilitate the application of PEP-1-1 to variously aggregated SAMs, by means of a few simple steps to make the SAM directly readable into the GAMS program. We are currently working on extensions of the PEP-1-1 model, to guide researchers in building dynamic or multi-country world models[5].

In what follows, we present the model equations and state their underlying hypotheses. Appendices contain the complete list of equations, sets, variables and parameters. Our presentation begins with the production equations, from which are derived factor-demand equations and price equations. This follows the method of exposition suggested in Decaluwé, Martens and Savard (2001).

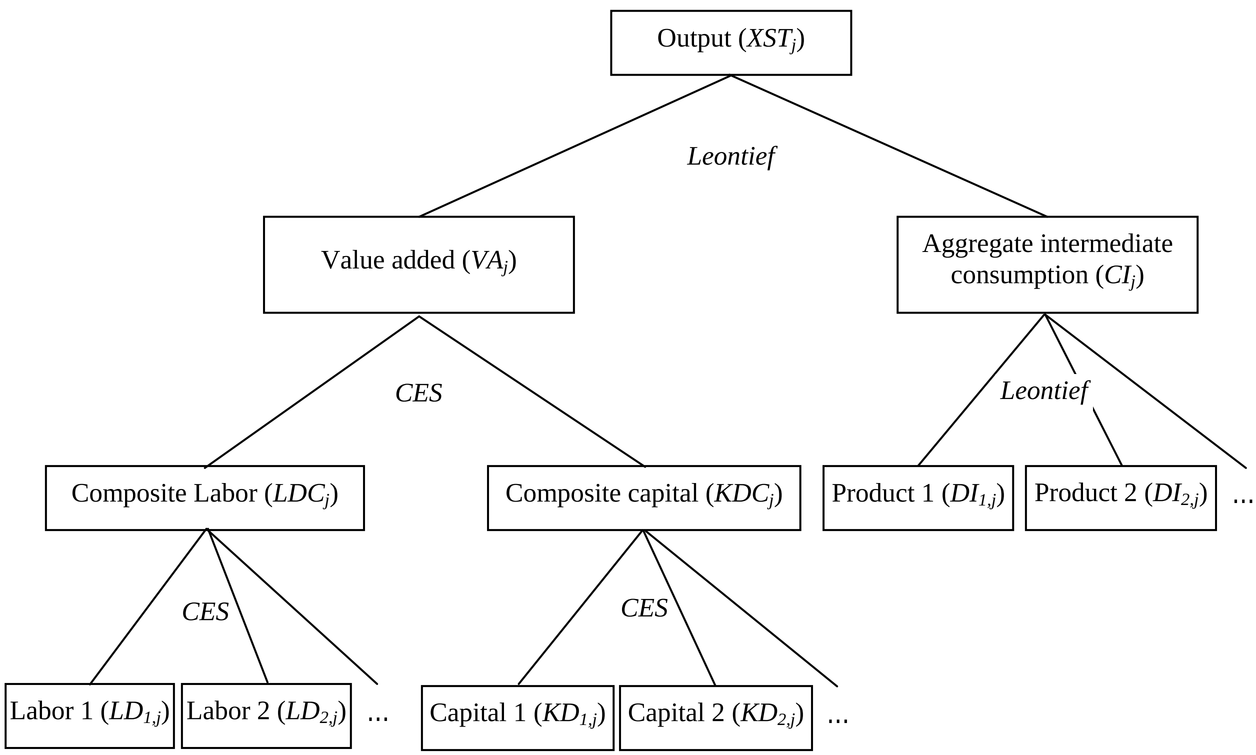
5 PEP-1-1 stands for single-country, single-period model. Forthcoming are: (i) single-country, multi-period, recursive dynamic model PEP-1-t, (ii) single-period world model PEP-w-1, and finally, (iii) multi-period, recursive dynamic world model PEP-w-t.

1. PRODUCTION

The set of productive activities is represented by indices $j, jj \in J = \{J_1, ..., J_j, ...\}$.

Firms are assumed to operate in a perfectly competitive environment. So each industry's representative firm maximizes profits subject to its production technology, while it considers the prices of goods and services and factors as given (price-taking behavior). Chart 1 describes the nested structure of production.

Chart 1: Nested structure of production



Such nested structures are common in CGE models. It is usually expected that the elasticity of substitution is greater at lower levels of the hierarchy. There are other possible specifications, and, in the end, it is up to the modeler to decide which combination of specifications best fits the situation.

At the top level (equations 1 and 2), the sectoral output of each productive activity *j* combines value added and total intermediate consumption in fixed shares. In other words, the two aggregate inputs are considered to be strictly complementary, without any possibility of substitution, following a Leontief production function.

1. $VA_j = v_j XST_j$

2. $CI_j = io_j XST_j$

where

CI_j : Total intermediate consumption of industry *j*

VA_j : Value added of industry *j*

XST_j : Total aggregate output of industry *j*

io_j : Coefficient (Leontief – intermediate consumption)

v_j : Coefficient (Leontief – value added)

At the second level, each industry's value added consists of composite labor and composite capital, following a constant elasticity of substitution (CES) specification.

$$3. \qquad VA_j = B_j^{VA}\left[\beta_j^{VA} LDC_j^{-\rho_j^{VA}} + \left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}}\right]^{-\frac{1}{\rho_j^{VA}}}$$

where

KDC_j : Industry *j* demand for composite capital

LDC_j : Industry *j* demand for composite labor

B_j^{VA} : Scale parameter (CES – value added)

β_j^{VA} : Share parameter (CES – value added)

ρ_j^{VA} : Elasticity parameter (CES – value added) ; $-1 < \rho_j^{VA} < \infty$

Profit maximization (or cost minimization) by the firms leads them to employ labor and capital to the point where the value marginal product of each is equal to its price (the wage rate and the rental rate of capital respectively). With a CES production function, such behavior is described by the demand for labor relative to capital of equation 4 (Appendix C1).

$$4. \qquad LDC_j = \left[\frac{\beta_j^{VA}}{1-\beta_j^{VA}} \frac{RC_j}{WC_j}\right]^{\sigma_j^{VA}} KDC_j$$

where

RC_j : Rental rate of industry *j* composite capital

WC_j : Wage rate of industry *j* composite labor

σ_j^{VA} : Elasticity of transformation (CES – value added) ; $0 < \sigma_j^{VA} < \infty$

In accordance with the algebra of CES aggregator functions, $\rho_j^{VA} = \dfrac{1-\sigma_j^{VA}}{\sigma_j^{VA}}$ (Appendix C1).

At the bottom level on the value added side, the various categories of labor, indexed as $l \in L = \{L_1,...,L_l,...\}$, are combined following a constant elasticity of substitution (CES) technology (equation 5), which reflects the imperfect substitutability between different types of labor. The firm chooses its labor composition so as to minimize its labor cost given the relative wage rates. Labor demand of each type derives from the first-order conditions of cost minimization by the representative firm, subject to the CES technology (equation 6) (Appendix C2). Likewise, composite capital is a CES combination of the different categories of capital, indexed as $k \in K = \{K_1,...,K_k,...\}$. As in the case of labor, it is assumed that different categories of capital (land, buildings, machinery and equipment, etc.) are imperfect substitutes (equation 7). The demand for each type of capital results from cost minimization (equation 8).

5. $$LDC_j = B_j^{LD}\left[\sum_l \beta_{l,j}^{LD}\ LD_{l,j}^{-\rho_j^{LD}}\right]^{-\frac{1}{\rho_j^{LD}}}$$

6. $$LD_{l,j} = \left[\frac{\beta_{l,j}^{LD}\ WC_j}{WTI_{l,j}}\right]^{\sigma_j^{LD}} \left(B_j^{LD}\right)^{\sigma_j^{LD}-1} LDC_j$$

7. $$KDC_j = B_j^{KD}\left[\sum_k \beta_{k,j}^{KD}\ KD_{k,j}^{-\rho_j^{KD}}\right]^{-\frac{1}{\rho_j^{KD}}}$$

8. $$KD_{k,j} = \left[\frac{\beta_{k,j}^{KD}\ RC_j}{RTI_{k,j}}\right]^{\sigma_j^{KD}} \left(B_j^{KD}\right)^{\sigma_j^{KD}-1} KDC_j$$

where

$KD_{k,j}$: Demand for type *k* capital by industry *j*

$LD_{l,j}$: Demand for type *l* labor by industry *j*

$WTI_{l,j}$: Wage rate paid by industry *j* for type *l* labor, including payroll taxes

$RTI_{k,j}$: Rental rate paid by industry *j* for type *k* capital, including capital taxes

B_j^{KD} : Scale parameter (CES – composite capital)

B_j^{LD} : Scale parameter (CES – composite labor)

$\beta_{k,j}^{KD}$: Share parameter (CES – composite capital)

$\beta_{l,j}^{LD}$: Share parameter (CES – composite labor)

ρ_j^{KD} : Elasticity parameter (CES – composite capital); $-1 < \rho_j^{KD} < \infty$

ρ_j^{LD} : Elasticity parameter (CES – composite labor); $-1 < \rho_j^{LD} < \infty$

σ_j^{KD} : Elasticity of substitution (CES – composite capital); $0 < \sigma_j^{KD} < \infty$

σ_j^{LD} : Elasticity of substitution (CES – composite labor); $0 < \sigma_j^{LD} < \infty$

In accordance with the algebra of the CES production function, $\rho_j^{KD} = \frac{1-\sigma_j^{KD}}{\sigma_j^{KD}}$ and $\rho_j^{LD} = \frac{1-\sigma_j^{LD}}{\sigma_j^{LD}}$ (Appendix C2).

Finally, returning to the second level, but on the intermediate consumption side, aggregate intermediate consumption is made up of various goods and services. Here it is assumed that intermediate inputs are perfectly complementary, and are combined following a Leontief production function. No substitutions are possible.

9. $DI_{i,j} = aij_{i,j} CI_j$

where

$DI_{i,j}$: Intermediate consumption of commodity *i* by industry *j*

$aij_{i,j}$: Input-output coefficient

2. INCOME AND SAVINGS

The PEP-1-1 model offers the possibility of several categories of households and businesses, respectively indexed as $h, hj \in H \subset AG = \{H_1, ..., H_h, ...\}$, and $f, fj \in F \subset AG = \{F_1, ..., F_f, ...\}$, together with government, designated as *GVT*, and the rest of the world, *ROW*. Elements of the set *AG* of all agents are designated as:

$$ag, agj \in AG = H \cup F \cup \{GVT, ROW\} = \{H_1, ..., H_h, ..., F_1, ..., F_f, ... GVT, ROW\}.$$

2.1 Households

Household incomes come from three sources: labor income, capital income, and transfers received from other agents.

10. $YH_h = YHL_h + YHK_h + YHTR_h$

where

YH_h : Total income of type *h* households

YHK_h : Capital income of type *h* households

YHL_h : Labor income of type *h* households

$YHTR_h$: Transfer income of type *h* households

Each household type receives a fixed share of the earnings of each type of labor (equation 11). Likewise, total capital income is distributed between agents, including households, in fixed proportions (equation 12). Finally, transfer income is simply the sum of all transfers received by type *h* households (equation 13).

11. $YHL_h = \sum_l \lambda_{h,l}^{WL} \left(W_l \sum_j LD_{l,j} \right)$

12. $YHK_h = \sum_k \lambda_{h,k}^{RK} \left(\sum_j R_{k,j} KD_{k,j} \right)$

13. $YHTR_h = \sum_{ag} TR_{h,ag}$

where

$R_{k,j}$: Rental rate of type *k* capital in industry *j*

$TR_{h,ag}$: Transfers from agent *ag* to type *h* households

W_l : Wage rate of type *l* labor

$\lambda_{ag,k}^{RK}$: Share of type *k* capital income received by agent *ag*

$\lambda_{h,l}^{WL}$: Share of type *l* labor income received by type *h* households

Subtracting direct taxes and household transfers to government yields type *h* household disposable income (equation 14). Indeed, since household transfers to government are mostly contributions to various social programs, our calculation of disposable income is consistent with national accounts. Whatever disposable income is left after savings and transfers to other agents is entirely dedicated to consumption (equation 15).

Finally, household savings are a linear function of disposable income. This differs from the frequently used specification where savings are a fixed proportion of income. Equation 16, by contrast, allows for the marginal propensity to save to be different from the average propensity. This choice is motivated by the fact that it is common for certain household categories to have negative savings. Now, if it is assumed that the marginal propensity to save is equal to the average propensity, and if that parameter is calibrated on negative observed savings, then there results an undesirable consequence: a fall in the income of these households increases their savings, or a rise in income leads to more endebtedness. Our formulation avoids this pitfall, but introduces an extra free parameter: instead of just having to calibrate the average propensity to save, the modeler must also determine the marginal propensity. Usually, it is the latter that is handled as a free parameter, and its value is determined from the literature or by econometric estimation. Once the marginal propensity has been determined, the savings function intercept is calibrated from the SAM. For household categories with negative savings, the intercept will be negative, while the slope (the marginal propensity) is positive. In addition, equation 16 makes it possible to fully of partially index the intercept to changes in the consumer price index. This is especially useful for testing the model's homogeneity, in which case price elasticity η is set to 1.

14. $YDH_h = YH_h - TDH_h - TR_{gvt,h}$

15. $CTH_h = YDH_h - SH_h - \sum_{agng} TR_{agng,h}$

16. $SH_h = PIXCON^{\eta} sh0_h + sh1_h YDH_h$

where

CTH_h : Consumption budget of type *h* households

$PIXCON$: Consumer price index

SH_h : Savings of type *h* households

TDH_h : Income taxes of type *h* households

YDH_h : Disposable income of type *h* households

η : Price elasticity of indexed transfers and parameters

$sh0_h$: Intercept (type *h* household savings)

$sh1_h$: Slope (type *h* household savings)

agng: Index of non-government agents;

$agng \in AGNG \subset AG = H \cup F \cup \{ROW\} = \{H_1,...,H_h,...,F_1,...,F_f,...,ROW\}$

2.2 Businesses

Business income consists, on one hand, of its share of capital income, and, on the other hand, of transfers received from other agents (including the transfer part of interest on consumer debt).

17. $YF_f = YFK_f + YFTR_f$

18. $YFK_f = \sum_k \lambda_{f,k}^{RK} \left(\sum_j R_{k,j} KD_{k,j} \right)$

19. $YFTR_f = \sum_{ag} TR_{f,ag}$

where

YF_f : Total income of type *f* businesses

YFK_f : Capital income of type *f* businesses

$YFTR_f$: Transfer income of type *f* businesses

Deducting business income taxes from total income yields the disposable income of each type of business (equation 20). Likewise, business savings are the residual that remains after subtracting transfers to other agents from disposable income (equation 21).

20. $YDF_f = YF_f - TDF_f$

21. $SF_f = YDF_f - \sum_{ag} TR_{ag,f}$

where

SF_f : Savings of type *f* businesses

TDF_f : Income taxes of type *f* businesses

YDF_f : Disposable income of type *f* businesses

2.3 Government

In the PEP-1-1 model, it is possible to take into account a large variety of tax instruments. Indeed, equation 22 says that the government draws its income from household and business income taxes, taxes on products and on imports (*TPRCTS*), and other taxes on production (*TPRODN*). According to the 1993 *System of National Accounts* (SNA93), taxes on products (not « production ») and imports consist of indirect taxes on consumption, taxes and duties on imports, and export taxes, while other taxes on production consist of payroll taxes, taxes on capital, and taxes on production (see Appendix B1). In addition to these various forms of fiscal revenue, government receives part of the remuneration of capital and transfers from other agents. Equations 22 to 34 describe the different government revenue sources.

22. $YG = YGK + TDHT + TDFT + TPRODN + TPRCTS + YGTR$

23. $YGK = \sum_k \lambda_{gvt,k}^{RK} \left(\sum_j R_{k,j} KD_{k,j} \right)$

24. $TDHT = \sum_h TDH_h$

25. $TDFT = \sum_f TDF_f$

26. $TPRODN = TIWT + TIKT + TIPT$

27. $TIWT = \sum_{l,j} TIW_{l,j}$

28. $TIKT = \sum_{k,j} TIK_{k,j}$

29. $TIPT = \sum_j TIP_j$

30. $TPRCTS = TICT + TIMT + TIXT$

31. $TICT = \sum_i TIC_i$

32. $TIMT = \sum_m TIM_m$

33. $TIXT = \sum_x TIX_x$

34. $YGTR = \sum_{agng} TR_{gvt,agng}$

where

$TDFT$: Total government revenue from business income taxes

$TDHT$: Total government revenue from household income taxes

TIC_i : Government revenue from indirect taxes on product i

$TICT$: Total government receipts of indirect taxes on commodities

$TIK_{k,j}$: Government revenue from taxes on type k capital used by industry j

$TIKT$: Total government revenue from from taxes on capital

TIM_m : Government revenue from import duties on product m

$TIMT$: Total government revenue from import duties

TIP_j : Government revenue from taxes on industry j production (excluding taxes directly related to the use of capital and labor)

$TIPT$: Total government revenue from production taxes (excluding taxes directly related to the use of capital and labor)

$TIW_{l,j}$: Government revenue from payroll taxes on type l labor in industry j

$TIWT$: Total government revenue from payroll taxes

TIX_x : Government revenue from export taxes on product x

$TIXT$: Total government revenue from export taxes

$TPRCTS$: Total government revenue from taxes on products and imports

$TPRODN$: Total government revenue from other taxes on production[6]

YG : Total government income

YGK : Government capital income

$YGTR$: Government transfer income

Similarly to what has been done with household savings, income taxes are described as a linear function of total income, whether it be for households (equation 35) or for businesses (equation 36). That way,

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6 That is, taxes on production other than taxes on products and taxes and duties on imports (see Appendix B1).

when a non-zero intercept is applied, the marginal rate of taxation is different from the average rate. Such an arrangement can be useful for simulating fiscal changes: for instance, marginal rates of taxation can be computed from fiscal parameters[7]; given these marginal rates, the intercept is then calibrated from SAM values. Moreover, the intercept may be partially of fully indexed to changes in the consumer price index.

35. $TDH_h = PIXCON^{\eta} ttdh0_h + ttdh1_h YH_h$

36. $TDF_f = PIXCON^{\eta} ttdf0_f + ttdf1_f YFK_f$

where

$ttdf0_f$: Intercept (income taxes of type *f* businesses)

$ttdf1_f$: Marginal income tax rate of type *f* businesses

$ttdh0_h$: Intercept (income taxes of type *h* households)

$ttdh1_h$: Marginal income tax rate of type *h* households

As mentioned earlier, the model allows for taxes on production factors (payroll taxes and capital taxes), as well as for taxes on production itself (together, these three forms of taxation constitute « other taxes on production » in the SNA93 – see Appendix B1). First, as regards taxes on factors of production, the model notation distinguishes tax rates by industry, and also by type of labor or capital. Each rate then applies to the corresponding transactions (equations 37 and 38). Next, a tax may be applied to the total value of production (equation 39).

37. $TIW_{l,j} = ttiw_{l,j} W_l LD_{l,j}$

38. $TIK_{k,j} = ttik_{k,j} R_{k,j} KD_{k,j}$

39. $TIP_j = ttip_j PP_j XST_j$

where

PP_j : Industry *j* unit cost, including taxes directly related to the use of capital and labor, but excluding other taxes on production

$ttik_{k,j}$: Tax rate on type *k* capital used in industry *j*

$ttip_j$: Tax rate on the production of industry *j*

$ttiw_{l,j}$: Tax rate on type *l* worker compensation in industry *j*

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7 This opens the door to applying the « marginal effective tax rates » (METR) approach in the CGE (Decaluwé *et al.*, 2005 and

Finally, the government can implement three types of taxes on products (for the definition of « taxes on products », see Appendix B1). Equations 40 and 41 describe how these taxes are levied in the cases of non-imported and imported products. It should be noted that these taxes apply on the sales value including margins (trade and transport margins are discussed in Appendix B3) and custom duties whenever the latter exist. Other taxes collected are taxes and duties on imported products (equation 42), and export taxes (equation 43).

40. $$TIC_{nm} = ttic_{nm}\left(PL_{nm} + \sum_{i} PC_{i}\ tmrg_{i,nm}\right)DD_{nm}$$

41. $$TIC_{m} = ttic_{m}\left[\left(PL_{m} + \sum_{i} PC_{i}\ tmrg_{i,m}\right)DD_{m} + \left((1 + ttim_{m})PWM_{m}\ e\ + \sum_{i} PC_{i}\ tmrg_{i,m}\right)IM_{m}\right]$$

42. $$TIM_{m} = ttim_{m} PWM_{m} e\ IM_{m}$$

43. $$TIX_{x} = ttix_{x}\left(PE_{x} + \sum_{i} PC_{i}\ tmrg_{i,x}^{X}\right)EXD_{x}$$

where

DD_i : Domestic demand for commodity *i* produced locally

e : Exchange rate[8]; price of foreign currency in terms of local currency

EX_x : Quantity of product *x* exported

IM_m : Quantity of product *m* imported

PE_x : Price received for exported commodity *x* (excluding export taxes)

PL_i : Price of local product *i* (excluding all taxes on products)

PWM_m : World price of imported product *m* (expressed in foreign currency)

$ttic_i$: Tax rate on commodity *i*

$ttim_m$: Rate of taxes and duties on imports of commodity *m*

$ttix_x$: Export tax rate on exported commodity *x*

$tmrg_{i,ij}$: Rate of margin *i* applied to commodity *ij*

2006).

8 In the standard model, exchange rate *e* is chosen as the numeraire.

$tmrg_{i,x}^{X}$: Rate of margin i applied to export x

The current government budget surplus or deficit (positive or negative savings) is the difference between its revenue and its expenditures. The latter consist of transfers to agents and current expenditures on goods and services.

44. $$SG = YG - \sum_{agng} TR_{agng,gvt} - G$$

where

SG : Government savings

G : Current government expenditures on goods and services

2.4 Rest of the world

The rest of the world receives payments for the value of imports, part of the income of capital, and transfers from domestic agents (equation 45). Foreign spending in the domestic economy consists of the value of exports, and transfers to domestic agents. The difference between foreign receipts and spending is the amount of rest-of-the-world savings (equation 46), which are equal in absolute value to the current accoung balance, but of opposite sign (equation 47).

45. $$YROW = e\sum_{m} PWM_m IM_m + \sum_{k} \lambda_{row,k}^{RK} \left(\sum_{j} R_{k,j} KD_{k,j} \right) + \sum_{agd} TR_{row,agd}$$

46. $$SROW = YROW - \sum_{x} PE_x^{FOB} EXD_x - \sum_{agd} TR_{agd,row}$$

47. $$SROW = -CAB$$

where

CAB : Current account balance

PE_x^{FOB} : FOB price of exported product x (in the national currency)

$SROW$: Rest-of-the-world savings

$YROW$: Rest-of-the-world income

2.5 Transfers

The way to treat transfers in a CGE model is not obvious. In most cases, indeed, these are payments without any real counterpart, and they are not explicitly related to a specific form of economic behavior. For lack of information on the precise nature of each type of transfer, they should be treated in the most neutral way possible, to prevent them from becoming a factor modifying economic agents' behavior. So household transfers to non-government agents and business transfers are simply proportional to disposable income. As for household transfers to government, they are akin to social program contributions: as such, they are treated in the same way as household income taxes. All other transfers are initially set equal to their SAM values, and indexed, fully or partially, to the consumer price index.

48. $TR_{agng,h} = \lambda^{TR}_{agng,h} YDH_h$

49. $TR_{gvt,h} = PIXCON^{\eta} tr0_h + tr1_h YH_h$

50. $TR_{ag,f} = \lambda^{TR}_{ag,f} YDF_f$

51. $TR_{agng,gvt} = PIXCON^{\eta} TR^{0}_{agng,gvt}$

52. $TR_{agd,row} = PIXCON^{\eta} TR^{0}_{agd,row}$

where

$\lambda^{TR}_{ag,agj}$: Share parameter (transfer functions)

$tr0_{gvt,h}$: Intercept (transfers by type *h* households to government)

$tr1_{gvt,h}$: Marginal rate of transfers by type *h* households to government

3. DEMAND

The demand for goods and services, whether domestically produced or imported, consists of household consumption demand, investment demand, demand by public administrations, and demand as transport or trade margins (for the treatment of margins in the SNA93, see Appendix B3).

It is assumed that households have Stone-Geary utility functions (from which derives the Linear Expenditure System). A characteristic of these utility functions is that there is a minimum level of consumption of each commodity (which may be zero for some commodities). Contrary to Cobb-Douglas utility functions, often used in the literature, this specification imposes neither zero cross-price elasticities between all pairs of goods, nor unit income-elasticities for all goods. Thus, it offers a degree of flexibility with respect to substitution possibilities in response to relative price changes. Type *h* household demand for each good (equation 53) is determined by utility maximization subject to the budget constraint (Appendix C4).

53. $$C_{i,h}PC_i = C_{i,h}^{MIN}PC_i + \gamma_{i,h}^{LES}\left(CTH_h - \sum_{ij} C_{ij,h}^{MIN}PC_{ij}\right)$$

where

$C_{i,h}$: Consumption of commodity *i* by type *h* households

$C_{i,h}^{MIN}$: Minimum consumption of commodity *i* by type *h* households

PC_i : Purchaser price of composite comodity *i* (including all taxes and margins)

$\gamma_{i,h}^{LES}$: Marginal share of commodity *i* in type *h* household consumption budget

Investment demand includes both gross fixed capital formation (GFCF) and changes in inventories. The two components of investment demand are quite different. In particular, GFCF cannot be negative (even though *net* investment, that is, gross investment minus depreciation, may be), whereas changes in inventories in the SAM may be positive or negative: roughly speaking, GFCF can be thought of as irreversible, while inventory accumulation is almost fully reversible. Endogenizing negative inventory changes is difficult to achieve satisfactorily in a CGE model. To avoid these complications, inventory changes are exogenous in PEP-1-1, fixed in volume. GFCF, on the contrary, is endogenous in the default closure of PEP-1-1, where total investment expenditure is determined by the savings-investment equilibrium constraint (equation 92), with savings endogenous. GFCF expenditure, obtained by subtracting the cost of changes in inventories from total investment expenditure (equation 54), is distributed among commodities in fixed shares (equation 55); implicitly, the production function of new capital is Cobb-Douglas. So, for a given amount of investment expenditures, the quantity demanded of each commodity *i* for investment purposes is inversely related to its purchaser price. The same hypothesis is made regarding government current expenditures on goods and services (equation 56). With a given current expenditure budget, the quantity demanded of each commodity varies inversely with its price.

54. $$GFCF = IT - \sum_i PC_i \, VSTK_i$$

55. $$PC_i INV_i = \gamma_i^{INV} GFCF$$

56. $$PC_i CG_i = \gamma_i^{GVT} G$$

where

$GFCF$:	Gross fixed capital formation
INV_i :	Final demand of commodity *i* for investment purposes
IT :	Total investment expenditures
$VSTK_i$:	Inventory change of commodity *i*
γ_i^{INV} :	Share of commodity *i* in total investment expenditures
CG_i :	Public consumption of commodity *i* (volume)
γ_i^{GVT} :	Share of commodity *i* in total current public expenditures on goods and services

In addition to being required for final demand, goods and services are used as inputs in the production process. Intermediate demand for each commodity is the sum of industry demands.

57. $$DIT_i = \sum_j DI_{i,j}$$

where

DIT_i :	Total intermediate demand for commodity *i*

Finally, some services, such as transport and retail and wholesale trade, are used to move commodities and make them available to the market. So margin rates are applied to the volume of domestic production and imports to determine the quantities of these margin services required to distribute commodities to buyers (trade and transport margins are discussed in Appendix B3).

58. $$MRGN_i = \sum_{ij} tmrg_{i,ij} DD_{ij} + \sum_m tmrg_{i,m} \; IM_m + \sum_x tmrg_{i,x}^X \; EXD_x$$

where

$MRGN_i$:	Demand for commodity *i* as a trade or transport margin

4. PRODUCER SUPPLIES OF PRODUCTS AND INTERNATIONAL TRADE

In this section, we define the trade relations with the rest of the world, that is, the supply of exports and the demand for imports. This is achieved through specifying domestic buyers' behavior with respect to the different supply sources, and domestic producers' supply behavior. The latter comprises two aspects: first, how composite output translates into the supply of products, and, second, how the supply of each product is directed to destination markets. The small-country hypothesis is adopted, in the sense that the world price of traded goods (imports and exports) is exogenous.

The section on production describes how industries combine inputs to produce total aggregate output XST_j. Industries usually produce more than one product. The Leontief-style one-to-one correspondence between products and industries is a special case, and the 1993 *System of National Accounts* recommends constructing input-output tables in the rectangular format, where there are generally more goods than industries. If an industry's aggregate output were simply the sum of its products, the profit-maximizing firm would concentrate all of its output on the product with the highest price. Here, however, it is assumed that, although an industry can reorganize its production to change the proportions of goods produced, the different products are not perfectly « transformable » into one another. This is represented by means of a constant elasticity of transformation (CET) function that describes how easily the product-mix can be adjusted in response to price changes (equation 59):

59. $$XST_j = B_j^{XT}\left[\sum_i \beta_{j,i}^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}}$$

where

$XS_{j,i}$: Industry *j* production of commodity *i*

B_j^{XT} : Scale parameter (CET – total output)

$\beta_{j,i}^{XT}$: Share parameter (CET – total output)

ρ_j^{XT} : Elasticity parameter (CET – total output) ; $1 < \rho_j^{XT} < \infty$

Producers allocate output among products so as to maximize sales revenue, given product prices, subject to equation 59. Individual product supply functions (equation 60) are derived from the first-order conditions of revenue maximizing (Appendix C5).

60. $$XS_{j,i} = \frac{XST_j}{\left(B_j^{XT}\right)^{1+\sigma_j^{XT}}} \left[\frac{P_{j,i}}{\beta_{j,i}^{XT} \; PT_j} \right]^{\sigma_j^{XT}}$$

where

$P_{j,i}$: Basic price of industry *j*'s production of commodity *i*

σ_j^{XT} : Elasticity of transformation (CET – total output) ; $0 < \sigma_j^{XT} < \infty$

In accordance with the algebra of the CET transformation function, $\rho_j^{XT} = \dfrac{1+\sigma_j^{XT}}{\sigma_j^{XT}}$ (Appendix C5).

Next, the output of every product of an industry is shared out among markets (domestic or export), again with the goal of maximizing the firm's total revenue, given the demand in each market and the various taxes that apply. It is assumed that production directed to one market is somewhat different from production directed to another market. This imperfect substitutability is represented in PEP-1-1 by means of a constant elasticity of transformation (CET) aggregator function that describes how readily production can be redirected from one market to another.[9]

61. $$XS_{j,x} = B_{j,x}^{X} \left[\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X}} + \left(1-\beta_{j,x}^{X}\right) DS_{j,x}^{\rho_{j,x}^{X}} \right]^{\frac{1}{\rho_{j,x}^{X}}}$$

where

$DS_{j,i}$: Supply of commodity *i* by sector *j* to the domestic market

$B_{j,x}^{X}$: Scale parameter (CET – exports and local sales)

$\beta_{j,x}^{X}$: Share parameter (CET – exports and local sales)

$\rho_{j,x}^{X}$: Elasticity parameter (CET – exports and local sales) ; $1 < \rho_{j,x}^{X} < \infty$

Obviously, for products that are not exported to the world market, total output is simply equal to supply on the domestic market.

62. $$XS_{j,nx} = DS_{j,nx}$$

9 Equation 61 implies that there is a single trading partner, the « Rest-of-the-world ». It is left up to the modeler to adapt the model, whenever appropriate, to distinguish more than one trading partner.

Relative supply functions are derived from the first-order conditions of revenue maximizing subject to the CET aggregator function (equation 61) (Appendix C6).

63. $$EX_{j,x} = \left[\frac{1-\beta_{j,x}^{X}}{\beta_{j,x}^{X}} \frac{PE_x}{PL_x} \right]^{\sigma_{j,x}^{X}} DS_{j,x}$$

where

$\sigma_{j,x}^{X}$: Elasticity of transformation (CET – exports and local sales) ; $0 < \sigma_{j,x}^{X} < \infty$

In accordance with the algebra of the CET transformation function, $\rho_{j,x}^{X} = \frac{1+\sigma_{j,x}^{X}}{\sigma_{j,x}^{X}}$ (Appendix C6).

To summarize, producers' supply behavior is represented by nested CET functions: on the upper level, aggregate output is allocated to individual products; on the lower level, the supply of each product is distributed between the domestic market and exports.

Many CGE models presume that producers can always sell as much as they wish on the world market at the (exogenous) current price; we take a different view (and, in so doing, we depart from the « pure » form of the small-country hypothesis): equation 64 says that a local producer can increase his share of the world market only by offering a price PE_x^{FOB} that is advantageous relative to the (exogenous) world price PWX_x. The ease with which his share can be increased depends on the degree of substitutability of the proposed product to competing products; in other words, it depends on the price-elasticity of export demand. Equation 64 also makes it possible to simulate an exogenous variation in world demand for the product, through a change in the variable EXD_x^{O}.

64. $$EXD_x = EXD_x^{O} \left(\frac{e\ PWX_x}{PE_x^{FOB}} \right)^{\sigma_x^{XD}}$$

where

EXD_x : World demand for exports of product x

PE_x^{FOB} : FOB price of exported commodity x (in local currency)

PWX_x : World price of exported product x (expressed in foreign currency)

σ_x^{XD} : Price-elasticity of the world demand for exports of product x

Buyer behavior is symmetrical to producer behavior, in that it is assumed that local products are imperfect substitutes for imports, or, in other words, that goods are heterogenous with respect to their origin. So commodities demanded on the domestic market are composite goods, combinations of locally produced goods and imports. The imperfect substitutability between the two is represented by a constant elasticity of substitution (CES) aggregator function (equation 65).

65. $$Q_m = B_m^M \left[\beta_m^M IM_m^{-\rho_m^M} + \left(1-\beta_m^M\right) DD_m^{-\rho_m^M} \right]^{\frac{-1}{\rho_m^M}}$$

where

Q_m : Quantity demanded of importable composite commodity *m*

B_m^M : Scale parameter (CES – composite commodity)

β_m^M : Share parameter (CES – composite commodity)

ρ_m^M : Elasticity parameter (CES – composite commodity); $-1 < \rho_m^M < \infty$

Naturally, for goods with no competition from imports, the demand for the composite commodity is the demand for the domestically produced good.

66. $$Q_{nm} = DD_{nm}$$

Just as sellers seek to maximize revenue, buyers minimize expenses, subject to the CES aggregation function (equation 65). Relative demand functions derive from the first-order optimum conditions (Appendix C7).

67. $$IM_m = \left[\frac{\beta_m^M}{1-\beta_m^M} \frac{PD_m}{PM_m} \right]^{\sigma_m^M} DD_m$$

where

PD_m : Price of local product *m* sold on the domestic market (including all taxes and margins)

PM_m : Price of imported product *m* (including all taxes and margins)

σ_m^M : Elasticity of substitution (CES – composite commodity); $0 < \sigma_m^M < \infty$

In accordance with the algebra of CES aggregator functions, $\rho_m^M = \frac{1-\sigma_m^M}{\sigma_m^M}$ (Appendix C7).

While equation 67 specifies the (relative) demand for imports, the supply function of imports in PEP-1-1 is implicit. According to the small-country hypothesis, the price-elasticity of import supply is assumed to be infinite at the going world price: this is what is implied by fixing the world price of imports as exogenous in equation 82 below.

5. PRICES

5.1 Production

The different prices and price indexes naturally depend on the hypotheses and functional forms already stated. In aggregations, the price of an aggregate is a weighted sum of the prices of its components. The weights are determined by equating the value of the aggregate to the sum of the values of its components, given the quantity of the aggregate (which is determined from the aggregator function). The weight assigned the price of each component is therefore the ratio of its volume (or quantity) to the volume (or quantity) of the aggregate[10]. Only in Leontief fixed-proportions aggregations are the weights invariant to relative price changes; in other cases, component proportions, and, consequently, component price weights, change in response to relative price changes, and they change more or less sharply, depending on the elasticity of substitution or transformation. For instance, the unit cost of an industry's output (including taxes directly related to the use of capital and labor, but excluding other taxes on production) is a weighted sum of the prices of value added and aggregate intermediate consumption (equation 68).

68. $$PP_j = \frac{PVA_j VA_j + PCI_j CI_j}{XST_j}$$

Here, the weights are VA_j / XST_j and CI_j / XST_j. Multiplying both sides of equation 68 by XST_j yields the value accounting identity $PP_j XST_j = PVA_j VA_j + PCI_j CI_j$. The same principle applies to the prices of other aggregates. The price of aggregate intermediate consumption is a combination of the commodity prices of the industry's intermediate inputs (equation 70), just as the price of value added is a combination of the prices of composite labor and composite capital (equation 71). So is it with the prices of composite factors. The price of an industry's composite labor is a weighted sum of the wage rates (including payroll taxes) of the different categories of labor used by that industry (equation 72). In the same way, the price of an industry's composite capital is a weighted sum of the rental rates of the different types of capital used by that industry (equation 74).

Since various forms of taxation appear in the model, it is necessary to define the relationship between prices before taxes, and prices including taxes. The basic price of production (for a definition of « basic

10 Note that, in general, the weights do not add up to 1. An alternative, mathematically equivalent, modeling approach would have been to determine aggregate prices from the cost (or value) functions dual to the aggregation functions, and then to compute the quantity indexes of the aggregates from the identities between the value of an aggregate and the sum of its components' values.

price », see Appendix B2) is obtained from the unit cost by adding taxes on production (other than taxes on labor or capital, already included in the unit cost) (equation 69). Likewise, wages paid by industry differ from wages received by workers by the amount of payroll taxes (equation 73). The same applies to the rental rate of capital (equation 75). The principles of price aggregation and tax accounting yield the following price equations, in addition to equation 68, stated earlier.

69. $PT_j = \left(1 + ttip_j\right) PP_j$

70. $PCI_j = \dfrac{\sum_i PC_i DI_{i,j}}{CI_j}$

71. $PVA_j = \dfrac{WC_j LDC_j + RC_j KDC_j}{VA_j}$

72. $WC_j = \dfrac{\sum_l WTI_{l,j} LD_{l,j}}{LDC_j}$

73. $WTI_{l,j} = W_l \left(1 + ttiw_{l,j}\right)$

74. $RC_j = \dfrac{\sum_k RTI_{k,j} KD_{k,j}}{KDC_j}$

75. $RTI_{k,j} = R_{k,j} \left(1 + ttik_{k,j}\right)$

where

PT_j : Basic price of industry *j*'s output

PCI_j : Intermediate consumption price index of industry *j*

Finally, the rental rate received by the owners of capital, $R_{k,j}$, is determined in one of two manners, depending on the option selected by the user regarding the mobility of capital. In PEP-1-1, parameter *kmob* acts as a switch. If *kmob* = 1, capital is assumed to be perfectly mobile, and the aggregate supply of type *k* capital KS_k is exogenous, fixed at its initial SAM value, so the use of capital by the industries is constrained by the supply-demand equilibrium condition (equation 91). In that case, the allocation of capital between industries is the result of the arbitrage process that makes the rental rate received by owners equal across industries (equation 76). If *kmob* = 0, capital is assumed to be industry-specific, equation 76 is not in effect, the $KD_{k,j}$ are exogenous, fixed at their initial SAM values, and rents paid on the use of capital are Ricardian rents.

76. $R_{k,j} = RK_k, \quad if \ capital \ is \ mobile$

where

RK_k : Rental rate of type k capital (if capital is mobile)

5.2 International trade

Exporting industries have the possibility of selling their output on the international market or the domestic market. So the price of their aggregate production is a weighted sum of the price obtained on each market, following the price aggregation principle. The weight assigned to each market is proportional to the quantity sold on that market (equation 77); these weights vary in response to relative price changes, more or less sharply, depending on the elasticity of transformation in the CET. The basic price (for a definition, see Appendix B2) obtained by industry *j* for exportable product *x* is a weighted sum of its basic price on the domestic market and its basic price on the export market (equation 78). The FOB price paid by purchasers on the export market is different from the one received by the producer, since margins and export taxes must be added on (equation 80). And of course, for products not exported by an industry, the price obtained is the domestic market price (equation 79).

77. $$PT_j = \frac{\sum_i P_{j,i} \ XS_{j,i}}{XST_j}$$

78. $$P_{j,x} = \frac{PE_x \ EX_{j,x} + PL_x \ DS_{j,x}}{XS_{j,x}}$$

79. $$P_{j,nx} = PL_{nx}$$

80. $$\left(PE_x + \sum_i PC_i \ tmrg_{i,x}^X\right)\left(1 + ttix_x\right) = PE_x^{FOB}$$

As was previously explained, commodities purchased on the domestic market are composites. For goods facing import competition, the price of the composite is a weighted sum of the price paid for domestically produced, and imported goods (equation 83). The price paid for the local product is the sum of the price received by the producer, margins, and indirect taxes (equation 81). Similarly, the price paid for the imported product is the world price, translated into the local currency, plus taxes and duties on imports, margins, and domestic indirect taxes (equation 82). The price of commodities for which there is no competing import is simply the price paid for the local product (equation 84).

81. $PD_i = (1 + ttic_i)\left(PL_i + \sum_{ij} PC_i \; tmrg_{ij,i}\right)$

82. $PM_m = (1 + ttic_m)\left((1 + ttim_m) e \; PWM_m + \sum_i PC_i \; tmrg_{i,m}\right)$

83. $PC_m = \frac{PM_m IM_m + PD_m DD_m}{Q_m}$

84. $PC_{nm} = PD_{nm}$

5.3 Price indexes

Finally, four price indexes have been defined: the GDP deflator (equation 85), the consumer price index (equation 86), the investment price index (equation 87), and the public expenditures price index (equation 88). The first is a Fisher index, the second is a Laspeyres index, and the third and fourth are exact price indexes, dual to the Cobb-Douglas functions which describe commodity demand for investment purposes and for public consumption (Appendix C8).

85. $PIXGDP = \sqrt{\frac{\sum_j PVA_j VAO_j}{\sum_j PVAO_j VAO_j} \frac{\sum_j PVA_j VA_j}{\sum_j PVAO_j VA_j}}$

86. $PIXCON = \frac{\sum_i PC_i \sum_h C_{i,h}^0}{\sum_{ij} PC_{ij}^0 \sum_h C_{ij,h}^0}$

87. $PIXINV = \prod_i \left(\frac{PC_i}{PC_i^0}\right)^{\gamma_i^{INV}}$

88. $PIXGVT = \prod_i \left(\frac{PC_i}{PC_i^0}\right)^{\gamma_i^{GVT}}$

where

$PIXGDP$: GDP deflator

$PIXGVT$: Public expenditures price index

$PIXINV$: Investment price index

6. EQUILIBRIUM

Whether it be for the goods and services market or the factor market, supply and demand equilibrium must be verified. Thus, equation 89 defines the equilibrium between the supply and demand of each commodity on the domestic market. Equations 90 and 91 ensure the equilibrium between total demand for each factor and available supply. Likewise, total investment expenditure must be equal to the sum of agents' savings (equation 92). The sum of supplies of every commodity by local producers must be equal to domestic demand for that commodity produced locally (equation 93). And finally, supply to the export market of each good must be matched by demand (equation 94).

89. $Q_i = \sum_h C_{i,h} + CG_i + INV_i + VSTK_i + DIT_i + MRGN_i$

90. $\sum_j LD_{l,j} = LS_l$

91. $\sum_j KD_{k,j} = KS_k$

92. $IT = \sum_h SH_h + \sum_f SF_f + SG + SROW$

93. $\sum_j DS_{j,i} = DD_i$

94. $\sum_j EX_{j,x} = EXD_x$

where

LS_l : Supply of type *l* labor

KS_k : Supply of type *k* capital

7. GROSS DOMESTIC PRODUCT

GDP at basic prices is equal to payments made to factors, plus taxes on production other than taxes on labor or capital already included in factor costs (equation 95). On the other hand, GDP at market prices from the final demand perspective is the sum of net final expenditures: household consumption, current public expenditures on goods and services, investment expenditures, plus the value of exports, minus the value of imports (equation 98). As for GDP at market prices from the income perspective (equation 97), it is equal to the sum total of income paid to labor and to capital, plus taxes on products and imports (*TPRCTS* – equation 30), plus other taxes on production (*TPRODN* – equation 26). GDP at market prices exceeds GDP at basic prices by exactly the amount of taxes on products and imports (*TPRCTS*). For a discussion of GDP concepts according to the SNA93, see Appendix B4.

95. $$GDP^{BP} = \sum_{j} PVA_j VA_j + TIPT$$

96. $$GDP^{MP} = GDP^{BP} + TPRCTS$$

97. $$GDP^{IB} = \sum_{l,j} W_l \, LD_{l,j} + \sum_{k,j} R_{k,j} KD_{k,j} + TPRODN + TPRCTS$$

98. $$GDP^{FD} = \sum_{i} PC_i \left[\sum_{h} C_{i,h} + CG_i + INV_i + VSTK_i \right] + \sum_{x} PE_x^{FOB} EXD_x - e \sum_{m} PWM_m IM_m$$

where

GDP^{BP} : GDP at basic prices

GDP^{FD} : GDP at purchasers’ prices from the perspective of final demand

GDP^{IB} : GDP at market prices (income-based)

GDP^{MP} : GDP at market prices

REFERENCES

DECALUWÉ, Bernard, André LEMELIN, David BAHAN (2006) « Oferta endógena de trabajo y capital parcialmente móvil en un MEGC birregional: Versión estática del modelo de equilibrio general computable del Ministerio de Hacienda de Québec », *Investigación Económica,* 258, octubre-diciembre.

DECALUWÉ, Bernard, André LEMELIN, David BAHAN et Nabil ANNABI (2005) « Offre de travail endogène et mobilité du capital dans un MEGC bi-régional: la version statique du modèle d'équilibre général calculable du Ministère des Finances du Québec », texte d'une conférence donnée à Séville, lors de l'atelier international *The State-of-the-Art in Regional Modeling*, 21-23 octobre 2004, co-organisé par le Global Economic Modeling Network (ECOMOD) et la Fundación Centro de Estudio Andaluces (centrA), Ministère des Finances du Québec, collection *Feuille d'argent*, Travaux de recherche 2005-001, 62 pages.
http://www.finances.gouv.qc.ca/documents/feuille/fr/2005_001.pdf

DECALUWÉ, Bernard, André LEMELIN, Véronique ROBICHAUD, David BAHAN, et Daniel FLOREA (2004). « Le modèle d'équilibre général calculable du ministère des Finances, de l'Économie et de la recherche du Québec: un modèle bi-régional du Québec et du Reste-du-Canada », chapitre 14, p. 285-297 dans CLOUTIER, L. Martin et Christian DEBRESSON, avec la collaboration d'Érik DIETZENBACHER, *Changement climatique, flux technologiques, financiers et commerciaux – nouvelles directions d'analyse entrée-sortie*, Actes de la Quatorzième Conférence internationale de techniques d'analyse entrée-sortie, tenue à Montréal, 10-15 octobre 2002, Presses de l'Université du Québec.

DECALUWÉ, Bernard, André LEMELIN, Véronique ROBICHAUD et David BAHAN (2003). *Modèle d'équilibre général du ministère des Finances du Québec (MEGFQ): caractéristiques et structure du modèle*, ministère des Finances du Québec, Collection *Feuille d'argent*, Travaux de recherche 2003-002
http://www.finances.gouv.qc.ca/documents/feuille/fr/2003_002.pdf
Disponible en version anglaise sous le titre: *General equilibrium model of the ministère des Finances du Québec (GEMFQ): Characteristics and structure of the model*
http://www.finances.gouv.qc.ca/documents/Feuille/en/2003_002_eng.pdf

DECALUWÉ, B., A. MARTENS et L. SAVARD (2001). *La politique économique du développement et les modèles d'équilibre général calculable*, Les Presses de l'Université de Montréal, Montréal.

INTER-SECRETARIAT WORKING GROUP ON NATIONAL ACCOUNTS (1993). « System of National Accounts 1993 (SNA93) », Eurostat, International Monetary Fund, OECD, United Nations, World Bank; Bruxelles-Luxembourg, New York, Paris, Washington (D.C.), 711 p.
unstats.un.org/unsd/sna1993/toctop.asp

APPENDIX A: EQUATIONS, SETS, VARIABLES AND PARAMETERS

A1. Equations

A1.1 PRODUCTION

1. $$VA_j = v_j XST_j$$

2. $$CI_j = io_j XST_j$$

3. $$VA_j = B_j^{VA}\left[\beta_j^{VA} LDC_j^{-\rho_j^{VA}} + \left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}}\right]^{-\frac{1}{\rho_j^{VA}}}$$

4. $$LDC_j = \left[\frac{\beta_j^{VA}}{1-\beta_j^{VA}} \frac{RC_j}{WC_j}\right]^{\sigma_j^{VA}} KDC_j$$

5. $$LDC_j = B_j^{LD}\left[\sum_l \beta_{l,j}^{LD} \ LD_{l,j}^{-\rho_j^{LD}}\right]^{-\frac{1}{\rho_j^{LD}}}$$

6. $$LD_{l,j} = \left[\frac{\beta_{l,j}^{LD} \ WC_j}{WTI_{l,j}}\right]^{\sigma_j^{LD}} \left(B_j^{LD}\right)^{\sigma_j^{LD}-1} LDC_j$$

7. $$KDC_j = B_j^{KD}\left[\sum_k \beta_{k,j}^{KD} \ KD_{k,j}^{-\rho_j^{KD}}\right]^{-\frac{1}{\rho_j^{KD}}}$$

8. $$KD_{k,j} = \left[\frac{\beta_{k,j}^{KD} \ RC_j}{RTI_{k,j}}\right]^{\sigma_j^{KD}} \left(B_j^{KD}\right)^{\sigma_j^{KD}-1} KDC_j$$

9. $$DI_{i,j} = aij_{i,j} CI_j$$

A1.2 INCOME AND SAVINGS

A1.2.1 Households

10. $YH_h = YHL_h + YHK_h + YHTR_h$

11. $YHL_h = \sum_l \lambda_{h,l}^{WL} \left(W_l \sum_j LD_{l,j} \right)$

12. $YHK_h = \sum_k \lambda_{h,k}^{RK} \left(\sum_j R_{k,j} KD_{k,j} \right)$

13. $YHTR_h = \sum_{ag} TR_{h,ag}$

14. $YDH_h = YH_h - TDH_h - TR_{gvt,h}$

15. $CTH_h = YDH_h - SH_h - \sum_{agng} TR_{agng,h}$

16. $SH_h = PIXCON^{\eta} sh0_h + sh1_h YDH_h$

A1.2.2 Firms

17. $YF_f = YFK_f + YFTR_f$

18. $YFK_f = \sum_k \lambda_{f,k}^{RK} \left(\sum_j R_{k,j} KD_{k,j} \right)$

19. $YFTR_f = \sum_{ag} TR_{f,ag}$

20. $YDF_f = YF_f - TDF_f$

21. $SF_f = YDF_f - \sum_{ag} TR_{ag,f}$

A1.2.3 Government

22. $YG = YGK + TDHT + TDFT + TPRODN + TPRCTS + YGTR$

23. $YGK = \sum_k \lambda_{gvt,k}^{RK} \left(\sum_j R_{k,j} KD_{k,j} \right)$

24. $TDHT = \sum_h TDH_h$

25. $TDFT = \sum_{f} TDF_{f}$

26. $TPRODN = TIWT + TIKT + TIPT$

27. $TIWT = \sum_{l,j} TIW_{l,j}$

28. $TIKT = \sum_{k,j} TIK_{k,j}$

29. $TIPT = \sum_{j} TIP_{j}$

30. $TPRCTS = TICT + TIMT + TIXT$

31. $TICT = \sum_{i} TIC_{i}$

32. $TIMT = \sum_{m} TIM_{m}$

33. $TIXT = \sum_{x} TIX_{x}$

34. $YGTR = \sum_{agng} TR_{gvt,agng}$

35. $TDH_{h} = PIXCON^{\eta} ttdh0_{h} + ttdh1_{h} YH_{h}$

36. $TDF_{f} = PIXCON^{\eta} ttdf0_{f} + ttdf1_{f} YFK_{f}$

37. $TIW_{l,j} = ttiw_{l,j} W_{l} LD_{l,j}$

38. $TIK_{k,j} = ttik_{k,j} R_{k,j} KD_{k,j}$

39. $TIP_{j} = ttip_{j} PP_{j} XST_{j}$

40. $TIC_{nm} = ttic_{nm} \left(PL_{nm} + \sum_{i} PC_{i}\ tmrg_{i,nm} \right) DD_{nm}$

41. $TIC_{m} = ttic_{m} \left[\left(PL_{m} + \sum_{i} PC_{i}\ tmrg_{i,m} \right) DD_{m} + \left(\left(1 + ttim_{m} \right) PWM_{m}\ e\ + \sum_{i} PC_{i}\ tmrg_{i,m} \right) IM_{m} \right]$

42. $TIM_{m} = ttim_{m} PWM_{m} e\ IM_{m}$

43. $TIX_{x} = ttix_{x} \left(PE_{x} + \sum_{i} PC_{i}\ tmrg_{i,x}^{X} \right) EXD_{x}$

44. $SG = YG - \sum_{agng} TR_{agng,gvt} - G$

A1.2.4 Rest of the world

45. $YROW = e\sum_{m} PWM_m IM_m + \sum_{k} \lambda_{row,k}^{RK} \left(\sum_{j} R_{k,j} KD_{k,j} \right) + \sum_{agd} TR_{row,agd}$

46. $SROW = YROW - \sum_{x} PE_x^{FOB} EXD_x - \sum_{agd} TR_{agd,row}$

47. $SROW = -CAB$

A1.2.5 Transfers

48. $TR_{agng,h} = \lambda_{agng,h}^{TR} YDH_h$

49. $TR_{gvt,h} = PIXCON^{\eta} tr0_h + tr1_h YH_h$

50. $TR_{ag,f} = \lambda_{ag,f}^{TR} YDF_f$

51. $TR_{agng,gvt} = PIXCON^{\eta} TR_{agng,gvt}^{0}$

52. $TR_{agd,row} = PIXCON^{\eta} TR_{agd,row}^{0}$

A1.3 DEMAND

53. $C_{i,h} PC_i = C_{i,h}^{MIN} PC_i + \gamma_{i,h}^{LES} \left(CTH_h - \sum_{ij} C_{ij,h}^{MIN} PC_{ij} \right)$

54. $GFCF = IT - \sum_{i} PC_i\ VSTK_i$

55. $PC_i INV_i = \gamma_i^{INV} GFCF$

56. $PC_i CG_i = \gamma_i^{GVT} G$

57. $DIT_i = \sum_{j} DI_{i,j}$

58. $MRGN_i = \sum_{ij} tmrg_{i,ij} DD_{ij} + \sum_{m} tmrg_{i,m}\ IM_m + \sum_{x} tmrg_{i,x}^{X}\ EXD_x$

A1.4 PRODUCER SUPPLIES OF PRODUCTS AND INTERNATIONAL TRADE

59. $$XST_j = B_j^{XT}\left[\sum_i \beta_{j,i}^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}}$$

60. $$XS_{j,i} = \frac{XST_j}{\left(B_j^{XT}\right)^{1+\sigma_j^{XT}}}\left[\frac{P_{j,i}}{\beta_{j,i}^{XT}\ PT_j}\right]^{\sigma_j^{XT}}$$

61. $$XS_{j,x} = B_{j,x}^{X}\left[\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X}} + \left(1-\beta_{j,x}^{X}\right) DS_{j,x}^{\rho_{j,x}^{X}}\right]^{\frac{1}{\rho_{j,x}^{X}}}$$

62. $$XS_{j,nx} = DS_{j,nx}$$

63. $$EX_{j,x} = \left[\frac{1-\beta_{j,x}^{X}}{\beta_{j,x}^{X}}\frac{PE_x}{PL_x}\right]^{\sigma_{j,x}^{X}} DS_{j,x}$$

64. $$EXD_x = EXD_x^{O}\left(\frac{e\ PWX_x}{PE_x^{FOB}}\right)^{\sigma_x^{XD}}$$

65. $$Q_m = B_m^{M}\left[\beta_m^{M} IM_m^{-\rho_m^{M}} + \left(1-\beta_m^{M}\right) DD_m^{-\rho_m^{M}}\right]^{\frac{-1}{\rho_m^{M}}}$$

66. $$Q_{nm} = DD_{nm}$$

67. $$IM_m = \left[\frac{\beta_m^{M}}{1-\beta_m^{M}}\frac{PD_m}{PM_m}\right]^{\sigma_m^{M}} DD_m$$

A1.5 PRICES

A1.5.1 Production

68. $$PP_j = \frac{PVA_j VA_j + PCI_j CI_j}{XST_j}$$

69. $PT_j = \left(1 + ttip_j\right) PP_j$

70. $PCI_j = \dfrac{\sum_i PC_i DI_{i,j}}{CI_j}$

71. $PVA_j = \dfrac{WC_j LDC_j + RC_j KDC_j}{VA_j}$

72. $WC_j = \dfrac{\sum_l WTI_{l,j} LD_{l,j}}{LDC_j}$

73. $WTI_{l,j} = W_l \left(1 + ttiw_{l,j}\right)$

74. $RC_j = \dfrac{\sum_k RTI_{k,j} KD_{k,j}}{KDC_j}$

75. $RTI_{k,j} = R_{k,j} \left(1 + ttik_{k,j}\right)$

76. $R_{k,j} = RK_k, \quad \text{if capital is mobile}$

A1.5.2 International trade

77. $PT_j = \dfrac{\sum_i P_{j,i} \; XS_{j,i}}{XST_j}$

78. $P_{j,x} = \dfrac{PE_x \; EX_{j,x} + PL_x \; DS_{j,x}}{XS_{j,x}}$

79. $P_{j,nx} = PL_{nx}$

80. $\left(PE_x + \sum_i PC_i \; tmrg_{i,x}^{X}\right)\left(1 + ttix_x\right) = PE_x^{FOB}$

81. $PD_i = \left(1 + ttic_i\right)\left(PL_i + \sum_{ij} PC_i \; tmrg_{ij,i}\right)$

82. $PM_m = \left(1 + ttic_m\right)\left(\left(1 + ttim_m\right) e \; PWM_m + \sum_i PC_i \; tmrg_{i,m}\right)$

83. $$PC_m = \frac{PM_m IM_m + PD_m DD_m}{Q_m}$$

84. $$PC_{nm} = PD_{nm}$$

A1.5.3 Price indexes

85. $$PIXGDP = \sqrt{\frac{\sum_j PVA_j VAO_j}{\sum_j PVAO_j VAO_j} \frac{\sum_j PVA_j VA_j}{\sum_j PVAO_j VA_j}}$$

86. $$PIXCON = \frac{\sum_i PC_i \sum_h C_{i,h}^0}{\sum_{ij} PC_{ij}^0 \sum_h C_{ij,h}^0}$$

87. $$PIXINV = \prod_i \left(\frac{PC_i}{PC_i^0} \right)^{\gamma_i^{INV}}$$

88. $$PIXGVT = \prod_i \left(\frac{PC_i}{PC_i^0} \right)^{\gamma_i^{GVT}}$$

A1.6 EQUILIBRIUM

89. $$Q_i = \sum_h C_{i,h} + CG_i + INV_i + VSTK_i + DIT_i + MRGN_i$$

90. $$\sum_j LD_{l,j} = LS_l$$

91. $$\sum_j KD_{k,j} = KS_k$$

92. $$IT = \sum_h SH_h + \sum_f SF_f + SG + SROW$$

93. $$\sum_j DS_{j,i} = DD_i$$

94. $$\sum_j EX_{j,x} = EXD_x$$

A1.7 GROSS DOMESTIC PRODUCT

95. $$GDP^{BP} = \sum_{j} PVA_j VA_j + TIPT$$

96. $$GDP^{MP} = GDP^{BP} + TPRCTS$$

97. $$GDP^{IB} = \sum_{l,j} W_l \, LD_{l,j} + \sum_{k,j} R_{k,j} KD_{k,j} + TPRODN + TPRCTS$$

98. $$GDP^{FD} = \sum_{i} PC_i \left[\sum_{h} C_{i,h} + CG_i + INV_i + VSTK_i \right] + \sum_{x} PE_x^{FOB} EXD_x - e \sum_{m} PWM_m IM_m$$

A2. Sets

A2.1 INDUSTRIES AND COMMODITIES

All industries: $j, jj \in J = \{J_1,...,J_j,...\}$

All commodities: $i, ij \in I = \{I_1,...,I_i,...\}$

Imported commodities: $m \in M \subset I$; $M = \{M_1,...,M_m,...\}$

Non imported commodities: $nm \in NM \subset I$; $NM = \{NM_1,...,NM_{nm},...\}$; $NM \cap M = \emptyset$

Exported commodities: $x \in X \subset I$; $X = \{X_1,...,X_x,...\}$

Non exported commodities: $nx \in NX \subset I$; $NX = \{NX_1,...,NX_{nx},...\}$; $NX \cap X = \emptyset$

A2.2 PRODUCTION FACTORS

Labor categories: $l \in L = \{L_1,...,L_l,...\}$

Capital categories: $k \in K = \{K_1,...,K_k,...\}$

A2.3 AGENTS

All agents: $ag, agj \in AG = H \cup F \cup \{GVT, ROW\} = \{H_1,...,H_h,...,F_1,...,F_f,...,GVT, ROW\}$

Household categories: $h, hj \in H \subset AG = \{H_1,...,H_h,...\}$

Firm categories: $f, fj \in F \subset AG = \{F_1,...,F_f,...\}$

Non governmental agent:

$agng \in AGNG \subset AG = H \cup F \cup \{ROW\} = \{H_1,...,H_h,...,F_1,...,F_f,...,ROW\}$

Domestic agents: $agd \in AGD \subset AG = H \cup F \cup \{GVT\} = \{H_1,...,H_h,...,F_1,...,F_f,...,GVT\}$

A3. Variables

NOTE: In what follows, the word “taxes” should be understood as “taxes, minus subsidies”.

A3.1 VOLUME VARIABLES

$C_{i,h}$: Consumption of commodity *i* by type *h* households

$C_{i,h}^{MIN}$: Minimum consumption of commodity *i* by type *h* households

CG_i : Public consumption of commodity *i*

CI_j : Total intermediate consumption of industry *j*

DD_i : Domestic demand for commodity *i* produced locally

$DI_{i,j}$: Intermediate consumption of commodity *i* by industry *j*

DIT_i : Total intermediate demand for commodity *i*

$DS_{j,i}$: Supply of commodity *i* by sector *j* to the domestic market

$EX_{j,x}$: Quantity of product *x* exported by sector *j*

EXD_x : World demand for exports of product *x*

IM_m : Quantity of product *m* imported

INV_i : Final demand of commodity *i* for investment purposes

$KD_{k,j}$: Demand for type *k* capital by industry *j*

KDC_j : Industry *j* demand for composite capital

KS_k : Supply of type *k* capital

$LD_{l,j}$: Demand for type *l* labor by industry *j*

LDC_j : Industry *j* demand for composite labor

LS_l : Supply of type *l* labor

$MRGN_i$: Demand for commodity *i* as a trade or transport margin

Q_i : Quantity demanded of composite commodity *i*

VA_j : Value added of industry *j*

$VSTK_i$: Inventory change of commodity *i*

$XS_{j,i}$: Industry *j* production of commodity *i*

XST_j : Total aggregate output of industry *j*

A3.2 PRICE VARIABLES

e : Exchange rate[11]; price of foreign currency in terms of local currency

$P_{j,i}$: Basic price of industry *j*'s production of commodity *i*

PC_i : Purchaser price of composite comodity *i* (including all taxes and margins)

PCI_j : Intermediate consumption price index of industry *j*

PD_i : Price of local product *i* sold on the domestic market (including all taxes and margins)

PE_x : Price received for exported commodity *x* (excluding export taxes)

PE_x^{FOB} : FOB price of exported commodity *x* (in local currency)

$PIXCON$: Consumer price index

$PIXGDP$: GDP deflator

$PIXGVT$: Public expenditures price index

$PIXINV$: Investment price index

PL_i : Price of local product *i* (excluding all taxes on products)

PM_m : Price of imported product *m* (including all taxes and tariffs)

PP_j : Industry *j* unit cost, including taxes directly related to the use of capital and labor, but excluding other taxes on production

PT_j : Basic price of industry *j*'s output

PVA_j : Price of industry *j* value added (including taxes on production directly related to the use of capital and labour)

PWM_m : World price of imported product *m* (expressed in foreign currency)

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[11] The default choice of numeraire in PEP-1-1 is the exchange rate *e*. This is implemented by fixing the value of *e* as exogenous. But the choice of numeraire in a CGE model is arbitrary (although the interpretation of results can be more or less easy, depending on which numeraire is selected).

PWX_x : World price of exported product *x* (expressed in foreign currency)

$R_{k,j}$: Rental rate of type *k* capital in industry *j*

RC_j : Rental rate of industry *j* composite capital

RK_k : Rental rate of type *k* capital (if capital is mobile)

$RTI_{k,j}$: Rental rate paid by industry *j* for type *k* capital, including capital taxes

W_l : Wage rate of type *l* labor

WC_j : Wage rate of industry *j* composite labor

$WTI_{l,j}$: Wage rate paid by industry *j* for type *l* labor, including payroll taxes

A3.3 NOMINAL (VALUE) VARIABLES

CAB : Current account balance

CTH_h : Consumption budget of type *h* households

G : Current government expenditures on goods and services

GDP^{BP} : GDP at basic prices

GDP^{FD} : GDP at purchasers' prices from the perspective of final demand

GDP^{IB} : GDP at market prices (income-based)

GDP^{MP} : GDP at market prices

$GFCF$: Gross fixed capital formation

IT : Total investment expenditures

SF_f : Savings of type *f* businesses

SG : Government savings

SH_h : Savings of type *h* households

$SROW$: Rest-of-the-world savings

TDF_f : Income taxes of type *f* businesses

$TDFT$: Total government revenue from business income taxes

TDH_h : Income taxes of type *h* households

$TDHT$: Total government revenue from household income taxes

TIC_i : Government revenue from indirect taxes on product i

$TICT$: Total government receipts of indirect taxes on commodities

$TIK_{k,j}$: Government revenue from taxes on type k capital used by industry j

$TIKT$: Total government revenue from from taxes on capital

TIM_m : Government revenue from import duties on product m

$TIMT$: Total government revenue from import duties

TIP_j : Government revenue from taxes on industry j production (excluding taxes directly related to the use of capital and labor)

$TIPT$: Total government revenue from production taxes (excluding taxes directly related to the use of capital and labor)

$TIW_{l,j}$: Government revenue from payroll taxes on type l labor in industry j

$TIWT$: Total government revenue from payroll taxes

TIX_x : Government revenue from export taxes on product x

$TIXT$: Total government revenue from export taxes

$TPRCTS$: Total government revenue from taxes on products and imports

$TPRODN$: Total government revenue from other taxes on production[12]

$TR_{ag,agj}$: Transfers from agent agj to agent ag

YDF_f : Disposable income of type f businesses

YDH_h : Disposable income of type h households

YF_f : Total income of type f businesses

YFK_f : Capital income of type f businesses

$YFTR_f$: Transfer income of type f businesses

YG : Total government income

YGK : Government capital income

$YGTR$: Government transfer income

YH_h : Total income of type *h* households

YHK_h : Capital income of type *h* households

YHL_h : Labor income of type *h* households

$YHTR_h$: Transfer income of type *h* households

$YROW$: Rest-of-the-world income

A4. Parameters

$aij_{i,j}$: Input-output coefficient

B_j^{KD} : Scale parameter (CES – composite capital)

B_j^{LD} : Scale parameter (CES – composite labor)

B_m^{M} : Scale parameter (CES – composite commodity)

B_j^{VA} : Scale parameter (CES – value added)

$B_{j,x}^{X}$: Scale parameter (CET – exports and local sales)

B_j^{XT} : Scale parameter (CET – total output)

$\beta_{k,j}^{KD}$: Share parameter (CES – composite capital)

$\beta_{l,j}^{LD}$: Share parameter (CES – composite labor)

β_m^{M} : Share parameter (CES – composite commodity)

β_j^{VA} : Share parameter (CES – value added)

$\beta_{j,x}^{X}$: Share parameter (CET – exports and local sales)

$\beta_{j,i}^{XT}$: Share parameter (CET – total output)

η : Price elasticity of indexed transfers and parameters

γ_i^{GVT} : Share of commodity *i* in total current public expenditures on goods and services

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12 That is, taxes on production other than taxes on products and taxes and duties on imports (see Appendix B1).

γ_i^{INV} : Share of commodity *i* in total investment expenditures

$\gamma_{i,h}^{LES}$: Marginal share of commodity *i* in type *h* household consumption budget

io_j : Coefficient (Leontief – intermediate consumption)

$\lambda_{ag,k}^{RK}$: Share of type *k* capital income received by agent *ag*

$\lambda_{ag,agj}^{TR}$: Share parameter (transfer functions)

$\lambda_{h,l}^{WL}$: Share of type *l* labor income received by type *h* households

ρ_j^{KD} : Elasticity parameter (CES – composite capital); $-1 < \rho_j^{KD} < \infty$

ρ_j^{LD} : Elasticity parameter (CES – composite labor); $-1 < \rho_j^{LD} < \infty$

ρ_m^{M} : Elasticity parameter (CES – composite commodity); $-1 < \rho_m^{M} < \infty$

ρ_j^{VA} : Elasticity parameter (CES – value added) ; $-1 < \rho_j^{VA} < \infty$

$\rho_{j,x}^{X}$: Elasticity parameter (CET – exports and local sales) ; $1 < \rho_{j,x}^{X} < \infty$

ρ_j^{XT} : Elasticity parameter (CET – total output) ; $1 < \rho_j^{XT} < \infty$

σ_j^{KD} : Elasticity of substitution (CES – composite capital); $0 < \sigma_j^{KD} < \infty$

σ_j^{LD} : Elasticity of substitution (CES – composite labor); $0 < \sigma_j^{LD} < \infty$

σ_m^{M} : Elasticity of substitution (CES – composite commodity); $0 < \sigma_m^{M} < \infty$

σ_j^{VA} : Elasticity of transformation (CES – value added) ; $0 < \sigma_j^{VA} < \infty$

$\sigma_{j,x}^{X}$: Elasticity of transformation (CET – exports and local sales) ; $0 < \sigma_{j,x}^{X} < \infty$

σ_x^{XD} : Price elasticity of the world demand for exports of product *x*

σ_j^{XT} : Elasticity of transformation (CET – total output) ; $0 < \sigma_j^{XT} < \infty$

$sh0_h$: Intercept (type *h* household savings)

$sh1_h$: Slope (type *h* household savings)

$tmrg_{i,ij}$: Rate of margin *i* applied to commodity *ij*

$tmrg_{i,x}^{X}$: Rate of margin *i* applied to exported commodity *x*

$tr0_h$: Intercept (transfers by type *h* households to government)

$tr1_h$: Marginal rate of transfers by type *h* households to government

$ttdf0_f$: Intercept (income taxes of type *f* businesses)

$ttdf1_f$: Marginal income tax rate of type *f* businesses

$ttdh0_h$: Intercept (income taxes of type *h* households)

$ttdh1_h$: Marginal income tax rate of type *h* households

$ttic_i$: Tax rate on commodity *i*

$ttik_{k,j}$: Tax rate on type *k* capital used in industry *j*

$ttim_m$: Rate of taxes and duties on imports of commodity *m*

$ttip_j$: Tax rate on the production of industry *j*

$ttiw_{l,j}$: Tax rate on type *l* worker compensation in industry *j*

$ttix_x$: Export tax rate on exported commodity *x*

v_j : Coefficient (Leontief – value added)

APPENDIX B: MODEL VARIABLES AND CONCEPTS OF THE 1993 SYSTEM OF NATIONAL ACCOUNTS (SNA93)

This Appendix is dedicated to clarifying the following concepts, on the basis of the 1993 System of National Accounts (INTER-SECRETARIAT WORKING GROUP ON NATIONAL ACCOUNTS, 1993)[13]:

Taxes on production and imports

Taxes on products

Other taxes on production

Basic prices and value added at basic prices

Producers' prices and value added at producers' prices

Purchasers' prices

GDP at basic prices

GDP at market prices

B1. « Indirect » taxes

Paragraph 7.49 of the SNA93 says that taxes on ***products*** are a ***subset*** of taxes on ***production***, rather than a separate category. Taxes on production, together with taxes and duties on imports, constitute the set of ***Taxes on production and imports***.

> 7.49. ***Taxes on production and imports consist of***:
>
> ***taxes on products*** payable on goods and services when they are produced, delivered, sold, transferred or otherwise disposed of by their producers; they include
>
> ***taxes and duties on imports*** that become payable when goods enter the economic territory by crossing the frontier or when services are delivered to resident units by non-resident units; when outputs are valued at basic prices, taxes on domestically produced products are not recorded in the accounts of the System as being payable by their producers
>
> plus:
>
> ***other taxes on production***, consisting mainly of taxes on the ownership or use of land, buildings or other assets used in production or on the labour employed, or compensation of employees paid.
>
> Taxes on the personal use of vehicles, etc., by households are recorded under current taxes on income, wealth, etc.

[13] *http://unstats.un.org/unsd/sna1993/toctop.asp.*

The word « other » is important here. This implies the following classification scheme:

<table>
<tr><td rowspan="3">Taxes on production and imports</td><td colspan="2">Taxes and duties on imports</td></tr>
<tr><td rowspan="2">Taxes on production</td><td>Taxes on products</td></tr>
<tr><td>Other taxes on production</td></tr>
</table>

Paragraph 7.50 defines taxes on production, and says that they correspond *grosso modo* to « ***indirect taxes*** », although that expression is no longer part of the national accounts terminology (it is nonetheless still used, by Statistics Canada, among others – see box below).

7.50. ***At the level of an individual enterprise, taxes on production are recorded as being payable out of its value added***. Similarly, in business accounting, taxes on production, except invoiced VAT, are usually regarded as costs of production that may be charged against sales or other receipts when calculating profits for tax or other purposes. ***They correspond grosso modo to "indirect taxes" as traditionally understood, indirect taxes being taxes that supposedly can be passed on, in whole or in part, to other institutional units by increasing the prices of the goods or services sold***. However, it is extremely difficult, if not impossible, to determine the real incidence of different kinds of taxes, and the use of the terms "direct" and "indirect" taxes has fallen out of favour in economics and is no longer used in the System.

B1.1 OTHER TAXES ON PRODUCTION

As indicated in 7.49, and detailed in 7.70 below, ***Other taxes on production*** include payroll taxes, taxes on capital, as well as property or real estate taxes. But they ***do not include taxes on products***, defined in Paragraph 7.62 below.

3. Other taxes on production (D.29)

7.70. ***These consist of all taxes, except taxes on products, that enterprises incur as a result of engaging in production***. Such taxes ***do not include any taxes on the profits or other income*** received by the enterprise and are payable irrespective of the profitability of the production. They may be payable on the land, fixed assets or labour employed in the production process or on certain activities or transactions. Other taxes on production include the following:

(a) ***Taxes on payroll or work force***: these consist of taxes payable by enterprises assessed either as a proportion of the wages and salaries paid or as a fixed amount per person employed. They do not include compulsory social security contributions paid by employers or any taxes paid by the employees themselves out of their wages or salaries (GFS, 3; OECD, 3000);

(b) ***Recurrent taxes on land, buildings or other structures***: these consist of taxes payable regularly, usually each year, in respect of the use or ownership of land, buildings or other structures utilized by enterprises in production, whether the enterprises own or rent such assets (GFS, 4.1; OECD, 4100);

(c) ***Business and professional licences***: these consist of taxes paid by enterprises in order to obtain a licence to carry on a particular kind of business or profession. However, if the government carries out checks on the suitability, or safety of the business premises, on the reliability, or safety, of the equipment employed, on the professional competence of the staff employed, or on the quality or standard of goods or services produced, as a condition for granting such a licence, the payments are not unrequited and should be treated as payments for services rendered, unless the amounts charged for the licences are out of all proportion to the costs of the checks carried out by governments (GFS, 5.5.1; OECD, 5210). (See also paragraph 8.54 (c) of chapter VIII for the treatment of licences obtained by households for their own personal use.);

(d) ***Taxes on the use of fixed assets or other activities***: these include taxes levied periodically on the use of vehicles, ships, aircraft or other machinery or equipment used by enterprises for purposes of production, whether such assets are owned or rented. These taxes are often described as licences, and are usually fixed amounts which do not depend on the actual rate of usage (GFS, 5.5.2 and 5.5.3; OECD, 5200);

(e) ***Stamp taxes***: these consist of stamp taxes which do not fall on particular classes of transactions already identified, for example, stamps on legal documents or cheques. These are treated as taxes on the production of business or financial services. However, stamp taxes on the sale of specific products, such as alcoholic beverages or tobacco, are treated as taxes on products (GFS, 7.2; OECD, 6200);

(f) ***Taxes on pollution***: these consist of taxes levied on the emission or discharge into the environment of noxious gases, liquids or other harmful substances. They do not include payments made for the collection and disposal of waste or noxious substances by public authorities, which constitute intermediate consumption of enterprises (GFS, 7.3; OECD, 5200);

(g) ***Taxes on international transactions***: these consist of taxes on travel abroad, foreign remittances or similar transactions with non-residents (GFS, 6.5 and 6.6; OECD, 5127).

B1.2 TAXES ON PRODUCTS

Taxes on products are defined in 7.62.

2. Taxes on products (D.21)

7.62. ***A tax on a product*** is a tax that is ***payable per unit of some good or service***. The tax may be a specific amount of money per unit of quantity of a good or service (the quantity units being measured either in terms of discrete units or continuous physical variables such as volume, weight, strength, distance, time, etc.), or it may be calculated ad valorem as a specified percentage of the price per unit or value of the goods or services transacted. A tax on a product usually becomes payable when it is produced, sold or imported, but it may also become payable in other circumstances, such as when a good is exported, leased, transferred, delivered, or used for own consumption or own capital formation. An enterprise may or may not itemize the amount of a tax on a product separately on the invoice or bill which they charge their customers.

B1.3 CORRESPONDING VARIABLES IN PEP-1-1

Variables In PEP-1-1 which correspond to these concepts are:

TIC_i : Government revenue from indirect taxes on product *i;*
part of SNA93 « taxes on products »

$TIK_{k,j}$: Government revenue from taxes on type *k* capital used by industry *j;*
part of SNA93 « other taxes on production »

TIM_m : Government revenue from import duties on product *m;*
part of SNA93 « taxes and duties on imports »

TIP_j : Government revenue from taxes on industry *j* production (excluding taxes directly related to the use of capital and labor)*;*
part of SNA93 « other taxes on production »

$TIW_{l,j}$: Government revenue from payroll taxes on type *l* labor in industry *j;*
part of SNA93 « other taxes on production »

TIX_x : Government revenue from export taxes on product *x;*
part of SNA93 « taxes on products »

$TPRCTS$: Total government revenue from taxes on products and imports*;*
correspond to SNA93 « taxes on products »

$TPRODN$: Total government revenue from other taxes on production*;*
correspond to SNA93 « other taxes on production »

B2. Price concepts

B2.1 PRODUCERS' PRICES AND BASIC PRICES

Value added is defined at the level of individual producer units, and is aggregated to sectors or industries, or to the whole economy. It is calculated by subtracting the value of intermediate consumption from the value of production. Now, the values of intermediate consumption and production depend on which price concepts are used in their measurement. Consequently, SNA93 defines price concepts in reference to the valuation of production and intermediate consumption.

Price concepts differ according to which taxes are included or excluded. ***Taxes on imports*** are recorded only at the level of the total economy, as they are not payable out of the values added of domestic producers: consequently, they are not relevant in the valuation of production. Output valued at ***basic prices*** excludes all taxes (subsidies) on products payable (receivable) on the goods or services produced as outputs, but ***includes other taxes on production*** (see Paragraph 7.53 below). ***Producers' prices*** also ***include all taxes or subsidies on products*** payable or receivable on outputs, except invoiced VAT or similar deductible taxes as invoiced VAT is never included in the value of output.

> 7.52. In the generation of income account, taxes on imports are recorded only at the level of the total economy as they are not payable out of the values added of domestic producers. Moreover, at the level of an individual institutional unit or sector, ***only those taxes on products that have not been deducted from the value of the output of that unit or sector need to be recorded under uses in its generation of income account. These vary*** depending upon the way in which output is valued. ***When output is valued at basic prices, all taxes (subsidies) on products payable (receivable) on the goods or services produced as outputs are deducted from (added to) the value of that output at producers' prices. They do not, therefore, have to be recorded under uses in the generation of income account of the units or sectors concerned***, being recorded only at the level of the total economy, in the same way as taxes on imports. ***When output is valued at producers' prices, all taxes or subsidies on products payable or receivable on outputs have to be recorded under uses in the generation of income accounts of the units or sectors concerned, except invoiced VAT or similar deductible taxes as invoiced VAT is never included in the value of output***. Non-deductible VAT and similar taxes are recorded under uses only at the level of the total economy, like taxes on imports.

> 7.53. ***Other taxes or subsidies on production*** - i.e., taxes payable on the land, assets, labour, etc., employed in production - ***are not taxes payable per unit of output*** and cannot be deducted from the producer's price. They are recorded as being payable out of the values added of the individual producers or sectors concerned.

The implications of the above are presented in the two following figures: in Figure B1, value added is recorded at basic prices; in Figure B2, it is recorded at producer prices. In both cases, the starting point is production at producers' prices. But in the calculation of value added at basic prices, taxes on products are immediately deducted to obtain production at basic prices. It follows that taxes on products do not appear in the « Generation of income account » when value added is at basic prices, but they do when value added is at producer prices.

Figure B1 – Value added of a single producer unit at basic prices

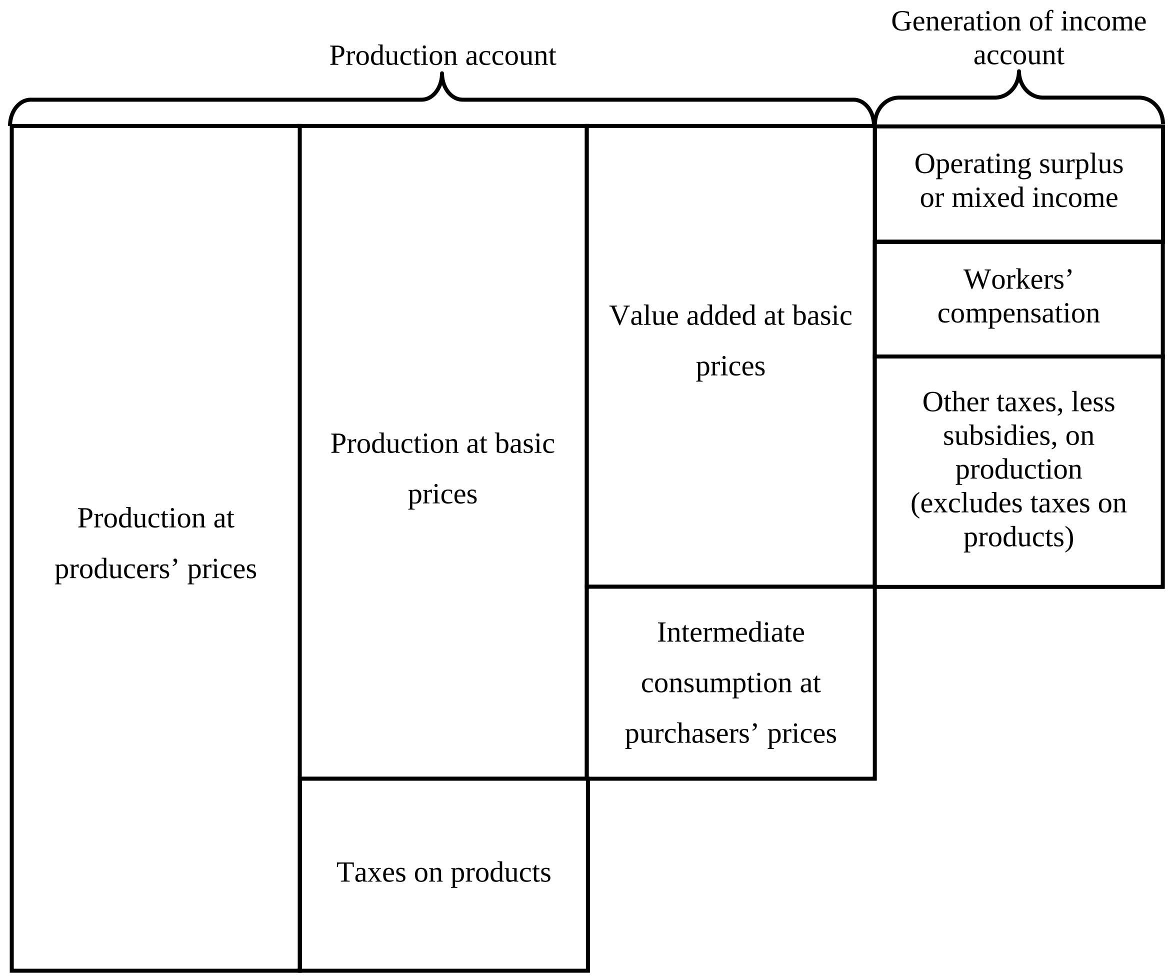
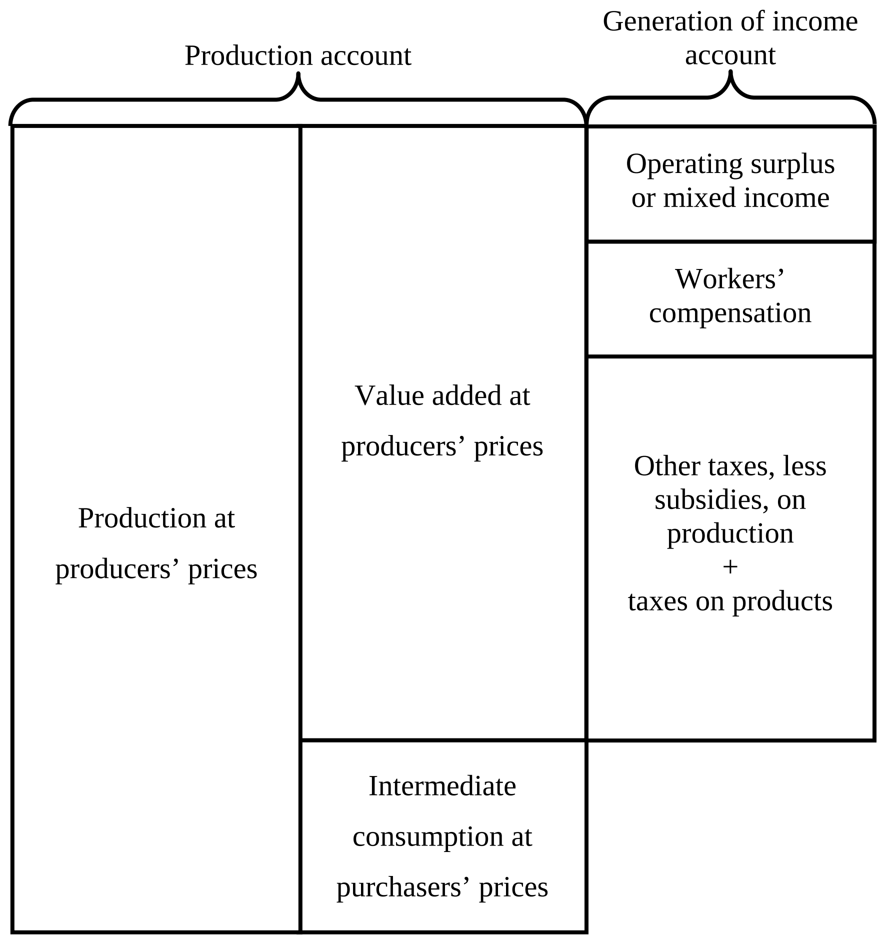


Figure B2 – Value added of a single producer unit at producers' prices



B2.2 PURCHASERS' PRICES

The reader will have noticed that, in computing value added, whether at basic prices or at producers' prices, intermediate consumption ***at purchasers' prices*** is deducted from the value of production. This is justified, particularly in the case of value added at basic prices, by the fact that « From the point of view of the producer, purchasers' prices for inputs and basic prices for outputs represent the prices actually paid and received » (SNA93, Paragraph 6.226).

As a general principle, the SNA records ***product uses at purchasers' prices***, which include taxes and margins.

> 3.81. Usually, the producer and the user of a given product perceive its value differently owing to the existence of taxes and subsidies on products, transport costs to be paid and the occurrence of trade margins. In order to keep as close as possible to the views of the economic transactors themselves, ***the System records all uses at purchasers' prices including these elements, but excludes them from the value of output of the product.***

The most explicit definition of purchasers' prices is given in Paragraph 3.83 (and repeated in 6.215):

> 3.83. Use of products is recorded at purchasers' prices. The ***purchaser's price*** is defined as the ***amount payable by the purchaser, excluding any deductible VAT or similar deductible tax***, in order to take delivery of a unit of a good or service at the time and place required by the purchaser. The purchaser's price of a good ***includes any transport charges*** paid separately by the purchaser to take delivery at the required time and place.

But there may be more than one definition of purchasers' price for a given product, when it goes through several successive transactions before its final use.

> 6.216. When ***comparing the purchaser's price with the producer's or basic price,*** it is important to specify whether they refer to the same transaction or two different transactions. For certain purposes, including input-output analysis, it may be convenient to compare the ***price paid by the final purchaser of a good after it has passed through the wholesale and retail distribution chains*** with the producer's price received by its original producer. In this case the prices refer to two different transactions taking place at quite different times and locations: they must differ at least by the amount of the ***wholesale and retail trade margins***.

Paragraph 6.217 is useful in that it indirectly confirms that the difference between producer price and basic price is indeed « the value of any taxes less subsidies on the product (other than VAT) ».

> 6.217. ***When the prices refer to the same transaction,*** that is, the purchaser buys directly from the producer, the purchaser's price may exceed the ***producer's price*** by:
> (a) The value of any ***non-deductible VAT***, payable by the purchaser; and
> (b) The value of any ***transport charges*** on a good paid separately by the purchaser and not included in the producer's price.
> ***It follows that the purchaser's price may exceed the basic price by the amount of the two items just listed plus the value of any taxes less subsidies on the product (other than VAT).***

The existence of a value-added tax (VAT) is a complicating factor (see Paragraphs 6.207 to 6.214, not reproduced here).

B2.3 CORRESPONDING VARIABLES IN PEP-1-1

Variables In PEP-1-1 which correspond to these concepts are:

P_j : Basic price of industry *j*'s output
⇒ SNA93 « basic price » of aggregate output

PCI_j : Intermediate consumption price index of industry *j*
⇒ SNA93 « purchaser price » index of intermediate consumption

PC_i : Purchaser price of composite comodity *i* (including all taxes and margins)
⇒ SNA93 « purchaser price »

PD_i : Price of local product *i* sold on the domestic market (including all taxes and margins)
⇒ SNA93 « purchaser price »

PM_m : Price of imported product *m* (including all taxes and margins)
⇒ SNA93 « purchaser price »

PE_x : Price received for exported commodity *x* (excluding export taxes)
⇒ SNA93 « basic price » of exports

PE_x^{FOB} : FOB price of exported commodity *x* (in local currency)
⇒ SNA93 « producer price » ***and*** « purchaser price » of exported products

PL_i : Price of local product *i* (excluding all taxes on products)
⇒ SNA93 « basic price » of local products sold on the domestic market

PP_j : Industry *j* unit cost, including taxes directly related to the use of capital and labor, but excluding other taxes on production
⇒ SNA93 « basic price », ***minus*** taxes on production not directly related to the use of capital and labor

There is no variable in PEP-1-1 corresponding to producer prices, except for PE_x^{FOB}. Subtracting margins from PD_i would yield producer prices of local products sold domestically.

B3. Trade and transport margins

In the SNA93, the output of wholesale and retail trade is not measured by the value of their sales, but rather by the value of the services rendered as intermediaries between producers and buyers. The value of these services is the gross trade margin. In the SNA, a purchase is recorded as two simultaneous flows:

one is the value of what is purchased, and the other is the trade margin that is included in the price paid by the buyer.[14]

> 3.30. The System's recording of transactions for wholesalers and retailers does not mirror the way in which those involved view them. The purchases of goods for resale by wholesalers and retailers are not recorded explicitly, and they are viewed as selling, not the goods, but the services of storing and displaying a selection of goods in convenient locations and making them easily available for customers. This partitioning implements the System's measure of output for traders, which is by the value of the margins on goods they purchase for resale.

Transport services are treated both as margins and as directly purchased services. But transport services produced for own use within enterprises are not recorded separately (see SNA93 Paragraph 6.103).

Trade and transport margins appear in the supply part of the supply-and-use table. SNA93 Table 2.10, reproduced after section B4.3 below, provides an example of how trade margins are recorded in the System.

> 2.214. The upper part of the table shows the origin of the resources of goods and services. In the rows, the various types of products are presented according to a classification which can be used at various levels of detail. In the columns, starting from the right side, imports are shown first. Then a matrix showing the output of industries, according to the activity classification, appears. This is the make matrix. It may be valued either at basic prices or at producers' prices in the absence of a value added tax (VAT), or at producers' prices in the presence of VAT. The actual figures in the table are at basic prices which is the preferred method of valuation for output. The column for total industries records the total output of industries for each kind of product. The output of a given industry may cover a number of different products, the principal and the secondary ones.
>
> *Text refers to:* table 2.10.

> 2.215. Taxes, less subsidies, on products - with varying content according to the valuation of output - and trade and transport margins are recorded in two columns in order to get total supply of each type of product valued at purchasers' prices. The corresponding trade and transport services are deducted globally at the intersection between the relevant rows and the column for trade and transport margins. Thus the total of the latter is zero.
>
> *Text refers to:* table 2.10.

In column (2) of the SNA93 Table 2.10, the production of trade and transport services as margins is subtracted from total supply (–68 for trade, and –10 for transport), so that the column total is zero. In PEP-1-1, margins are treated in a different, but arithmetically equivalent, manner. In the underlying SAM,

---

14 Also see SNA93 Paragraphs 6.110 to 6.114 (not reproduced here).

there is a « Trade and transport margins » column in the use table (bottom part of the supply-and-use table): margins are formally treated as a (fictitious) industry. Its sales are the amount of trade and transport margins included in the supply of goods and services (78 = 2 + 2 + 74); its intermediate purchases are trade services (68) and transport services (10); no value added is generated. The only difference with Table 2.10 is that total supply of trade services (line 6 of the supply table – top part of Table 2.10) is then increased by 68, and that of transport services (line 7) by 10, while total uses of these services are increased by the same amounts (lines 6 and 7 of the use table – bottom part of Table 2.10). See variable $MRGN_i$ in equations 58 and 89.

B4. GDP concepts

B4.1 GDP AT BASIC PRICES AND AT PRODUCERS' PRICES

GDP is the sum of value added of all resident producer units. So, just as value added can be evaluated at basic prices or producer prices, so can GDP be evaluated at both sets of prices.

> 2.172. Basically, ***GDP is a concept of value added***. It is the ***sum of gross value added of all resident producer units (institutional sectors or, alternatively, industries) plus that part (possibly the total) of taxes, less subsidies, on products which is not included in the valuation of output.**** Gross value added is the difference between output and intermediate consumption.
>
> **** If basic prices are used for valuing output, GDP is equal to the sum of gross value added of all resident producer units plus all taxes on products (less subsidies on products)***. If producers' prices are used for valuing output, GDP is equal to the sum of gross value added of all resident producer units plus taxes and duties on imports, less import subsidies - in absence of a value added tax system - or plus taxes and duties on imports (less import subsidies) and value added type taxes - when such a taxation system does exist.

GDP at basic prices is computed from gross value added at basic prices, defined as:

> 6.226. ***Gross value added at basic prices is defined as output valued at basic prices less intermediate consumption valued at purchasers' prices.*** Although the outputs and inputs are valued using different sets of prices, for brevity the value added is described by the prices used to value the outputs. From the point of view of the producer, purchasers' prices for inputs and basic prices for outputs represent the prices actually paid and received. Their use leads to a measure of gross value added which is particularly relevant for the producer. The resulting measure has also some convenient properties for aggregation purposes as explained later, ***although there is no named aggregate in the System which corresponds to the sum of the gross values added of all enterprises measured at basic prices.***

GDP at producers' prices is computed from gross value added at producers' prices, defined as:

> 6.227. ***Gross value added at producers' prices is defined as output valued at producers' prices less intermediate consumption valued at purchasers' prices.*** As already explained, in the absence of VAT, the total value of the intermediate inputs consumed is the same whether they are valued at producers' or at purchasers' prices, in which case this measure of gross value added is the same as one

> which uses producers' prices to value both inputs and outputs. It is an economically meaningful measure that is equivalent to the traditional measure of gross value added at market prices. However, in the presence of VAT, the producer's price excludes invoiced VAT, and it would be inappropriate to describe this measure as being at "market" prices.

The difference between the two measures of GDP is the sum of taxes included in value added at producers' prices and at basic prices:

> 6.228. Both this measure of gross value added and that described in the previous section use purchasers' prices to value intermediate inputs. ***The difference between the two measures is entirely attributable to their differing treatments of taxes or subsidies on products payable on outputs (other than invoiced VAT)***. By definition, the value of output at producers' prices exceeds that at basic prices by the amount, if any, of the taxes, less subsidies, on the output so that the two associated measures of gross value added must differ by the same amount.

B4.2 GDP AT PURCHASERS' PRICES FROM THE PERSPECTIVE OF FINAL DEMAND

GDP can also be computed from final demand. Since final demand is valued at purchasers' prices, this measure of GDP is different from gross value added aggregations:

> 2.173. Next, ***GDP is also equal to the sum of the final uses of goods and services (all uses except intermediate consumption) measured in purchasers' prices, less the value of imports of goods and services.***

B4.3 GDP FROM THE INCOME PERSPECTIVE

Finally, GDP can be computed as the sum of incomes distributed by resident producer units:

> 2.174. Finally, ***GDP is also equal to the sum of primary incomes distributed by resident producer units.***

This is detailed in Paragraph 2.222. This paragraph refers to Table 2.10, which may be downloaded from the SNA93 site (reproduced on the following page).

> 2.222. The three approaches to GDP (1,854) appear in the supply and use table, as well as in the integrated economic accounts:
>
> From the production side, GDP is equal to total output (3,604) minus total intermediate consumption (1,883) plus taxes, less subsidies, on products (133) not included in the value of output.
>
> From the demand side, GDP is equal to final consumption expenditure (1,015 + 16 + 156 + 212) plus gross capital formation (376 + 28 + 10) plus exports (540) minus imports (499).
>
> ***From the income side, GDP is equal to compensation of employees (762) plus taxes, less subsidies, on production and imports (191), plus mixed income, gross (442), plus operating surplus, gross (459).***

Table 2.10. Supply and use (reduced format)

Resources		Supply of products: Total supply at purchasers' prices (1)	Trade and transport margins (2)	Taxes less subsidies on products (3)	Output in industries (by ISIC categories): Agri-culture hunting, forestry fishing (A+B) (4)	Mining and quarrying (C) (5)	Manufac-turing, electricity (D+E) (6)	Construc-tion (F) (7)	Wholesale, retail trade, repair motor vehicles and hshld goods, hotels, restaurants (G+H) (8)	Transport, storage, and com-munication (I) (9)	Financial interme-diation, real estate, other business services (J+K) (10)	Education, health, personal services, pub. admin. and defense (M+N+O+P+L) (11)	Total industry, in basic prices (12)	(13)	Imports of goods and services (14)
Goods and services (by CPC sections)															
1. Agriculture, forestry and fishery products	(0)	128	2	2	87	0	0	0	0	0	0	0	87		37
2. Ores and minerals	(1)	103	2	0	0	30	10	0	1	0	0	0	41		60
3. Electricity, gas and water	(17-18)	160	0	5	0	2	152	0	0	0	0	0	154		1
4. Manufacturing	(2-4)	2 160	74	89	2	2	1 666	11	16	8	7	2	1 714		283
5. Construction work and construction, land	(5)	262	0	17	0	0	7	232	0	5	0	0	244		1
6. Trade services, restaurant and hotel services	(6)	179	−68	3	0	0	8	1	149	7	0	0	165		79
7. Transport, storage and communication services	(7)	111	−10	5	0	0	0	0	21	75	0	0	96		20
8. Business services	(8)	590	0	8	0	1	0	0	2	3	465	98	569		13
9. Community, social and pers. serv. excl. public admin.	(9)	375	0	4	0	0	1	0	2	2	6	355	366		5
10. Public administration	(91)	168	0	0	0	0	0	0	0	0	0	168	168		0
11. *Total*		4 236	0	133	89	35	1 844	244	191	100	478	623	3 604		499

Uses		Use of products, purchasers' price: Total uses in purchasers' prices (1)	(2)	Taxes less subsidies on products (3)	Intermediate consumption in industries (by ISIC categories): Agri-culture hunting, forestry fishing (A+B) (4)	Mining and quarrying (C) (5)	Manufac-turing, electricity (D+E) (6)	Construc-tion (F) (7)	Wholesale, retail trade, repair motor vehicles and hshld goods, hotels, restaurants (G+H) (8)	Transport, storage, and com-munication (I) (9)	Financial interme-diation, real estate, other business services (J+K) (10)	Education, health, personal services, pub. admin. and defense (M+N+O+P+L) (11)	Total industry (12)	Total economy (13)	Exports of goods and services (14)	Final consumption expenditure: Households (14)	NPISH individual (15)	Genearl government individual (16)	Genearl government collective (17)	Gross capital formation: Gross fixed capital formation (18)	Changes in inven-tories (19)	Net purchases of valuables (20)
Goods and services (by CPC sections)																						
1. Agriculture, forestry and fishery products	(0)	128			3	0	71	0	3	1	5	5	88		7	28	0	0	2	2	1	
2. Ores and minerals	(1)	103			1	3	91	0	0	0	1	0	96		6	2	0	0	0		−1	
3. Electricity, gas and water	(17-18)	160			2	2	96	1	5	3	4	10	123		1	36	0	0	0		0	
4. Manufacturing	(2-4)	2 160			32	7	675	80	36	21	45	96	992		422	567	0	0	3	161	5	10
5. Construction work and construction, land	(5)	262			1	2	7	5	2	1	3	19	40		6	3	0	0	0	190	23	
6. Trade services, restaurant and hotel services	(6)	179			2	1	34	1	9	6	4	4	61		67	51	0	0	0		0	
7. Transport, storage and communication services	(7)	111			2	1	29	3	19	12	5	7	78		19	14	0	0	0		0	
8. Business services	(8)	590			3	1	117	23	25	15	54	71	309		8	250	0	0	0	23	0	
9. Community, social and pers. serv. exc. pub. ad.	(9)	375			1	0	7	1	1	1	11	73	95		4	58	14	0	204	0	0	
10. Public administration	(91)	168			0	0	0	0	0	0	0	1	1		0	6	2	156	3		0	
11. *Total uses in purchasers' prices*		4 236			47	17	1 127	114	100	60	132	286	883		540	1 015	16	156	212	376	28	10
12. *Total gross value added/GDP*				133	42	18	717	130	91	40	346	337	1 721	1 854								
13. Compensation of employees					9	13	336	58	44	16	54	232	762	762								
14. Taxes less subsidies on production and imports				133	−2	−2	46	5	0	−6	12	5	58	191								
15. Taxes less subsidies on products				133										133								
16. Other taxes less subsidies on production					−2	−2	46	5	0	−6	12	5	58	58								
17. Mixed income, net					14	0	227	35	36	3	99	18	432	432								
18. Operating surplus, net					10	4	30	21	−4	12	127	47	247	247								
19. Consumption of fixed capital					11	3	78	11	15	15	54	35	222	222								
20. Mixed income, gross					17	0	228	36	36	7	99	19	442	442								
21. Operating surplus, gross					18	7	107	31	11	23	181	81	459	459								
22. *Total*					89	35	1 844	244	191	100	478	623	3 604									
23. *Labour inputs (hours worked)*					2 058	292	31 982	5 024	7 078	2 032	3 700	17 203	69 369									
24. *Gross fixed capital formation*					11	6	117	9	20	39	144	30	376									
25. *Stocks of fixed assets*					159	90	1 788	160	298	572	2 260	456	5 783									

GDP

B4.4 WHAT ABOUT GDP AT FACTOR COST?

This concepts no longer exists in the System of National Accounts. Nonetheless, GDP at factor costs can easily be computed. In the absence of *other taxes on production*, GDP at factor costs is identical to GDP at basic prices.

> 6.229. ***Gross value added at factor cost is not a concept used explicitly in the System***. Nevertheless, it can easily be derived from either of the measures of gross value added presented above by subtracting the value of any taxes, less subsidies, on production payable out of gross value added as defined. For example, the only taxes on production remaining to be paid out of gross value added at basic prices consist of "other taxes on production". These consist mostly of current taxes (or subsidies) on the labour or capital employed in the enterprise, such as payroll taxes or current taxes on vehicles or buildings. Gross value added at factor cost can, therefore, be derived from gross value added at basic prices by subtracting "other taxes, less subsidies, on production".

> 6.230. The conceptual difficulty with gross value added at factor cost is that there is no observable vector of prices such that gross value added at factor cost is obtained directly by multiplying the price vector by the vector of quantities of inputs and outputs that defines the production process. By definition, "other taxes or subsidies on production" are not taxes or subsidies on products that can be eliminated from the input and output prices. Thus, despite its traditional name, gross value added at factor cost is not strictly a measure of value added.

> 6.231. Gross value added at factor cost is essentially a measure of income and not output. It represents the amount remaining for distribution out of gross value added, however defined, after the payment of all taxes on production and the receipt of all subsidies on production. It makes no difference which measure of gross value added is used because the measures considered above differ only in respect of the amounts of the taxes or subsidies on production which remain payable out of gross value added.

> 6.232. Claims on gross value added, other than payments of taxes, less subsidies, to government used to be described as "factor incomes". While the concept of factor income is no longer used in the System, gross value added at factor cost could be interpreted as measuring the value of the fund out of which so-called "factor incomes" can be paid: it follows that it is equal to the total value of the "factor" incomes generated by production.

Statistics Canada, among other statistical agencies, has abandoned the concept of « GDP at factor costs »:

> **What is the difference between the GDP at factor cost and the GDP at basic prices?**
>
> Whereas in the past, Statistics Canada published net domestic product at factor cost, this practice changed with the publication of the estimates of the first quarter of 2001 of the national economic and financial accounts. To bring the Canadian System of National Economic Accounts into line with international standards, the valuation of production is now done according to basic prices.
>
> The concept of GDP at basic prices differs from the concept of GDP at factor costs in that the former includes net indirect taxes (indirect taxes less subsidies) attached to factors of production. For example, whereas property taxes, capital taxes and payroll taxes were not included in the valuation of GDP at factor costs, they are included in the

valuation of GDP at basic prices. These production expenses are included in GDP at basic prices, subtracting from them any subsidies attached to factors of production, such as subsidies allocated for job creation and training.

The concept of GDP at basic prices also differs from GDP at market prices, but in this case the difference concerns the taxes and subsidies on the products themselves, such as sales taxes, fuel taxes, duties and taxes on imports, excise taxes on tobacco and alcohol products and subsidies paid on agricultural commodities, transportation services and energy. Whereas production at basic prices excludes taxes and subsidies on products, GDP at market prices includes taxes net of subsidies on products.

Source: http://www.statcan.gc.ca/nea-cen/faq-foq/gdp-pib-eng.htm

APPENDIX C: MATHEMATICAL DERIVATIONS

C1. Relative demand of capital and labor

C1.1 COST-MINIMIZING PROBLEM

The production function of value added is given by equation 3:

$$3. \qquad VA_j = B_j^{VA}\left[\beta_j^{VA} LDC_j^{-\rho_j^{VA}} + \left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}}\right]^{-\frac{1}{\rho_j^{VA}}}$$

The producer's problem is to minimize the cost of value added

$$99. \qquad WC_j LDC_j + RC_j KDC_j$$

subject to 3 and the constraint $VA_j = \overline{VA_j}$. Form the Lagrangian

$$100. \qquad \Lambda = WC_j LDC_j + RC_j KDC_j - \lambda\left(VA_j - \overline{VA_j}\right)$$

$$101. \qquad \Lambda = WC_j LDC_j + RC_j KDC_j - \lambda\left(B_j^{VA}\left[\beta_j^{VA} LDC_j^{-\rho_j^{VA}} + \left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}}\right]^{-\frac{1}{\rho_j^{VA}}} - \overline{VA_j}\right)$$

First order conditions are:

$$102. \qquad \frac{\partial \Lambda}{\partial \lambda} = -\left(VA_j - \overline{VA_j}\right) = 0$$

$$103. \qquad \frac{\partial \Lambda}{\partial LDC_j} = WC_j - \lambda\left(\frac{\partial VA_j}{\partial LDC_j}\right) = 0$$

$$104. \qquad \frac{\partial \Lambda}{\partial KDC_j} = RC_j - \lambda\left(\frac{\partial VA_j}{\partial KDC_j}\right) = 0$$

with

105. $$\frac{\partial VA_j}{\partial LDC_j} = \frac{\partial}{\partial LDC_j} B_j^{VA}\left[\beta_j^{VA} LDC_j^{-\rho_j^{VA}} + \left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}}\right]^{-\frac{1}{\rho_j^{VA}}}$$

106. $$\frac{\partial VA_j}{\partial LDC_j} = B_j^{VA}\left(\frac{-1}{\rho_j^{VA}}\right)\left[\beta_j^{VA} LDC_j^{-\rho_j^{VA}} + \left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}}\right]^{\frac{-1}{\rho_j^{VA}}-1}\left(-\rho_j^{VA}\beta_j^{VA} LDC_j^{-\rho_j^{VA}-1}\right)$$

107. $$\frac{\partial VA_j}{\partial LDC_j} = B_j^{VA}\left[\beta_j^{VA} LDC_j^{-\rho_j^{VA}} + \left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}}\right]^{\frac{-1}{\rho_j^{VA}}-1}\left(\beta_j^{VA} LDC_j^{-\rho_j^{VA}-1}\right)$$

Likewise,

108. $$\frac{\partial VA_j}{\partial KDC_j} = B_j^{VA}\left[\beta_j^{VA} LDC_j^{-\rho_j^{VA}} + \left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}}\right]^{\frac{-1}{\rho_j^{VA}}-1}\left(\left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}-1}\right)$$

C1.2 RELATIVE DEMAND

It follows from equations 103, 104, 107, and 108 that

109. $$\frac{WC_j}{RC_j} = \frac{B_j^{VA}\left[\beta_j^{VA} LDC_j^{-\rho_j^{VA}} + \left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}}\right]^{\frac{-1}{\rho_j^{VA}}-1}\left(\beta_j^{VA} LDC_j^{-\rho_j^{VA}-1}\right)}{B_j^{VA}\left[\beta_j^{VA} LDC_j^{-\rho_j^{VA}} + \left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}}\right]^{\frac{-1}{\rho_j^{VA}}-1}\left(\left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}-1}\right)}$$

110. $$\frac{WC_j}{RC_j} = \frac{\beta_j^{VA} LDC_j^{-\rho_j^{VA}-1}}{\left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}-1}}$$

111. $$\left(\frac{LDC_j}{KDC_j}\right)^{-\rho_j^{VA}-1} = \frac{1-\beta_j^{VA}}{\beta_j^{VA}}\frac{WC_j}{RC_j}$$

112. $$\frac{LDC_j}{KDC_j} = \left(\frac{1-\beta_j^{VA}}{\beta_j^{VA}} \frac{WC_j}{RC_j} \right)^{-\frac{1}{\rho_j^{VA}+1}}$$

Substituting $\sigma_j^{VA} = \frac{1}{\rho_j^{VA}+1}$, equation 4 follows

4. $$LDC_j = \left[\frac{\beta_j^{VA}}{1-\beta_j^{VA}} \frac{RC_j}{WC_j} \right]^{\sigma_j^{VA}} KDC_j$$

C1.3 Elasticity of substitution

In C1.2 above, nothing was said about the interpretation of $\sigma_j^{VA} = \frac{1}{\rho_j^{VA}+1}$. Here we show that σ_j^{VA} is indeed the elasticity of substitution. The elasticity of substitution between composite labor and composite capital is defined as

113. $$\frac{\partial \ln\left(\frac{LDC_j}{KDC_j} \right)}{\partial \ln\left(\frac{\partial VA_j \big/ \partial KDC_j}{\partial VA_j \big/ \partial LDC_j} \right)} = \frac{\left(\frac{\partial VA_j \big/ \partial KDC_j}{\partial VA_j \big/ \partial LDC_j} \right)}{\left(\frac{LDC_j}{KDC_j} \right)} \frac{\partial\left(\frac{LDC_j}{KDC_j} \right)}{\partial\left(\frac{\partial VA_j \big/ \partial KDC_j}{\partial VA_j \big/ \partial LDC_j} \right)}$$

where $\frac{\partial VA_j \big/ \partial KDC_j}{\partial VA_j \big/ \partial LDC_j}$ is the marginal rate of substitution between composite labor and composite capital ($MRS_{LDC,KDC}^{j}$):

114. $$MRS_{LDC,KDC}^{j} = -\frac{d\ LDC_j}{d\ kDC_j} = \frac{\partial VA_j \big/ \partial KDC_j}{\partial VA_j \big/ \partial LDC_j}$$

Substitute from equations 107 and 108,

115. $$MRS_{LDC,KDC}^{j} = \frac{B_j^{VA}\left[\beta_j^{VA} LDC_j^{-\rho_j^{VA}} + \left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}}\right]^{\frac{-1}{\rho_j^{VA}}-1}\left(\left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}-1}\right)}{B_j^{VA}\left[\beta_j^{VA} LDC_j^{-\rho_j^{VA}} + \left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}}\right]^{\frac{-1}{\rho_j^{VA}}-1}\left(\beta_j^{VA} LDC_j^{-\rho_j^{VA}-1}\right)}$$

116. $$MRS_{LDC,KDC}^{j} = \frac{\left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}-1}}{\beta_j^{VA} LDC_j^{-\rho_j^{VA}-1}}$$

The elasticity of substitution is therefore

117. $$\frac{\partial \ln\left(\frac{LDC_j}{KDC_j}\right)}{\partial \ln\left(\frac{\partial VA_j / \partial KDC_j}{\partial VA_j / \partial LDC_j}\right)} = \frac{\left(\frac{\left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}-1}}{\beta_j^{VA} LDC_j^{-\rho_j^{VA}-1}}\right)}{\left(\frac{LDC_j}{KDC_j}\right)} \frac{\partial\left(\frac{LDC_j}{KDC_j}\right)}{\partial\left(\frac{\left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}-1}}{\beta_j^{VA} LDC_j^{-\rho_j^{VA}-1}}\right)}$$

118. $$\frac{\partial \ln\left(\frac{LDC_j}{KDC_j}\right)}{\partial \ln\left(\frac{\partial VA_j / \partial KDC_j}{\partial VA_j / \partial LDC_j}\right)} = \frac{\left(\frac{1-\beta_j^{VA}}{\beta_j^{VA}}\right)\left(\frac{KDC_j}{LDC_j}\right)^{-\rho_j^{VA}-1}}{\left(\frac{LDC_j}{KDC_j}\right)} \left[\frac{\partial\left(\left(\frac{1-\beta_j^{VA}}{\beta_j^{VA}}\right)\left(\frac{KDC_j}{LDC_j}\right)^{-\rho_j^{VA}-1}\right)}{\partial\left(\frac{LDC_j}{KDC_j}\right)}\right]^{-1}$$

Noting that $\left(\frac{KDC_j}{LDC_j}\right)^{-\rho_j^{VA}-1} = \left(\frac{LDC_j}{KDC_j}\right)^{\rho_j^{VA}+1}$, simplify as

119. $$\frac{\partial \ln\left(\frac{LDC_j}{KDC_j}\right)}{\partial \ln\left(\frac{\partial VA_j / \partial KDC_j}{\partial VA_j / \partial LDC_j}\right)} = \frac{\left(\frac{1-\beta_j^{VA}}{\beta_j^{VA}}\right)\left(\frac{LDC_j}{KDC_j}\right)^{\rho_j^{VA}+1}}{\left(\frac{LDC_j}{KDC_j}\right)} \left[\frac{\partial\left(\left(\frac{1-\beta_j^{VA}}{\beta_j^{VA}}\right)\left(\frac{LDC_j}{KDC_j}\right)^{\rho_j^{VA}+1}\right)}{\partial\left(\frac{LDC_j}{KDC_j}\right)}\right]^{-1}$$

120. $$\frac{\partial \ln\left(\frac{LDC_j}{KDC_j}\right)}{\partial \ln\left(\frac{\partial VA_j / \partial KDC_j}{\partial VA_j / \partial LDC_j}\right)} = \frac{\left(\frac{1-\beta_j^{VA}}{\beta_j^{VA}}\right)\left(\frac{LDC_j}{KDC_j}\right)^{\rho_j^{VA}+1}}{\left(\frac{LDC_j}{KDC_j}\right)} \left(\frac{1-\beta_j^{VA}}{\beta_j^{VA}}\right)^{-1} \left[\left(\rho_j^{VA}+1\right)\left(\frac{LDC_j}{KDC_j}\right)^{\rho_j^{VA}}\right]^{-1}$$

121. $$\frac{\partial \ln\left(\frac{LDC_j}{KDC_j}\right)}{\partial \ln\left(\frac{\partial VA_j / \partial KDC_j}{\partial VA_j / \partial LDC_j}\right)} = \left(\frac{LDC_j}{KDC_j}\right)^{\rho_j^{VA}} \left[\left(\rho_j^{VA}+1\right)\left(\frac{LDC_j}{KDC_j}\right)^{\rho_j^{VA}}\right]^{-1} = \frac{1}{\rho_j^{VA}+1}$$

Hence $\sigma_j^{VA} = \frac{1}{\rho_j^{VA}+1}$ is indeed the elasticity of substitution.

C2. Labor demand by category

C2.1 WAGE BILL MINIMIZING PROBLEM

The aggregator function of composite labor is given by equation 5:

5. $$LDC_j = B_j^{LD}\left[\sum_l \beta_{l,j}^{LD}\ LD_{l,j}^{-\rho_j^{LD}}\right]^{-\frac{1}{\rho_j^{LD}}}$$

The producer's problem is to minimize the wage bill $\sum_l WTI_{l,j} LD_{l,j}$ subject to equation 5 and the constraint $LDC_j = \overline{LDC_j}$. Form the Lagrangian

122. $\Lambda = \sum_{l} WTI_{l,j} LD_{l,j} - \lambda \left(LDC_J - \overline{LDC_j} \right)$

123. $\Lambda = \sum_{l} WTI_{l,j} LD_{l,j} - \lambda \left\{ B_j^{LD} \left[\sum_{l} \beta_{l,j}^{LD} \; LD_{l,j}^{-\rho_j^{LD}} \right]^{-\frac{1}{\rho_j^{LD}}} - \overline{LDC_j} \right\}$

First order conditions are:

124. $\dfrac{\partial \Lambda}{\partial \lambda} = -\left(LDC_J - \overline{LDC_j} \right) = 0$

125. $\dfrac{\partial \Lambda}{\partial LD_{lj,j}} = WTI_{lj,j} - \lambda \dfrac{\partial LDC_j}{\partial LD_{lj,j}} = 0$

with

126. $\dfrac{\partial LDC_j}{\partial LD_{lj,j}} = \dfrac{\partial}{\partial LD_{lj,j}} B_j^{LD} \left[\sum_{l} \beta_{l,j}^{LD} \; LD_{l,j}^{-\rho_j^{LD}} \right]^{-\frac{1}{\rho_j^{LD}}}$

127. $\dfrac{\partial LDC_j}{\partial LD_{lj,j}} = B_j^{LD} \left(-\dfrac{1}{\rho_j^{LD}} \right) \left[\sum_{l} \beta_{l,j}^{LD} \; LD_{l,j}^{-\rho_j^{LD}} \right]^{-\frac{1}{\rho_j^{LD}}-1} \left(-\rho_j^{LD} \beta_{lj,j}^{LD} \; LD_{lj,j}^{-\rho_j^{LD}-1} \right)$

128. $\dfrac{\partial LDC_j}{\partial LD_{lj,j}} = B_j^{LD} \left[\sum_{l} \beta_{l,j}^{LD} \; LD_{l,j}^{-\rho_j^{LD}} \right]^{-\frac{1}{\rho_j^{LD}}-1} \left(\beta_{lj,j}^{LD} \; LD_{lj,j}^{-\rho_j^{LD}-1} \right)$

C2.2 RELATIVE LABOR DEMAND

It follows from equations 125 and 128 that

129. $$\frac{WTI_{li,j}}{WTI_{lj,j}} = \frac{B_j^{LD}\left[\sum_l \beta_{l,j}^{LD}\ LD_{l,j}^{-\rho_j^{LD}}\right]^{-\frac{1}{\rho_j^{LD}}-1}\left(\beta_{li,j}^{LD}\ LD_{li,j}^{-\rho_j^{LD}-1}\right)}{B_j^{LD}\left[\sum_l \beta_{l,j}^{LD}\ LD_{l,j}^{-\rho_j^{LD}}\right]^{-\frac{1}{\rho_j^{LD}}-1}\left(\beta_{lj,j}^{LD}\ LD_{lj,j}^{-\rho_j^{LD}-1}\right)}$$

130. $$\frac{WTI_{li,j}}{WTI_{lj,j}} = \frac{\beta_{li,j}^{LD}\ LD_{li,j}^{-\rho_j^{LD}-1}}{\beta_{lj,j}^{LD}\ LD_{lj,j}^{-\rho_j^{LD}-1}}$$

131. $$\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)^{-\rho_j^{LD}-1} = \frac{\beta_{lj,j}^{LD}\ WTI_{li,j}}{\beta_{li,j}^{LD}\ WTI_{lj,j}}$$

132. $$\frac{LD_{li,j}}{LD_{lj,j}} = \left(\frac{\beta_{lj,j}^{LD}\ WTI_{li,j}}{\beta_{li,j}^{LD}\ WTI_{lj,j}}\right)^{\frac{1}{-\rho_j^{LD}-1}}$$

Substituting $\sigma_j^{LD} = \frac{1}{\rho_j^{LD}+1}$, it follows that

133. $$\frac{LD_{li,j}}{LD_{lj,j}} = \left(\frac{\beta_{lj,j}^{LD}\ WTI_{li,j}}{\beta_{li,j}^{LD}\ WTI_{lj,j}}\right)^{-\sigma_j^{LD}} = \left(\frac{\beta_{li,j}^{LD}\ WTI_{lj,j}}{\beta_{lj,j}^{LD}\ WTI_{li,j}}\right)^{\sigma_j^{LD}}$$

134. $$LD_{li,j} = \left(\frac{\beta_{li,j}^{LD}\ WTI_{lj,j}}{\beta_{lj,j}^{LD}\ WTI_{li,j}}\right)^{\sigma_j^{LD}} LD_{lj,j}$$

C2.3 UNIT COST OF COMPOSITE LABOR

Remembering that $\sigma_j^{LD} = \frac{1}{\rho_j^{LD}+1}$, and substituting equation 134 into 5,

$$135.\quad LDC_j = B_j^{LD}\left[\sum_l \beta_{l,j}^{LD}\left(\left(\frac{\beta_{l,j}^{LD}\,WTI_{lj,j}}{\beta_{lj,j}^{LD}\,WTI_{l,j}}\right)^{\frac{1}{\rho_j^{LD}+1}} LD_{lj,j}\right)^{-\rho_j^{LD}}\right]^{-\frac{1}{\rho_j^{LD}}}$$

$$136.\quad LDC_j = B_j^{LD}\left[\sum_l \beta_{l,j}^{LD}\left(\frac{\beta_{l,j}^{LD}\,WTI_{lj,j}}{\beta_{lj,j}^{LD}\,WTI_{l,j}}\right)^{-\frac{\rho_j^{LD}}{\rho_j^{LD}+1}} LD_{lj,j}^{-\rho_j^{LD}}\right]^{-\frac{1}{\rho_j^{LD}}}$$

$$137.\quad LDC_j = LD_{lj,j}B_j^{LD}\left[\sum_l \left(\beta_{l,j}^{LD}\right)^{1-\frac{\rho_j^{LD}}{\rho_j^{LD}+1}}\left(WTI_{l,j}\right)^{\frac{\rho_j^{LD}}{\rho_j^{LD}+1}}\left(\frac{WTI_{lj,j}}{\beta_{lj,j}^{LD}}\right)^{-\frac{\rho_j^{LD}}{\rho_j^{LD}+1}}\right]^{-\frac{1}{\rho_j^{LD}}}$$

$$138.\quad LDC_j = LD_{lj,j}\left(\frac{WTI_{lj,j}}{\beta_{lj,j}^{LD}}\right)^{\frac{1}{\rho_j^{LD}+1}} B_j^{LD}\left[\sum_l \left(\beta_{l,j}^{LD}\right)^{\frac{1}{\rho_j^{LD}+1}}\left(WTI_{l,j}\right)^{\frac{\rho_j^{LD}}{\rho_j^{LD}+1}}\right]^{-\frac{1}{\rho_j^{LD}}}$$

$$139.\quad LD_{lj,j} = \frac{LDC_j}{B_j^{LD}}\left[\sum_l \left(\beta_{l,j}^{LD}\right)^{\frac{1}{\rho_j^{LD}+1}}\left(WTI_{l,j}\right)^{\frac{\rho_j^{LD}}{\rho_j^{LD}+1}}\right]^{\frac{1}{\rho_j^{LD}}}\left(\frac{WTI_{lj,j}}{\beta_{lj,j}^{LD}}\right)^{-\frac{1}{\rho_j^{LD}+1}}$$

Now, the unit cost of composite labor is defined by equation 72

$$72.\quad WC_j = \frac{\sum_l WTI_{l,j}LD_{l,j}}{LDC_j}$$

Substituting $LD_{lj,j}$ from equation 139 for $LD_{l,j}$ in equation 72 yields

140. $$WC_j = \frac{1}{B_j^{LD}}\left[\sum_l \left(\beta_{l,j}^{LD}\right)^{\frac{1}{\rho_j^{LD}+1}} \left(WTI_{l,j}\right)^{\frac{\rho_j^{LD}}{\rho_j^{LD}+1}}\right]^{\frac{1}{\rho_j^{LD}}}\left[\sum_{lj} WTI_{lj,j}\left(\frac{WTI_{lj,j}}{\beta_{lj,j}^{LD}}\right)^{-\frac{1}{\rho_j^{LD}+1}}\right]$$

141. $$WC_j = \frac{1}{B_j^{LD}}\left[\sum_l \left(\beta_{l,j}^{LD}\right)^{\frac{1}{\rho_j^{LD}+1}} \left(WTI_{l,j}\right)^{\frac{\rho_j^{LD}}{\rho_j^{LD}+1}}\right]^{\frac{1}{\rho_j^{LD}}}\left[\sum_{lj} \left(\beta_{lj,j}^{LD}\right)^{\frac{1}{\rho_j^{LD}+1}} \left(WTI_{lj,j}\right)^{1-\frac{1}{\rho_j^{LD}+1}}\right]$$

where

142. $$\left[\sum_l \left(\beta_{l,j}^{LD}\right)^{\frac{1}{\rho_j^{LD}+1}} \left(WTI_{l,j}\right)^{\frac{\rho_j^{LD}}{\rho_j^{LD}+1}}\right] \equiv \left[\sum_{lj} \left(\beta_{lj,j}^{LD}\right)^{\frac{1}{\rho_j^{LD}+1}} \left(WTI_{lj,j}\right)^{\frac{\rho_j^{LD}}{\rho_j^{LD}+1}}\right]$$

Hence,

143. $$WC_j = \frac{1}{B_j^{LD}}\left[\sum_l \left(\beta_{l,j}^{LD}\right)^{\frac{1}{\rho_j^{LD}+1}} \left(WTI_{l,j}\right)^{\frac{\rho_j^{LD}}{\rho_j^{LD}+1}}\right]^{1+\frac{1}{\rho_j^{LD}}}$$

144. $$WC_j = \frac{1}{B_j^{LD}}\left[\sum_l \left(\beta_{l,j}^{LD}\right)^{\frac{1}{\rho_j^{LD}+1}} \left(WTI_{l,j}\right)^{\frac{\rho_j^{LD}}{\rho_j^{LD}+1}}\right]^{\frac{\rho_j^{LD}+1}{\rho_j^{LD}}}$$

Using $\sigma_j^{LD} = \frac{1}{\rho_j^{LD}+1}$,

145. $$WC_j = \frac{1}{B_j^{LD}}\left[\sum_l \left(\beta_{l,j}^{LD}\right)^{\sigma_j^{LD}} \left(WTI_{l,j}\right)^{1-\sigma_j^{LD}}\right]^{\frac{1}{1-\sigma_j^{LD}}}$$

C2.4 LABOR DEMAND FOR A SINGLE CATEGORY IN TERMS OF RELATIVE WAGE RATES

From equation 145, it follows that

$$146.\quad \left[\sum_l \left(\beta_{l,j}^{LD}\right)^{\sigma_j^{LD}} \left(WTI_{l,j}\right)^{1-\sigma_j^{LD}}\right]^{\frac{\sigma_j^{LD}}{1-\sigma_j^{LD}}} = \left(B_j^{LD}\right)^{\sigma_j^{LD}} WC_j^{\sigma_j^{LD}}$$

Substituting equation 146 into equation 139 yields

$$147.\quad LD_{lj,j} = \frac{LDC_j}{B_j^{LD}} \left(B_j^{LD}\right)^{\sigma_j^{LD}} WC_j^{\sigma_j^{LD}} \left(\frac{WTI_{lj,j}}{\beta_{lj,j}^{LD}}\right)^{-\sigma_j^{LD}}$$

After rearranging, equation 6 follows.

$$6.\quad LD_{l,j} = \left[\frac{\beta_{l,j}^{LD}\ WC_j}{WTI_{l,j}}\right]^{\sigma_j^{LD}} \left(B_j^{LD}\right)^{\sigma_j^{LD}-1} LDC_j$$

C2.6 ELASTICITY OF SUBSTITUTION

In C2.2 above, nothing was said about the interpretation of $\sigma_j^{LD} = \frac{1}{\rho_j^{LD}+1}$. Here we show that σ_j^{LD} is indeed the elasticity of substitution. The elasticity of substitution between type *li* and type *lj* labor is defined as

$$148.\quad \frac{\partial \ln\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)}{\partial \ln\left(\frac{\partial LDC_j / \partial LD_{lj,j}}{\partial LDC_j / \partial LD_{li,j}}\right)} = \frac{\left(\frac{\partial LDC_j / \partial LD_{lj,j}}{\partial LDC_j / \partial LD_{li,j}}\right)}{\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)} \frac{\partial\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)}{\partial\left(\frac{\partial LDC_j / \partial LD_{lj,j}}{\partial LDC_j / \partial LD_{li,j}}\right)}$$

where $\frac{\partial LDC_j / \partial LD_{lj,j}}{\partial LDC_j / \partial LD_{li,j}}$ is the marginal rate of substitution between *li* and *lj* in industry *j* ($MRS_{li,lj}^j$):

$$149.\quad MRS_{li,lj}^j = -\frac{dLD_{li,j}}{dLD_{lj,j}} = \frac{\partial LDC_j / \partial LD_{lj,j}}{\partial LDC_j / \partial LD_{li,j}}$$

Substitute from equation 128,

150. $$MRS_{li,lj}^{j}=-\frac{dLD_{li,j}}{dLD_{lj,j}}=\frac{B_j^{LD}\left[\sum_l \beta_{l,j}^{LD}\ LD_{l,j}^{-\rho_j^{LD}}\right]^{-\frac{1}{\rho_j^{LD}}-1}\left(\beta_{lj,j}^{LD}\ LD_{lj,j}^{-\rho_j^{LD}-1}\right)}{B_j^{LD}\left[\sum_l \beta_{l,j}^{LD}\ LD_{l,j}^{-\rho_j^{LD}}\right]^{-\frac{1}{\rho_j^{LD}}-1}\left(\beta_{li,j}^{LD}\ LD_{li,j}^{-\rho_j^{LD}-1}\right)}$$

151. $$MRS_{li,lj}^{j}=-\frac{dLD_{li,j}}{dLD_{lj,j}}=\frac{\left(\beta_{lj,j}^{LD}\ LD_{lj,j}^{-\rho_j^{LD}-1}\right)}{\left(\beta_{li,j}^{LD}\ LD_{li,j}^{-\rho_j^{LD}-1}\right)}=\frac{\beta_{lj,j}^{LD}}{\beta_{li,j}^{LD}}\left(\frac{LD_{lj,j}}{LD_{li,j}}\right)^{-\rho_j^{LD}-1}$$

The elasticity of substitution is therefore

152. $$\frac{\partial\ln\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)}{\partial\ln\left(\frac{\partial LDC_j/\partial LD_{lj,j}}{\partial LDC_j/\partial LD_{li,j}}\right)}=\frac{\frac{\beta_{lj,j}^{LD}}{\beta_{li,j}^{LD}}\left(\frac{LD_{lj,j}}{LD_{li,j}}\right)^{-\rho_j^{LD}-1}}{\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)}\frac{\partial\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)}{\partial\left(\frac{\beta_{lj,j}^{LD}}{\beta_{li,j}^{LD}}\left(\frac{LD_{lj,j}}{LD_{li,j}}\right)^{-\rho_j^{LD}-1}\right)}$$

153. $$\frac{\partial\ln\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)}{\partial\ln\left(\frac{\partial LDC_j/\partial LD_{lj,j}}{\partial LDC_j/\partial LD_{li,j}}\right)}=\frac{\frac{\beta_{lj,j}^{LD}}{\beta_{li,j}^{LD}}\left(\frac{LD_{lj,j}}{LD_{li,j}}\right)^{-\rho_j^{LD}-1}}{\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)}\left[\frac{\partial\left(\frac{\beta_{lj,j}^{LD}}{\beta_{li,j}^{LD}}\left(\frac{LD_{lj,j}}{LD_{li,j}}\right)^{-\rho_j^{LD}-1}\right)}{\partial\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)}\right]^{-1}$$

Noting that $\left(\frac{LD_{lj,j}}{LD_{li,j}}\right)^{-\rho_j^{LD}-1}=\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)^{\rho_j^{LD}+1}$, simplify as

154. $$\frac{\partial \ln\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)}{\partial \ln\left(\frac{\partial LDC_j / \partial LD_{lj,j}}{\partial LDC_j / \partial LD_{li,j}}\right)} = \frac{\frac{\beta_{lj,j}^{LD}}{\beta_{li,j}^{LD}}\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)^{\rho_j^{LD}+1}}{\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)} \left(\frac{\beta_{lj,j}^{LD}}{\beta_{li,j}^{LD}}\right)^{-1} \left[\frac{\partial\left(\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)^{\rho_j^{LD}+1}\right)}{\partial\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)}\right]^{-1}$$

155. $$\frac{\partial \ln\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)}{\partial \ln\left(\frac{\partial LDC_j / \partial LD_{lj,j}}{\partial LDC_j / \partial LD_{li,j}}\right)} = \left(\frac{LD_{li,j}}{LD_{lj,j}}\right)^{\rho_j^{LD}} \left[\left(\rho_j^{LD}+1\right)\left(\frac{LD_{li,j}}{LD_{lj,j}}\right)^{\rho_j^{LD}}\right]^{-1} = \frac{1}{\rho_j^{LD}+1}$$

Hence $\sigma_j^{LD} = \frac{1}{\rho_j^{LD}+1}$ is indeed the elasticity of substitution.

C3. Demand for capital by category

The CES aggregator function of composite capital has the same form as the aggregator function of composite labor. It is straightforward to rewrite the developments of C2 for the demand of type *k* capital by industry *j*.

C4. Stone-Geary utility and the demand for consumer goods

The utility function of the representative agent of type *h* households is a Stone-Geary utility function:

156. $$U_h = \prod_i \left(C_{i,h} - C_{i,h}^{MIN}\right)^{\gamma_{i,h}^{LES}}$$, where

157. $$\sum_i \gamma_{i,h}^{LES} = 1$$

Utility function 157 is equivalent to

158. $$\ln U_h = \sum_i \gamma_{i,h}^{LES} \ln\left(C_{i,h} - C_{i,h}^{MIN}\right)$$

The representative household maximizes utility subject to the budget constraint

159. $$\sum_i PC_i C_{i,h} = CTH_h$$

Form the Lagrangian:

160. $\Lambda = \sum_i \gamma_{i,h}^{LES} \ln\left(C_{i,h} - C_{i,h}^{MIN}\right) - \lambda\left(\sum_i PC_i C_{i,h} - CTH_h\right)$

The first-order conditions are:

161. $\frac{\partial \Lambda}{\partial \lambda} = -\left(\sum_i PC_i C_{i,h} - CTH_h\right) = 0$

162. $\frac{\partial \Lambda}{\partial C_{i,h}} = \frac{\gamma_{i,h}^{LES}}{\left(C_{i,h} - C_{i,h}^{MIN}\right)} - \lambda\ PC_i = 0$

First-order condition equation 162 is equivalent to

163. $\lambda\ PC_i \left(C_{i,h} - C_{i,h}^{MIN}\right) = \gamma_{i,h}^{LES}$

Summing equation 163 over *i*, remembering equation 157, yields

164. $\lambda \sum_i PC_i \left(C_{i,h} - C_{i,h}^{MIN}\right) = \sum_i \gamma_{i,h}^{LES} = 1$

165. $\sum_i PC_i \left(C_{i,h} - C_{i,h}^{MIN}\right) = \frac{1}{\lambda}$

And, given first-order condition equation 161,

166. $CTH_h - \sum_i PC_i C_{i,h}^{MIN} = \frac{1}{\lambda}$

Substituting equation 166 into equation 165 and rearranging, one obtains demand function equation 53:

53. $C_{i,h} PC_i = C_{i,h}^{MIN} PC_i + \gamma_{i,h}^{LES}\left(CTH_h - \sum_{ij} C_{ij,h}^{MIN} PC_{ij}\right)$

It should be mentioned in passing that methods of calibration of the Linear Expenditure System parameters often make use of the Frisch parameter, given by: $-\lambda\ CTH_h = -\frac{CTH_h}{CTH_h - \sum_i PC_i C_{i,h}^{MIN}}$.

C5. Allocation of aggregate output to product supplies

C5.1 SALES REVENUE MAXIMIZING PROBLEM

The aggregator function of production sold on the domestic market and exported is given by equation 59:

59. $$XST_j = B_j^{XT}\left[\sum_i \beta_{j,i}^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}}$$

The producer's problem is to maximize sales revenue $\sum_i P_{j,i} XS_{j,i}$ subject to equation 59 and the constraint $XST_j = \overline{XST_j}$. Form the Lagrangian

167. $$\Lambda = \sum_i P_{j,i} XS_{j,i} - \lambda\left(XST_j - \overline{XST_j}\right)$$

168. $$\Lambda = \sum_i P_{j,i} XS_{j,i} - \lambda\left(B_j^{XT}\left[\sum_i \beta_{j,i}^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}} - \overline{XST_j}\right)$$

First order conditions are:

169. $$\frac{\partial\Lambda}{\partial\lambda} = -\left(XST_j - \overline{XST_j}\right) = 0$$

170. $$\frac{\partial\Lambda}{\partial XS_{j,i}} = P_{j,i} - \lambda\frac{\partial XST_j}{\partial XS_{j,i}}$$

with

171. $$\frac{\partial XST_j}{\partial XS_{j,ij}} = \frac{\partial}{\partial XS_{j,ij}} B_j^{XT}\left[\sum_i \beta_{j,i}^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}}$$

172. $$\frac{\partial XST_j}{\partial XS_{j,ij}} = B_j^{XT}\left(\frac{1}{\rho_j^{XT}}\right)\left[\sum_i \beta_{j,i}^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}-1}\left(\rho_j^{XT}\beta_{j,ij}^{XT} XS_{j,ij}^{\rho_j^{XT}-1}\right)$$

173. $$\frac{\partial XST_j}{\partial XS_{j,ij}} = B_j^{XT}\left[\sum_i \beta_j^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}-1}\left(\beta_{j,ij}^{XT} XS_{j,ij}^{\rho_j^{XT}-1}\right)$$

C5.2 RELATIVE SUPPLY OF PRODUCTS

It follows from equations 170 and 173 that

174. $$\frac{P_{j,ii}}{P_{j,ij}} = \frac{B_j^{XT}\left[\sum_i \beta_j^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}-1}\left(\beta_{j,ii}^{XT} XS_{j,ii}^{\rho_j^{XT}-1}\right)}{B_j^{XT}\left[\sum_i \beta_j^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}-1}\left(\beta_{j,ij}^{XT} XS_{j,ij}^{\rho_j^{XT}-1}\right)}$$

175. $$\frac{P_{j,ii}}{P_{j,ij}} = \frac{\left(\beta_{j,ii}^{XT} XS_{j,ii}^{\rho_j^{XT}-1}\right)}{\left(\beta_{j,ij}^{XT} XS_{j,ij}^{\rho_j^{XT}-1}\right)} = \frac{\beta_{j,ii}^{XT}}{\beta_{j,ij}^{XT}}\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)^{\rho_j^{XT}-1}$$

176. $$\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)^{\rho_j^{XT}-1} = \frac{\beta_{j,ij}^{XT}}{\beta_{j,ii}^{XT}}\frac{P_{j,ii}}{P_{j,ij}}$$

177. $$\frac{XS_{j,ii}}{XS_{j,ij}} = \left(\frac{\beta_{j,ij}^{XT}}{\beta_{j,ii}^{XT}}\frac{P_{j,ii}}{P_{j,ij}}\right)^{\frac{1}{\rho_j^{XT}-1}}$$

Substituting $\sigma_j^{XT} = \dfrac{1}{\rho_j^{XT}-1}$, it follows that:

178. $$\frac{XS_{j,ii}}{XS_{j,ij}} = \left(\frac{\beta_{j,ij}^{XT}}{\beta_{j,ii}^{XT}}\frac{P_{j,ii}}{P_{j,ij}}\right)^{\sigma_j^{XT}}$$

179. $$XS_{j,ii} = \left(\frac{\beta_{j,ij}^{XT}}{\beta_{j,ii}^{XT}}\frac{P_{j,ii}}{P_{j,ij}}\right)^{\sigma_j^{XT}} XS_{j,ij}$$

C5.3 PRICE OF AGGREGATE OUTPUT

Remembering that $\sigma_j^{XT} = \dfrac{1}{\rho_j^{XT} - 1}$, and substituting equation 179 into equation 59,

180. $$XST_j = B_j^{XT}\left[\sum_i \beta_{j,i}^{XT}\left(\left(\frac{\beta_{j,ij}^{XT}}{\beta_{j,i}^{XT}}\frac{P_{j,i}}{P_{j,ij}}\right)^{\frac{1}{\rho_j^{XT}-1}} XS_{j,ij}\right)^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}}$$

181. $$XST_j = B_j^{XT}\left[\sum_i \beta_{j,i}^{XT}\left(\frac{\beta_{j,ij}^{XT}}{\beta_{j,i}^{XT}}\frac{P_{j,i}}{P_{j,ij}}\right)^{\frac{\rho_j^{XT}}{\rho_j^{XT}-1}} XS_{j,ij}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}}$$

182. $$XST_j = B_j^{XT} XS_{j,ij}\left[\sum_i \left(\beta_{j,i}^{XT}\right)^{1-\frac{\rho_j^{XT}}{\rho_j^{XT}-1}} \left(P_{j,i}\right)^{\frac{\rho_j^{XT}}{\rho_j^{XT}-1}}\left(\frac{\beta_{j,ij}^{XT}}{P_{j,ij}}\right)^{\frac{\rho_j^{XT}}{\rho_j^{XT}-1}}\right]^{\frac{1}{\rho_j^{XT}}}$$

183. $$XST_j = B_j^{XT} XS_{j,ij}\left[\sum_i \left(\beta_{j,i}^{XT}\right)^{-\frac{1}{\rho_j^{XT}-1}} \left(P_{j,i}\right)^{\frac{\rho_j^{XT}}{\rho_j^{XT}-1}}\left(\frac{P_{j,ij}}{\beta_{j,ij}^{XT}}\right)^{-\frac{\rho_j^{XT}}{\rho_j^{XT}-1}}\right]^{\frac{1}{\rho_j^{XT}}}$$

$$184. \quad XST_j = B_j^{XT} XS_{j,ij} \left(\frac{P_{j,ij}}{\beta_{j,ij}^{XT}} \right)^{-\frac{1}{\rho_j^{XT}-1}} \left[\sum_i \left(\beta_{j,i}^{XT} \right)^{1-\frac{\rho_j^{XT}}{\rho_j^{XT}-1}} \left(P_{j,i} \right)^{\frac{\rho_j^{XT}}{\rho_j^{XT}-1}} \right]^{\frac{1}{\rho_j^{XT}}}$$

$$185. \quad XS_{j,ij} = \frac{XST_j}{B_j^{XT}} \left[\sum_i \left(\beta_{j,i}^{XT} \right)^{-\frac{1}{\rho_j^{XT}-1}} \left(P_{j,i} \right)^{\frac{\rho_j^{XT}}{\rho_j^{XT}-1}} \right]^{-\frac{1}{\rho_j^{XT}}} \left(\frac{P_{j,ij}}{\beta_{j,ij}^{XT}} \right)^{\frac{1}{\rho_j^{XT}-1}}$$

Now, the price of the aggregate output is defined by equation 77

$$77. \quad PT_j = \frac{\sum_i P_{j,i} \; XS_{j,i}}{XST_j}$$

Substituting $XS_{j,ij}$ from equation 185 for $XS_{j,i}$ in equation 77 yields

$$186. \quad PT_j = \frac{1}{B_j^{XT}} \left[\sum_i \left(\beta_{j,i}^{XT} \right)^{-\frac{1}{\rho_j^{XT}-1}} \left(P_{j,i} \right)^{\frac{\rho_j^{XT}}{\rho_j^{XT}-1}} \right]^{-\frac{1}{\rho_j^{XT}}} \left[\sum_{ij} P_{j,ij} \left(\frac{P_{j,ij}}{\beta_{j,ij}^{XT}} \right)^{\frac{1}{\rho_j^{XT}-1}} \right]$$

$$187. \quad PT_j = \frac{1}{B_j^{XT}} \left[\sum_i \left(\beta_{j,i}^{XT} \right)^{-\frac{1}{\rho_j^{XT}-1}} \left(P_{j,i} \right)^{\frac{\rho_j^{XT}}{\rho_j^{XT}-1}} \right]^{-\frac{1}{\rho_j^{XT}}} \left[\sum_{ij} \left(\beta_{j,ij}^{XT} \right)^{-\frac{1}{\rho_j^{XT}-1}} \left(P_{j,ij} \right)^{\frac{\rho_j^{XT}}{\rho_j^{XT}-1}} \right]$$

where

$$188. \quad \left[\sum_i \left(\beta_{j,i}^{XT} \right)^{-\frac{1}{\rho_j^{XT}-1}} \left(P_{j,i} \right)^{\frac{\rho_j^{XT}}{\rho_j^{XT}-1}} \right] \equiv \left[\sum_{ij} \left(\beta_{j,ij}^{XT} \right)^{-\frac{1}{\rho_j^{XT}-1}} \left(P_{j,ij} \right)^{\frac{\rho_j^{XT}}{\rho_j^{XT}-1}} \right]$$

Hence,

189. $$PT_j = \frac{1}{B_j^{XT}}\left[\sum_i \left(\beta_{j,i}^{XT}\right)^{-\frac{1}{\rho_j^{XT}-1}} \left(P_{j,i}\right)^{\frac{\rho_j^{XT}}{\rho_j^{XT}-1}}\right]^{1-\frac{1}{\rho_j^{XT}}}$$

190. $$PT_j = \frac{1}{B_j^{XT}}\left[\sum_i \left(\beta_{j,i}^{XT}\right)^{-\frac{1}{\rho_j^{XT}-1}} \left(P_{j,i}\right)^{\frac{\rho_j^{XT}}{\rho_j^{XT}-1}}\right]^{\frac{\rho_j^{XT}-1}{\rho_j^{XT}}}$$

Given $\sigma_j^{XT} = \frac{1}{\rho_j^{XT}-1}$, we have

191. $$PT_j = \frac{1}{B_j^{XT}}\left[\sum_i \left(\beta_{j,i}^{XT}\right)^{-\sigma_j^{XT}} \left(P_{j,i}\right)^{\sigma_j^{XT}+1}\right]^{\frac{1}{\sigma_j^{XT}+1}}$$

C5.4 SUPPLY OF INDIVIDUAL PRODUCTS

Using $\sigma_j^{XT} = \frac{1}{\rho_j^{XT}-1}$ and equation 191, one obtains

192. $$\left[\sum_i \left(\beta_{j,i}^{XT}\right)^{-\frac{1}{\rho_j^{XT}-1}} \left(P_{j,i}\right)^{\frac{\rho_j^{XT}}{\rho_j^{XT}-1}}\right]^{-\frac{1}{\rho_j^{XT}}} = \left[\sum_i \left(\beta_{j,i}^{XT}\right)^{-\sigma_j^{XT}} \left(P_{j,i}\right)^{\sigma_j^{XT}+1}\right]^{-\frac{\sigma_j^{XT}}{\sigma_j^{XT}+1}}$$

193. $$\left[\sum_i \left(\beta_{j,i}^{XT}\right)^{-\frac{1}{\rho_j^{XT}-1}} \left(P_{j,i}\right)^{\frac{\rho_j^{XT}}{\rho_j^{XT}-1}}\right]^{-\frac{1}{\rho_j^{XT}}} = \left(B_j^{XT}\right)^{-\sigma_j^{XT}} PT_j^{-\sigma_j^{XT}}$$

Substituting into equation 185 yields

194. $$XS_{j,ij} = \frac{XST_j}{B_j^{XT}} \left(B_j^{XT}\right)^{-\sigma_j^{XT}} PT_j^{\;-\sigma_j^{XT}} \left(\frac{P_{j,ij}}{\beta_{j,ij}^{XT}}\right)^{\sigma_j^{XT}}$$

After rearranging, equation 60 follows:

60. $$XS_{j,i} = \frac{XST_j}{\left(B_j^{XT}\right)^{1+\sigma_j^{XT}}} \left[\frac{P_{j,i}}{\beta_{j,i}^{XT} \; PT_j}\right]^{\sigma_j^{XT}}$$

C5.5 ELASTICITY OF TRANSFORMATION

In C5.2 above, nothing was said about the interpretation of $\sigma_j^{XT} = \frac{1}{\rho_j^{XT} - 1}$. Here we show that σ_j^{XT} is indeed the elasticity of transformation.

Here, the elasticity of transformation between products is defined as[15]

195. $$-\frac{\partial \ln\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)}{\partial \ln\left(\frac{\partial XST_j / \partial XS_{j.ij}}{\partial XST_j / \partial XS_{j.ii}}\right)} = -\frac{\left(\frac{\partial XST_j / \partial XS_{j.ij}}{\partial XST_j / \partial XS_{j.ii}}\right)}{\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)} \frac{\partial\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)}{\partial\left(\frac{\partial XST_j / \partial XS_{j.ij}}{\partial XST_j / \partial XS_{j.ii}}\right)}$$

where $\frac{\partial XST_j / \partial XS_{j.ij}}{\partial XST_j / \partial XS_{j.ii}}$ is the marginal rate of transformation between industry *j*'s products *ii* and *ij* ($MRT_{ii,ij}^{j}$):

196. $$MRT_{ii,ij}^{j} = -\frac{d \; XS_{j.ii}}{d \; XS_{j.ij}} = \frac{\partial XST_j / \partial XS_{j.ij}}{\partial XST_j / \partial XS_{j.ii}}$$

Substitute from 175

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15 In microeconomic textbooks, the elasticity of transformation and the elasticity of substitution are identically defined. What differenciates them in practice is that the former is negative, while the latter is positive (think of movement along an two-factor isoquant, compared to movement along a two-goods transformation curve, in response to changes in relative prices: they are in opposite directions). Here the elasticity of transformation is defined with a minus sign, so it will take on positive values.

197. $$MRT_{ii,ij}^{j} = -\frac{d\ XS_{j.ii}}{d\ XS_{j.ij}} = \frac{B_j^{XT}\left[\sum_i \beta_j^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}-1}\left(\beta_{j,ij}^{XT} XS_{j,ij}^{\rho_j^{XT}-1}\right)}{B_j^{XT}\left[\sum_i \beta_j^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}-1}\left(\beta_{j,ii}^{XT} XS_{j,ii}^{\rho_j^{XT}-1}\right)}$$

198. $$MRT_{ii,ij}^{j} = -\frac{d\ XS_{j.ii}}{d\ XS_{j.ij}} = \frac{\left(\beta_{j,ij}^{XT} XS_{j,ij}^{\rho_j^{XT}-1}\right)}{\left(\beta_{j,ii}^{XT} XS_{j,ii}^{\rho_j^{XT}-1}\right)} = \frac{\beta_{j,ij}^{XT}}{\beta_{j,ii}^{XT}}\left(\frac{XS_{j,ij}}{XS_{j,ii}}\right)^{\rho_j^{XT}-1}$$

The elasticity of transformation is therefore

199. $$-\frac{\partial \ln\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)}{\partial \ln\left(\frac{\partial XST_j / \partial XS_{j.ij}}{\partial XST_j / \partial XS_{j.ii}}\right)} = -\frac{\left(\frac{\beta_{j,ij}^{XT}}{\beta_{j,ii}^{XT}}\left(\frac{XS_{j,ij}}{XS_{j,ii}}\right)^{\rho_j^{XT}-1}\right)}{\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)} \frac{\partial\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)}{\partial\left(\frac{\beta_{j,ij}^{XT}}{\beta_{j,ii}^{XT}}\left(\frac{XS_{j,ij}}{XS_{j,ii}}\right)^{\rho_j^{XT}-1}\right)}$$

200. $$-\frac{\partial \ln\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)}{\partial \ln\left(\frac{\partial XST_j / \partial XS_{j.ij}}{\partial XST_j / \partial XS_{j.ii}}\right)} = -\frac{\left(\frac{\beta_{j,ij}^{XT}}{\beta_{j,ii}^{XT}}\left(\frac{XS_{j,ij}}{XS_{j,ii}}\right)^{\rho_j^{XT}-1}\right)}{\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)}\left[\frac{\partial\left(\frac{\beta_{j,ij}^{XT}}{\beta_{j,ii}^{XT}}\left(\frac{XS_{j,ij}}{XS_{j,ii}}\right)^{\rho_j^{XT}-1}\right)}{\partial\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)}\right]^{-1}$$

Noting that $\left(\frac{XS_{j,ij}}{XS_{j,ii}}\right)^{\rho_j^{XT}-1} = \left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)^{1-\rho_j^{XT}}$, simplify as

201. $$-\frac{\partial \ln\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)}{\partial \ln\left(\frac{\partial XST_j / \partial XS_{j.ij}}{\partial XST_j / \partial XS_{j.ii}}\right)} = -\frac{\left(\frac{\beta_{j,ij}^{XT}}{\beta_{j,ii}^{XT}}\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)^{1-\rho_j^{XT}}\right)}{\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)}\left[\frac{\partial\left(\frac{\beta_{j,ij}^{XT}}{\beta_{j,ii}^{XT}}\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)^{1-\rho_j^{XT}}\right)}{\partial\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)}\right]^{-1}$$

202. $$-\frac{\partial \ln\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)}{\partial \ln\left(\frac{\partial XST_j / \partial XS_{j.ij}}{\partial XST_j / \partial XS_{j.ii}}\right)} = -\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)^{-\rho_j^{XT}}\left[\left(1-\rho_j^{XT}\right)\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)^{-\rho_j^{XT}}\right]^{-1}$$

203. $$-\frac{\partial \ln\left(\frac{XS_{j,ii}}{XS_{j,ij}}\right)}{\partial \ln\left(\frac{\partial XST_j / \partial XS_{j.ij}}{\partial XST_j / \partial XS_{j.ii}}\right)} = -\frac{1}{1-\rho_j^{XT}} = \frac{1}{\rho_j^{XT}-1}$$

Hence $\sigma_j^{XT} = \dfrac{1}{\rho_j^{XT}-1}$ is indeed the elasticity of transformation.

C6. Supply on domestic and export markets

C6.1 SALES REVENUE MAXIMIZING PROBLEM

The aggregator function of production sold on the domestic market and exported is given by equation 61:

61. $$XS_{j,x} = B_{j,x}^{X}\left[\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X}} + \left(1-\beta_{j,x}^{X}\right) DS_{j,x}^{\rho_{j,x}^{X}}\right]^{\frac{1}{\rho_{j,x}^{X}}}$$

The producer’s problem is to maximize sales revenue of exportable product *x*:

204. $$PE_{j,x} EX_{j,x} + PL_{j,x} D_{j,x}$$

subject to 61 and the constraint $XS_{j,x} = \overline{XS_{j,x}}$. Form the Lagrangian

First order conditions are:

with

and, similarly,

C6.2 RELATIVE SUPPLY ON THE DOMESTIC AND EXPORT MARKETS

It follows from 208. 209, 212, and 213 that

205. $\Lambda = PE_{j,x} EX_{j,x} + PL_{j,x} D_{j,x} - \lambda \left(XS_{j,x} - \overline{XS_{j,x}} \right)$

206. $\Lambda = PE_{j,x} EX_{j,x} + PL_{j,x} D_{j,x} - \lambda \left(B_{j,x}^{X} \left[\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X}} + \left(1 - \beta_{x}^{X}\right) D_{j,x}^{\rho_{j,x}^{X}} \right]^{\frac{1}{\rho_{j,x}^{X}}} - \overline{XS_{x}} \right)$

207. $\frac{\partial \Lambda}{\partial \lambda} = -\left(XS_{j,x} - \overline{XS_{j,x}} \right) = 0$

208. $\frac{\partial \Lambda}{\partial EX_{j,x}} = PE_{j,x} - \lambda \frac{\partial XS_{j,x}}{\partial EX_{j,x}}$

209. $\frac{\partial \Lambda}{\partial D_{j,x}} = PL_{j,x} - \lambda \frac{\partial XS_{j,x}}{\partial D_{j,x}}$

210. $\frac{\partial XS_{j,x}}{\partial EX_{j,x}} = \frac{\partial}{\partial EX_{j,x}} \left\{ B_{j,x}^{X} \left[\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X}} + \left(1 - \beta_{x}^{X}\right) D_{j,x}^{\rho_{j,x}^{X}} \right]^{\frac{1}{\rho_{j,x}^{X}}} \right\}$

211. $\frac{\partial XS_{j,x}}{\partial EX_{j,x}} = B_{j.x}^{X} \frac{1}{\rho_{j,x}^{X}} \left[\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X}} + \left(1 - \beta_{j,x}^{X}\right) D_{j,x}^{\rho_{j,x}^{X}} \right]^{\frac{1}{\rho_{j,x}^{X}} - 1} \left(\beta_{j,x}^{X} \rho_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X} - 1} \right)$

212. $\frac{\partial XS_{j,x}}{\partial EX_{j,x}} = B_{j.x}^{X} \left[\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X}} + \left(1 - \beta_{j,x}^{X}\right) D_{j,x}^{\rho_{j,x}^{X}} \right]^{\frac{1}{\rho_{j,x}^{X}} - 1} \left(\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X} - 1} \right)$

213. $$\frac{\partial\ XS_{j,x}}{\partial\ D_{j,x}} = B_{j.x}^{X}\left[\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X}} + \left(1-\beta_{j,x}^{X}\right) D_{j,x}^{\rho_{j,x}^{X}}\right]^{\frac{1}{\rho_{j,x}^{X}}-1} \left(\left(1-\beta_{j,x}^{X}\right) D_{j,x}^{\rho_{j,x}^{X}-1}\right)$$

214. $$\frac{PE_{j,x}}{PL_{j,x}} = \frac{\lambda\ B_{j.x}^{X}\left[\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X}} + \left(1-\beta_{j,x}^{X}\right) D_{j,x}^{\rho_{j,x}^{X}}\right]^{\frac{1}{\rho_{j,x}^{X}}-1} \left(\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X}-1}\right)}{\lambda\ B_{j.x}^{X}\left[\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X}} + \left(1-\beta_{j,x}^{X}\right) D_{j,x}^{\rho_{j,x}^{X}}\right]^{\frac{1}{\rho_{j,x}^{X}}-1} \left(\left(1-\beta_{j,x}^{X}\right) D_{j,x}^{\rho_{j,x}^{X}-1}\right)}$$

215. $$\frac{PE_{j,x}}{PL_{j,x}} = \frac{\left(\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X}-1}\right)}{\left(\left(1-\beta_{j,x}^{X}\right) D_{j,x}^{\rho_{j,x}^{X}-1}\right)}$$

216. $$\left(\frac{EX_{j,x}}{D_{j,x}}\right)^{\rho_{j,x}^{X}-1} = \frac{\left(1-\beta_{j,x}^{X}\right) PE_{j,x}}{\beta_{j,x}^{X} PL_{j,x}}$$

217. $$\frac{EX_{j,x}}{D_{j,x}} = \left[\frac{\left(1-\beta_{j,x}^{X}\right) PE_{j,x}}{\beta_{j,x}^{X} PL_{j,x}}\right]^{\frac{1}{\rho_{j,x}^{X}-1}}$$

Substituting $\sigma_{j,x}^{X} = \frac{1}{\rho_{j,x}^{X}-1}$, equation 63 follows:

218. $$EX_{j,x} = \left[\frac{1-\beta_{j,x}^{X}}{\beta_{j,x}^{X}} \frac{PE_{x}}{PL_{x}}\right]^{\sigma_{j,x}^{X}} DS_{j,x}$$

C6.3 ELASTICITY OF TRANSFORMATION

In C2.3 above, nothing was said about the interpretation of $\sigma_{j,x}^{X} = \frac{1}{\rho_{j,x}^{X}-1}$. Here we show that $\sigma_{j,x}^{X}$ is indeed the elasticity of transformation.

Here, the elasticity of transformation between production sold on the domestic market and production exported is defined as[16]

219. $$-\frac{\partial \ln\left(\frac{EX_{j,x}}{D_{j,x}}\right)}{\partial \ln\left(\frac{\partial XS_{j,x}/\partial D_{j,x}}{\partial XS_{j,x}/\partial EX_{j,x}}\right)} = -\frac{\left(\frac{\partial XS_{j,x}/\partial D_{j,x}}{\partial XS_{j,x}/\partial EX_{j,x}}\right)}{\left(\frac{EX_{j,x}}{D_{j,x}}\right)}\frac{\partial\left(\frac{EX_{j,x}}{D_{j,x}}\right)}{\partial\left(\frac{\partial XS_{j,x}/\partial D_{j,x}}{\partial XS_{j,x}/\partial EX_{j,x}}\right)}$$

where $\frac{\partial XS_{j,x}/\partial D_{j,x}}{\partial XS_{j,x}/\partial EX_{j,x}}$ is the marginal rate of transformation between production sold locally and exported ($MRT_{j,x}^{EX,D}$):

220. $$MRT_{j,x}^{EX,D} = -\frac{d\ EX_{j,x}}{d\ D_{j,x}} = \frac{\partial XS_{j,x}/\partial D_{j,x}}{\partial XS_{j,x}/\partial EX_{j,x}}$$

Substitute from equations 212 and 213

221. $$MRT_{j,x}^{EX,D} = -\frac{d\ EX_{j,x}}{d\ D_{j,x}} = \frac{B_{j.x}^{X}\left[\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X}} + \left(1-\beta_{j,x}^{X}\right) D_{j,x}^{\rho_{j,x}^{X}}\right]^{\frac{1}{\rho_{j,x}^{X}}-1}\left(\left(1-\beta_{j,x}^{X}\right) D_{j,x}^{\rho_{j,x}^{X}-1}\right)}{B_{j.x}^{X}\left[\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X}} + \left(1-\beta_{j,x}^{X}\right) D_{j,x}^{\rho_{j,x}^{X}}\right]^{\frac{1}{\rho_{j,x}^{X}}-1}\left(\beta_{j,x}^{X} EX_{j,x}^{\rho_{j,x}^{X}-1}\right)}$$

222. $$MRT_{j,x}^{EX,D} = -\frac{d\ EX_{j,x}}{d\ D_{j,x}} = \left(\frac{1-\beta_{j,x}^{X}}{\beta_{j,x}^{X}}\right)\left(\frac{D_{j,x}}{EX_{j,x}}\right)^{\rho_{j,x}^{X}-1}$$

The elasticity of transformation is therefore

[16] In microeconomic textbooks, the elasticity of transformation and the elasticity of substitution are identically defined. What differenciates them in practice is that the former is negative, while the latter is positive (think of movement along an two-factor isoquant, compared to movement along a two-goods transformation curve, in response to changes in relative prices: they are in opposite directions). Here the elasticity of transformation is defined with a minus sign, so it will take on positive values.

223. $$-\frac{\partial \ln\left(\frac{EX_{j,x}}{D_{j,x}}\right)}{\partial \ln\left(\frac{\partial XS_{j,x}/\partial D_{j,x}}{\partial XS_{j,x}/\partial EX_{j,x}}\right)} = -\frac{\left(\frac{1-\beta_{j,x}^{X}}{\beta_{j,x}^{X}}\right)\left(\frac{D_{j,x}}{EX_{j,x}}\right)^{\rho_{j,x}^{X}-1}}{\left(\frac{EX_{j,x}}{D_{j,x}}\right)} \frac{\partial\left(\frac{EX_{j,x}}{D_{j,x}}\right)}{\partial\left(\left(\frac{1-\beta_{j,x}^{X}}{\beta_{j,x}^{X}}\right)\left(\frac{D_{j,x}}{EX_{j,x}}\right)^{\rho_{j,x}^{X}-1}\right)}$$

224. $$-\frac{\partial \ln\left(\frac{EX_{j,x}}{D_{j,x}}\right)}{\partial \ln\left(\frac{\partial XS_{j,x}/\partial D_{j,x}}{\partial XS_{j,x}/\partial EX_{j,x}}\right)} = -\frac{\left(\frac{1-\beta_{j,x}^{X}}{\beta_{j,x}^{X}}\right)\left(\frac{D_{j,x}}{EX_{j,x}}\right)^{\rho_{j,x}^{X}-1}}{\left(\frac{EX_{j,x}}{D_{j,x}}\right)} \left[\frac{\partial\left(\left(\frac{1-\beta_{j,x}^{X}}{\beta_{j,x}^{X}}\right)\left(\frac{D_{j,x}}{EX_{j,x}}\right)^{\rho_{j,x}^{X}-1}\right)}{\partial\left(\frac{EX_{j,x}}{D_{j,x}}\right)}\right]^{-1}$$

Noting that $\left(\frac{D_{j,x}}{EX_{j,x}}\right)^{\rho_{j,x}^{X}-1} = \left(\frac{EX_{j,x}}{D_{j,x}}\right)^{1-\rho_{j,x}^{X}}$, simplify as

225. $$-\frac{\partial \ln\left(\frac{EX_{j,x}}{D_{j,x}}\right)}{\partial \ln\left(\frac{\partial XS_{j,x}/\partial D_{j,x}}{\partial XS_{j,x}/\partial EX_{j,x}}\right)} = -\frac{\left(\frac{1-\beta_{j,x}^{X}}{\beta_{j,x}^{X}}\right)\left(\frac{EX_{j,x}}{D_{j,x}}\right)^{1-\rho_{j,x}^{X}}}{\left(\frac{EX_{j,x}}{D_{j,x}}\right)} \left(\frac{1-\beta_{j,x}^{X}}{\beta_{j,x}^{X}}\right)^{-1} \left[\frac{\partial\left(\left(\frac{EX_{j,x}}{D_{j,x}}\right)^{1-\rho_{j,x}^{X}}\right)}{\partial\left(\frac{EX_{j,x}}{D_{j,x}}\right)}\right]^{-1}$$

226. $$-\frac{\partial \ln\left(\frac{EX_{j,x}}{D_{j,x}}\right)}{\partial \ln\left(\frac{\partial XS_{j,x}/\partial D_{j,x}}{\partial XS_{j,x}/\partial EX_{j,x}}\right)} = -\left(\frac{EX_{j,x}}{D_{j,x}}\right)^{\rho_{j,x}^{X}} \left[\left(1-\rho_{j,x}^{X}\right)\left(\frac{EX_{j,x}}{D_{j,x}}\right)^{\rho_{j,x}^{X}}\right]^{-1} = -\frac{1}{1-\rho_{j,x}^{X}} = \frac{1}{\rho_{j,x}^{X}-1}$$

Hence $\sigma_{j,x}^{X} = \frac{1}{\rho_{j,x}^{X}-1}$ is indeed the elasticity of transformation.

C7. Demand for local products and imports

C7.1 Expenditure minimizing problem

The aggregator function of composite commodity is given by equation 65:

65. $$Q_m = B_m^M \left[\beta_m^M IM_m^{-\rho_m^M} + \left(1-\beta_m^M\right) DD_m^{-\rho_m^M} \right]^{\frac{-1}{\rho_m^M}}$$

The buyer's problem is to minimize expenditure on the composite commodity

227. $$PM_m IM_m + PD_m D_m$$

subject to 65 and the constraint $Q_m = \overline{Q_m}$. Form the Lagrangian

228. $$\Lambda = PM_m IM_m + PD_m D_m - \lambda \left(Q_m - \overline{Q_m}\right)$$

229. $$\Lambda = PM_m IM_m + PD_m D_m - \lambda \left(B_m^M \left[\beta_m^M IM_m^{-\rho_m^M} + \left(1-\beta_m^M\right) D_m^{-\rho_m^M} \right]^{\frac{-1}{\rho_m^M}} - \overline{Q_m} \right)$$

First order conditions are:

230. $$\frac{\partial \Lambda}{\partial \lambda} = -\lambda \left(Q_m - \overline{Q_m}\right) = 0$$

231. $$\frac{\partial \Lambda}{\partial\, IM_m} = PM_m - \lambda \frac{\partial Q_m}{\partial\, IM_m} = 0$$

232. $$\frac{\partial \Lambda}{\partial\, D_m} = PD_m - \lambda \frac{\partial Q_m}{\partial\, D_m} = 0$$

with

233. $$\frac{\partial Q_m}{\partial\, IM_m} = B_m^M \frac{\partial}{\partial\, IM_m} \left[\beta_m^M IM_m^{-\rho_m^M} + \left(1-\beta_m^M\right) D_m^{-\rho_m^M} \right]^{\frac{-1}{\rho_m^M}}$$

234. $$\frac{\partial Q_m}{\partial\, IM_m} = B_m^M \left(\frac{-1}{\rho_m^M} \right) \left[\beta_m^M IM_m^{-\rho_m^M} + \left(1-\beta_m^M\right) D_m^{-\rho_m^M} \right]^{-\frac{1}{\rho_m^M}-1} \left(-\rho_m^M\right) \left(\beta_m^M IM_m^{-\rho_m^M - 1} \right)$$

$$235. \quad \frac{\partial Q_m}{\partial IM_m} = B_m^M \left[\beta_m^M IM_m^{-\rho_m^M} + \left(1-\beta_m^M\right) D_m^{-\rho_m^M} \right]^{-\frac{1}{\rho_m^M}-1} \left(\beta_m^M IM_m^{-\rho_m^M -1} \right)$$

Likewise,

$$236. \quad \frac{\partial Q_m}{\partial D_m} = B_m^M \left[\beta_m^M IM_m^{-\rho_m^M} + \left(1-\beta_m^M\right) D_m^{-\rho_m^M} \right]^{-\frac{1}{\rho_m^M}-1} \left(1-\beta_m^M\right) \left(D_m^{-\rho_m^M -1} \right)$$

C7.2 RELATIVE DEMAND FOR LOCAL PRODUCTS AND IMPORTS

It follows from equations 231, 232, 235, and 236 that

$$237. \quad \frac{PM_m}{PD_m} = \frac{B_m^M \left[\beta_m^M IM_m^{-\rho_m^M} + \left(1-\beta_m^M\right) D_m^{-\rho_m^M} \right]^{-\frac{1}{\rho_m^M}-1} \left(\beta_m^M IM_m^{-\rho_m^M -1} \right)}{B_m^M \left[\beta_m^M IM_m^{-\rho_m^M} + \left(1-\beta_m^M\right) D_m^{-\rho_m^M} \right]^{-\frac{1}{\rho_m^M}-1} \left(1-\beta_m^M\right) \left(D_m^{-\rho_m^M -1} \right)}$$

$$238. \quad \frac{PM_m}{PD_m} = \left(\frac{\beta_m^M}{1-\beta_m^M} \right) \left(\frac{IM_m}{D_m} \right)^{-\rho_m^M -1}$$

$$239. \quad \left(\frac{IM_m}{D_m} \right)^{-\rho_m^M -1} = \left(\frac{1-\beta_m^M}{\beta_m^M} \right) \frac{PM_m}{PD_m}$$

$$240. \quad \frac{IM_m}{D_m} = \left[\left(\frac{1-\beta_m^M}{\beta_m^M} \right) \frac{PM_m}{PD_m} \right]^{\frac{1}{-\rho_m^M -1}} = \left[\left(\frac{\beta_m^M}{1-\beta_m^M} \right) \frac{PD_m}{PM_m} \right]^{\frac{1}{\rho_m^M +1}}$$

Substituting $\sigma_m^M = \frac{1}{\rho_m^M + 1}$, equation 67 follows

$$241. \quad IM_m = \left[\frac{\beta_m^M}{1-\beta_m^M} \frac{PD_m}{PM_m} \right]^{\sigma_m^M} D_m$$

C7.3 ELASTICITY OF SUBSTITUTION

In C7.2 above, nothing was said about the interpretation of $\sigma_m^M = \frac{1}{\rho_m^M + 1}$. Here we show that σ_m^M is indeed the elasticity of substitution. The elasticity of substitution between imported and locally produced commodity m is defined as

$$242. \quad \frac{\partial \ln\left(\frac{IM_m}{D_m}\right)}{\partial \ln\left(\frac{\partial Q_m / \partial D_m}{\partial Q_m / \partial IM_m}\right)} = \frac{\left(\frac{\partial Q_m / \partial D_m}{\partial Q_m / \partial IM_m}\right)}{\left(\frac{IM_m}{D_m}\right)} \frac{\partial\left(\frac{IM_m}{D_m}\right)}{\partial\left(\frac{\partial Q_m / \partial D_m}{\partial Q_m / \partial IM_m}\right)}$$

where $\frac{\partial Q_m / \partial D_m}{\partial Q_m / \partial IM_m}$ is the marginal rate of substitution between imported and locally produced commodity m ($MRS_m^{IM,D}$):

$$243. \quad MRS_m^{IM,D} = -\frac{d\ IM_m}{d\ D_m} = \frac{\partial Q_m / \partial D_m}{\partial Q_m / \partial IM_m}$$

Substitute from 233 and 234

$$244. \quad MRS_m^{IM,D} = -\frac{d\ IM_m}{d\ D_m} = \frac{B_m^M \left[\beta_m^M IM_m^{-\rho_m^M} + \left(1-\beta_m^M\right) D_m^{-\rho_m^M}\right]^{-\frac{1}{\rho_m^M}-1} \left(1-\beta_m^M\right)\left(D_m^{-\rho_m^M - 1}\right)}{B_m^M \left[\beta_m^M IM_m^{-\rho_m^M} + \left(1-\beta_m^M\right) D_m^{-\rho_m^M}\right]^{-\frac{1}{\rho_m^M}-1} \left(\beta_m^M IM_m^{-\rho_m^M - 1}\right)}$$

$$245. \quad MRS_m^{IM,D} = -\frac{d\ IM_m}{d\ D_m} = \left(\frac{1-\beta_m^M}{\beta_m^M}\right)\left(\frac{D_m}{IM_m}\right)^{-\rho_m^M - 1}$$

The elasticity of substitution is therefore

246. $$\frac{\partial \ln\left(\frac{IM_m}{D_m}\right)}{\partial \ln\left(\frac{\partial Q_m/\partial D_m}{\partial Q_m/\partial IM_m}\right)} = \frac{\left(\left(\frac{1-\beta_m^M}{\beta_m^M}\right)\left(\frac{D_m}{IM_m}\right)^{-\rho_m^M-1}\right)}{\left(\frac{IM_m}{D_m}\right)} \frac{\partial\left(\frac{IM_m}{D_m}\right)}{\partial\left(\left(\frac{1-\beta_m^M}{\beta_m^M}\right)\left(\frac{D_m}{IM_m}\right)^{-\rho_m^M-1}\right)}$$

247. $$\frac{\partial \ln\left(\frac{IM_m}{D_m}\right)}{\partial \ln\left(\frac{\partial Q_m/\partial D_m}{\partial Q_m/\partial IM_m}\right)} = \frac{\left(\left(\frac{1-\beta_m^M}{\beta_m^M}\right)\left(\frac{D_m}{IM_m}\right)^{-\rho_m^M-1}\right)}{\left(\frac{IM_m}{D_m}\right)} \left[\frac{\partial\left(\left(\frac{1-\beta_m^M}{\beta_m^M}\right)\left(\frac{D_m}{IM_m}\right)^{-\rho_m^M-1}\right)}{\partial\left(\frac{IM_m}{D_m}\right)}\right]^{-1}$$

Noting that $\left(\frac{D_m}{IM_m}\right)^{-\rho_m^M-1} = \left(\frac{IM_m}{D_m}\right)^{\rho_m^M+1}$, simplify as

248. $$\frac{\partial \ln\left(\frac{IM_m}{D_m}\right)}{\partial \ln\left(\frac{\partial Q_m/\partial D_m}{\partial Q_m/\partial IM_m}\right)} = \frac{\left(\left(\frac{1-\beta_m^M}{\beta_m^M}\right)\left(\frac{IM_m}{D_m}\right)^{\rho_m^M+1}\right)}{\left(\frac{IM_m}{D_m}\right)} \left[\frac{\partial\left(\left(\frac{1-\beta_m^M}{\beta_m^M}\right)\left(\frac{IM_m}{D_m}\right)^{\rho_m^M+1}\right)}{\partial\left(\frac{IM_m}{D_m}\right)}\right]^{-1}$$

249. $$\frac{\partial \ln\left(\frac{IM_m}{D_m}\right)}{\partial \ln\left(\frac{\partial Q_m/\partial D_m}{\partial Q_m/\partial IM_m}\right)} = \frac{\left(\left(\frac{1-\beta_m^M}{\beta_m^M}\right)\left(\frac{IM_m}{D_m}\right)^{\rho_m^M+1}\right)}{\left(\frac{IM_m}{D_m}\right)} \left(\frac{1-\beta_m^M}{\beta_m^M}\right)^{-1} \left[\frac{\partial\left(\left(\frac{IM_m}{D_m}\right)^{\rho_m^M+1}\right)}{\partial\left(\frac{IM_m}{D_m}\right)}\right]^{-1}$$

250. $$\frac{\partial \ln\left(\frac{IM_m}{D_m}\right)}{\partial \ln\left(\frac{\partial Q_m / \partial D_m}{\partial Q_m / \partial IM_m}\right)} = \left(\frac{IM_m}{D_m}\right)^{\rho_m^M} \left[\left(\rho_m^M + 1\right)\left(\frac{IM_m}{D_m}\right)^{\rho_m^M}\right]^{-1} = \frac{1}{\rho_m^M + 1}$$

Hence $\sigma_m^M = \frac{1}{\rho_m^M + 1}$ is indeed the elasticity of substitution.

C8. Exact price indexes *PIXINV* and *PIXGVT*

The GFCF demand for commodities is given by equation 55, where total GFCF expenditure is distributed among commodities in fixed shares:

55. $$PC_i INV_i = \gamma_i^{INV} GFCF$$

251. $$INV_i = \frac{\gamma_i^{INV} GFCF}{PC_i}$$

Implicitly, the production function of new capital is a Cobb-Douglas function of the form:

252. $$\Delta K = A^K \prod_i INV_i^{\gamma_i^{INV}}$$

The indirect production function is obtained by substituting equation 251 into equation 252:

253. $$\Delta K = A^K \prod_i \left(\gamma_i^{INV} \frac{GFCF}{PC_i}\right)^{\gamma_i^{INV}} = A^K GFCF^{\sum_i \gamma_i^{INV}} \prod_i \left(\frac{\gamma_i^{INV}}{PC_i}\right)^{\gamma_i^{INV}}$$

254. $$\Delta K = A^K GFCF \prod_i \left(\frac{\gamma_i^{INV}}{PC_i}\right)^{\gamma_i^{INV}}$$

Let

$GFCF^0$: gross fixed capital formation expenditures in the SAM

PC_i^0 : initial commodity prices

*GFCF**: expenditure necessary to produce the same quantity of new capital at current prices PC_i

*GFCF** is calculated by solving

255. $$A^K GFCF^* \prod_i \left(\frac{\gamma_i^{INV}}{PC_i} \right)^{\gamma_i^{INV}} = A^K GFCF^0 \prod_i \left(\frac{\gamma_i^{INV}}{PC_i^0} \right)^{\gamma_i^{INV}}$$

An exact[17] GFCF price index is a formula which will yield the ratio $GFCF^* / GFCF^0$ as a function of the price ratios for any initial level of expenditures $GFCF^0$. The exact price index is therefore

256. $$\frac{GFCF^*}{GFCF^0} = \frac{A^K \prod_i \left(\frac{\gamma_i^{INV}}{PC_i^0} \right)^{\gamma_i^{INV}}}{A^K \prod_i \left(\frac{\gamma_i^{INV}}{PC_i} \right)^{\gamma_i^{INV}}} = \prod_i \left(\frac{PC_i}{PC_i^0} \right)^{\gamma_i^{INV}}$$

which is precisely *PIXINV* as given in equation 87.

87. $$PIXINV = \prod_i \left(\frac{PC_i}{PC_i^0} \right)^{\gamma_i^{INV}}$$

It can be shown in the same manner that *PIXGVT* in equation 88 is also an exact price index of government current expenditures on goods and services.

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17 Diewert, W. E. (1976) « Exact and superlative index numbers » Journal of Econometrics 4: 115-145. Reproduced in W. E. Diewert and A. O. Nakamura (1993), Essays in Index Number Theory, Vol. 1, North-Holland Publishing Co., Amsterdam, p. 223-257. Available on line at http://www.econ.ubc.ca/diewert/hmpgdie.htm.

APPENDIX D: NOTE ON THE RENTAL RATE AND THE RATE OF RETURN ON CAPITAL

Throughout this document, we have been careful to avoid inappropriate uses of the expression « rate of return on capital ». This short appendix aims at clarifying the meaning of « rate of return on capital », as distinct from the rental rate of capital.

The rental rate of capital in PEP-1-1 is

$R_{k,j}$: Rental rate of type *k* capital in industry *j*

This is the price received by owners for allowing the use of one unit of type *k* capital in industry *j* for one period. It enters the calculation of the capital income of households, businesses and government (equations 12, 18, and 23):

12. $$YHK_h = \sum_k \lambda_{h,k}^{RK} \left(\sum_j R_{k,j} KD_{k,j} \right)$$

18. $$YFK_f = \sum_k \lambda_{f,k}^{RK} \left(\sum_j R_{k,j} KD_{k,j} \right)$$

23. $$YGK = \sum_k \lambda_{gvt,k}^{RK} \left(\sum_j R_{k,j} KD_{k,j} \right)$$

In the case where capital is perfectly mobile, owners will allocate their capital to the most advantageous uses, so that the rental rate of each type of capital will be uniform across industries, equal to

RK_k : Rental rate of type *k* capital (if capital is mobile)

as described by equation 76:

76. $$R_{k,j} = RK_k , \quad if \ capital \ is \ mobile$$

The rental rate of capital $R_{k,j}$ is related to

$RTI_{k,j}$: Rental rate paid by industry *j* for type *k* capital, including capital taxes

by equation 75:

75. $$RTI_{k,j} = R_{k,j} \left(1 + ttik_{k,j}\right)$$

In turn, $RTI_{k,j}$ appears in the calculation of

RC_j : Rental rate of industry *j* composite capital

in equation 74:

74. $$RC_j = \frac{\sum_k RTI_{k,j} KD_{k,j}}{KDC_j}$$

Finally, RC_j is the rental rate of capital that enters equation 71, which determines the price of value added:

71. $$PVA_j = \frac{WC_j LDC_j + RC_j KDC_j}{VA_j}$$

Now, the ***rate of return*** on capital is a different concept. A rate of return is the ratio of income received on an asset to the value of that asset. In order to compute the rate of return on capital, our model would need a ***price of capital***, which does not appear in the static model PEP-1-1. So let

PK_k : Price of private investment in type *k* capital

PK_k is the replacement cost of capital. The rate of return on type *k* capital used in industry *j* is

$$Rho_{k,j} = \frac{R_{k,j} KD_{k,j}}{PK_k KD_{k,j}} = \frac{R_{k,j}}{PK_k}$$

Therefore, one must be careful to use the expression « return rate on capital » only when appropriate, that is, when the variable refered to is the ratio of capital income over the value of the asset.

APPENDIX E: WALRAS' LAW, REDUNDANT EQUATIONS, AND SLACK VARIABLES (LEON)

E1. Walras' Law and the *LEON* slack variable

Walras' Law states that, in a closed *n*-commodity system of supply and demand equilibrium, there is one redundant equation, due to the income-expenditure accounting identity. A CGE is such a closed system: basically, income generated in production is used to pay for the purchase of commodities. Therefore, in a CGE, it is necessary to eliminate one of the commodity supply-demand equilibrium conditions, to avoid creating a model with more equations than free variables (when counting equations and variables, GAMS does not distinguish redundant from non-redundant equations). The choice of the discarded equation is arbitrary. It is customary – at least in the PEP tradition – to introduce a slack variable, called *LEON* (in honor of Leon Walras), whose value is the imbalance in the discarded equation. This adds one equation and one variable, and so maintains the equality between number of variables and equations. Naturally, in a well-specified model, the equilibrium value of the slack variable is zero. Otherwise, a non-zero *LEON* signals an error in the model.

In PEP-1-1, supply-demand equilibrium conditions are represented by equation 89, of which there are as many as there are commodities, minus 1. *LEON* is the excess supply on the last commodity market, whose value is computed in the equation called *WALRAS* in the GAMS code.

E2. Demand for labor by category and the price of composite labor

There are, however, other redundant equations that are discarded from the model. For instance, equation 72 is inserted in the GAMS code as a mere comment, with the statement that « Given the way equation 6 is written, equation 72 is redundant ». Now, in Appendix C2, the following has been demonstrated:

- equations 5 and 134 (the latter derived from the first-order cost-minimization conditions) together imply equation 139;
- equations 139 and 72 together imply equation 6.

We shall now see that

- equation 139 implies 134;
- equation 6 implies 134;
- equation 139 implies 5;
- equations 5 and 6 together imply 72.

To summarize, equations 5 and 6 together make equation 72 redundant.

E2.1 EQUATION 140 IMPLIES EQUATIONS 135 AND 5

To begin, we demonstrate that equation 139 implies equations 134 and 5. It is straightforward to take the ratio of equation 139 for $LD_{li,j}$ over the same equation for $LD_{lj,j}$:

257. $$\frac{LD_{li,j}}{LD_{lj,j}}=\frac{\frac{LDC_j}{B_j^{LD}}\left[\sum_l\left(\beta_{l,j}^{LD}\right)^{\frac{1}{\rho_j^{LD}+1}}\left(WTI_{l,j}\right)^{\frac{\rho_j^{LD}}{\rho_j^{LD}+1}}\right]^{\frac{1}{\rho_j^{LD}}}\left(\frac{WTI_{li,j}}{\beta_{li,j}^{LD}}\right)^{-\frac{1}{\rho_j^{LD}+1}}}{\frac{LDC_j}{B_j^{LD}}\left[\sum_l\left(\beta_{l,j}^{LD}\right)^{\frac{1}{\rho_j^{LD}+1}}\left(WTI_{l,j}\right)^{\frac{\rho_j^{LD}}{\rho_j^{LD}+1}}\right]^{\frac{1}{\rho_j^{LD}}}\left(\frac{WTI_{lj,j}}{\beta_{lj,j}^{LD}}\right)^{-\frac{1}{\rho_j^{LD}+1}}}$$

258. $$\frac{LD_{li,j}}{LD_{lj,j}}=\frac{\left(\frac{WTI_{li,j}}{\beta_{li,j}^{LD}}\right)^{-\frac{1}{\rho_j^{LD}+1}}}{\left(\frac{WTI_{lj,j}}{\beta_{lj,j}^{LD}}\right)^{-\frac{1}{\rho_j^{LD}+1}}}$$

which, given $\sigma_j^{LD}=\frac{1}{\rho_j^{LD}+1}$, is equivalent to equation 134:

134. $$LD_{li,j}=\left(\frac{\beta_{li,j}^{LD}\,WTI_{lj,j}}{\beta_{lj,j}^{LD}\,WTI_{li,j}}\right)^{\sigma_j^{LD}}LD_{lj,j}$$

It is equally straightforward to verify in the same manner that equation 6 also implies equation 134.

Next, the development in Appendix C2 from equations 135 to 139 can be reversed, so that equations 135 and 139 are equivalent (each one implies the other). Finally, substituting for $\left(\dfrac{\beta_{l,j}^{LD}\ WTI_{lj,j}}{\beta_{lj,j}^{LD}\ WTI_{l,j}}\right)^{\frac{1}{\rho_j^{LD}+1}} LD_{lj,j}$ from equation 134 into equation 135, equation 5 follows.

E2.2 Equation 72 redundant given equations 5 and 6

Second, we prove that, given equations 5 and 6, equation 72 is redundant.

E2.2.1 Equations 5 and 6 together imply equation 144

The first step in the demonstration is to show that 5 and 6 together imply 144. Substitute 6 into 5, and there results:

$$259.\quad LDC_j = B_j^{LD}\left[\sum_l \beta_{l,j}^{LD}\left\{\left[\frac{\beta_{l,j}^{LD}\ WC_j}{WTI_{l,j}}\right]^{\sigma_j^{LD}}\left(B_j^{LD}\right)^{\sigma_j^{LD}-1} LDC_j\right\}^{-\rho_j^{LD}}\right]^{-\frac{1}{\rho_j^{LD}}}$$

Given $\sigma_j^{LD} = \dfrac{1}{\rho_j^{LD}+1}$,

$$260.\quad LDC_j = B_j^{LD}\left[\sum_l \beta_{l,j}^{LD}\left\{\left[\frac{\beta_{l,j}^{LD}\ WC_j}{WTI_{l,j}}\right]^{\frac{1}{\rho_j^{LD}+1}}\left(B_j^{LD}\right)^{-\frac{\rho_j^{LD}}{\rho_j^{LD}+1}} LDC_j\right\}^{-\rho_j^{LD}}\right]^{-\frac{1}{\rho_j^{LD}}}$$

$$261.\quad LDC_j = WC_j^{\frac{1}{\rho_j^{LD}+1}} \; LDC_j \; B_j^{LD} \left(B_j^{LD}\right)^{-\frac{\rho_j^{LD}}{\rho_j^{LD}+1}} \left[\sum_l \beta_{l,j}^{LD} \left\{ \left[\frac{\beta_{l,j}^{LD}}{WTI_{l,j}} \right]^{\frac{1}{\rho_j^{LD}+1}} \right\}^{-\rho_j^{LD}} \right]^{-\frac{1}{\rho_j^{LD}}}$$

$$262.\quad 1 = WC_j^{\frac{1}{\rho_j^{LD}+1}} \left(B_j^{LD}\right)^{\frac{1}{\rho_j^{LD}+1}} \left[\sum_l \beta_{l,j}^{LD} \left\{ \left[\frac{\beta_{l,j}^{LD}}{WTI_{l,j}} \right]^{\frac{-\rho_j^{LD}}{\rho_j^{LD}+1}} \right\} \right]^{-\frac{1}{\rho_j^{LD}}}$$

$$263.\quad WC_j^{\frac{1}{\rho_j^{LD}+1}} \left(B_j^{LD}\right)^{\frac{1}{\rho_j^{LD}+1}} = \left[\sum_l \beta_{l,j}^{LD} \left\{ \left[\frac{\beta_{l,j}^{LD}}{WTI_{l,j}} \right]^{\frac{-\rho_j^{LD}}{\rho_j^{LD}+1}} \right\} \right]^{\frac{1}{\rho_j^{LD}}}$$

$$264.\quad WC_j \; B_j^{LD} = \left[\sum_l \beta_{l,j}^{LD} \left\{ \left[\frac{\beta_{l,j}^{LD}}{WTI_{l,j}} \right]^{\frac{-\rho_j^{LD}}{\rho_j^{LD}+1}} \right\} \right]^{\frac{\rho_j^{LD}+1}{\rho_j^{LD}}}$$

And equation 144 follows:

$$144.\quad WC_j = \frac{1}{B_j^{LD}} \left[\sum_l \left(\beta_{l,j}^{LD}\right)^{\frac{1}{\rho_j^{LD}+1}} \; WTI_{l,j}^{\frac{\rho_j^{LD}}{\rho_j^{LD}+1}} \right]^{\frac{\rho_j^{LD}+1}{\rho_j^{LD}}}$$

E2.2.2 Equations 6 and 144 together imply equation 72

Next, from equation 6, it follows that

265. $\sum_l WTI_{l,j}\ LD_{l,j} = \sum_l WTI_{l,j} \left[\frac{\beta_{l,j}^{LD}\ WC_j}{WTI_{l,j}}\right]^{\sigma_j^{LD}} \left(B_j^{LD}\right)^{\sigma_j^{LD}-1} LDC_j$

266. $\sum_l WTI_{l,j}\ LD_{l,j} = \sum_l WTI_{l,j}^{\,1-\sigma_j^{LD}} \left[\beta_{l,j}^{LD}\ WC_j\right]^{\sigma_j^{LD}} \left(B_j^{LD}\right)^{\sigma_j^{LD}-1} LDC_j$

267. $\sum_l WTI_{l,j}\ LD_{l,j} = \left(B_j^{LD}\right)^{\sigma_j^{LD}-1} LDC_j\ WC_j^{\,\sigma_j^{LD}} \sum_l \left(\beta_{l,j}^{LD}\right)^{\sigma_j^{LD}} WTI_{l,j}^{\,1-\sigma_j^{LD}}$

268. $\left(B_j^{LD}\right)^{1-\sigma_j^{LD}} LDC_j^{-1}\ WC_j^{\,-\sigma_j^{LD}} \sum_l WTI_{l,j}\ LD_{l,j} = \sum_l \left(\beta_{l,j}^{LD}\right)^{\sigma_j^{LD}} WTI_{l,j}^{\,1-\sigma_j^{LD}}$

Since $\sigma_j^{LD} = \dfrac{1}{\rho_j^{LD}+1}$, we have

269. $\left(B_j^{LD}\right)^{\frac{\rho_j^{LD}}{\rho_j^{LD}+1}} LDC_j^{-1}\ WC_j^{\,-\frac{1}{\rho_j^{LD}+1}} \sum_l WTI_{l,j}\ LD_{l,j} = \sum_l \left(\beta_{l,j}^{LD}\right)^{\frac{1}{\rho_j^{LD}+1}} WTI_{l,j}^{\,\frac{\rho_j^{LD}}{\rho_j^{LD}+1}}$

Substitute into equation 144, and find

270. $WC_j = \dfrac{1}{B_j^{LD}} \left[\left(B_j^{LD}\right)^{\frac{\rho_j^{LD}}{\rho_j^{LD}+1}} LDC_j^{-1}\ WC_j^{\,-\frac{1}{\rho_j^{LD}+1}} \sum_l WTI_{l,j}\ LD_{l,j}\right]^{\frac{\rho_j^{LD}+1}{\rho_j^{LD}}}$

271. $WC_j = WC_j^{\,-\frac{1}{\rho_j^{LD}}}\ LDC_j^{\,-\frac{\rho_j^{LD}+1}{\rho_j^{LD}}} \left[\sum_l WTI_{l,j}\ LD_{l,j}\right]^{\frac{\rho_j^{LD}+1}{\rho_j^{LD}}}$

272. $WC_j^{\,\frac{\rho_j^{LD}+1}{\rho_j^{LD}}}\ LDC_j^{\,\frac{\rho_j^{LD}+1}{\rho_j^{LD}}} = \left[\sum_l WTI_{l,j}\ LD_{l,j}\right]^{\frac{\rho_j^{LD}+1}{\rho_j^{LD}}}$

which is equivalent to equation 72.

E2.3 SUMMARY OF THE LOGICAL STRUCTURE

The logical developments just presented are summarized in Figures E1 and E2[18]. Figure E1 illustrates the following relationships:

- equations 5 and 134 together imply 139;
- equation 139 implies 134;
- equation 139 is equivalent to 135;
- equations 139 and 72 together imply 6;
- equation 6 implies 134;
- equations 134 and 135 together imply 5;
- equations 5 and 6 together imply 144;
- equations 6 and 144 together imply 72.

Figure E2 is more compact, because intermediate equations 135 and 144 are kept implicit. Figure E2 illustrates the following relationships:

- equations 5 and 134 together imply 139;
- equation 139 implies 134;
- equation 139 implies 5;
- equations 139 and 72 together imply 6;
- equation 6 implies 134;
- equations 5 and 6 together imply 72.

[18] Logical implication (« If *A* then *B* ») is represented by solid arrows (⇨). Logical equivalence (« If *A* then *B* **and** If *B* then *A* ») is represented by double-headed solid arrows (⬄) Braces indicate that an implication follows from the combination of two or more equations.

Figure E1: Logical structure of labor demand by category

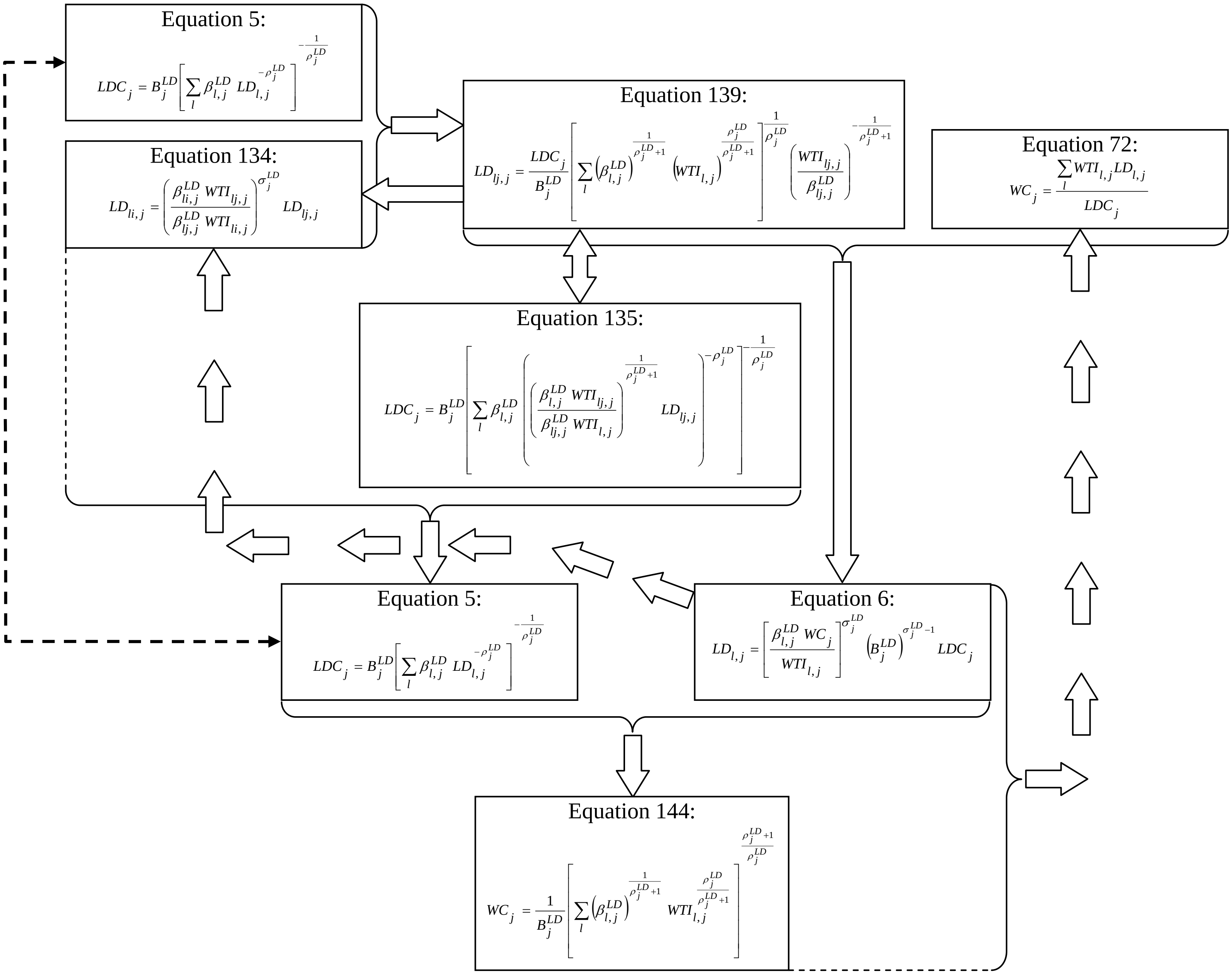
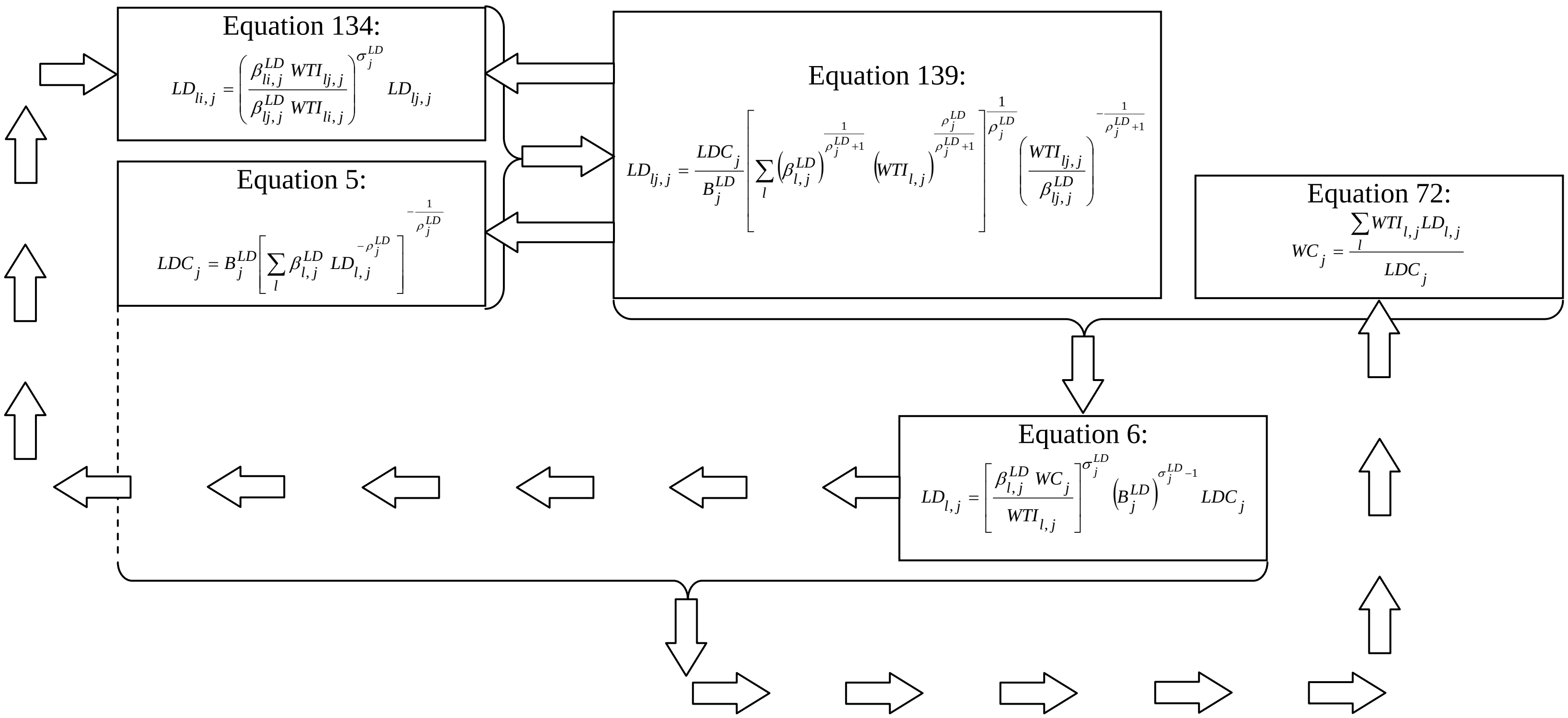


Figure E2: Logical structure of labor demand by category (compact form)



To summarize, given the logical structure described in Figures E1 and E2, the set of equations relating to labor demand by category and the price of composite labor may be formulated in three combinations:

- equations 5, 134, and 72;
- equations 139 and 72;
- equations 5 and 6.

The latter configuration is the one chosen in PEP-1-1, which implies that equation 72 is redundant. The number of equations is the same in all configurations. Let N_J be the number of industries, and N_L, the number of labor categories. Then

- equations 5 and 72 are families of N_J equations each;
- equations 139 and 6 are families of $N_J \times N_L$ equations each;
- but equation 134 is a family of $N_J \times (N_L - 1)$ equations.

In the case of equation 134, the equation with $LD_{lj,j}$ on the left hand side is a tautology:

$$273. \quad LD_{lj,j} \equiv \left(\frac{\beta_{lj,j}^{LD} \; WTI_{lj,j}}{\beta_{lj,j}^{LD} \; WTI_{lj,j}} \right)^{\sigma_j^{LD}} LD_{lj,j} \equiv LD_{lj,j}$$

So there are only $N_J \times (N_L - 1)$ independent equations 135.

The same configuration of equations that represents labor demand by category was also chosen to represent the demand for capital by category and the allocation of aggregate output to product supplies. The equations involved are, in the first case, equations 7, 8, and 74, and in the second case, equations 59, 60, and 77. Just as equation 72 is redundant given 5 and 6, so are 74 given 7 and 8, and 77 given 59 and 60. The underlying logical structure is similar to the one underlying the redundancy of equation 72.

On the other hand, when there are only two products or commodities involved, the configuration chosen in PEP-1-1 is analogous to the combination of equations 5, 72, and 135. The cases in point are: value added production (equations 3, 4, and 71), the allocation of production between domestic sales and exports (equations 61, 63, and 78), and the Armington demand for local products and imports (equations 65, 67, and 83). Of course, in these three cases, it would have been possible to apply the same configuration as for labor demand by category.

E3. Other slack variables

Just as the *LEON* variable was introduced as the imbalance of the equation chosen as redundant according to Walras’s Law, other slack variables could be introduced for each redundant equation that was discarded. In complex models, this device may be useful to verify that redundant equations are satisfied. It is illustrated in the variant *PEP_Base_1-1.v1-1_slack.gms* of the PEP-1-1 GAMS code.

APPENDIX F: MODEL PARAMETRIZATION

F1. What is model parametrization?

To implement a CGE model, values must be assigned to its parameters and exogenous variables. This process comprises two aspects: calibration of the parameters that can be determined from the information contained in the underlying social accounting matrix (SAM), and assignment of values to the so-called free parameters that remain.

Calibration can be considered a form of estimation. Unlike econometric estimation, however, calibration is not based on statistical inference procedures. Rather, it consists in determining parameter values on the basis of a detailed « snapshot » of the economy. Calibration goes back at least as far as the « structural analysis » approach of Leontief (1941, 1951, 1953) and Stone (1951, 1953, who coined the expression « social accounting matrix »). Indeed, Leontief input-output coefficients are computed from an input-output table as the ratios of input purchases over the value of production. And, given the extremely restrictive form of the Leontief fixed-proportions production function, all the parameters of the input-output model can be determined in that manner.

CGE models can be viewed as generalizations of input-output models. Of course, the latter are usually confined to the production system as represented by an input-output table, whereas CGE models are based on a SAM encompassing the whole of the economy. But the major difference lies in the fact that CGE models are, as their name indicates, models of economy-wide supply-and-demand equilibria regulated by the price system. In order for economic agents to be responsive to price changes, at least some of the functional forms which represent their behavior in CGE models have to be more flexible than simple fixed-proportions Leontief functions. However, more flexible functional forms have more parameters. Consequently, the information contained in the SAM is not sufficient to uniquely determine the values of all parameters.

The parameters that cannot be « calibrated » (that is, determined from the SAM) are called « free », and they must be assigned values by other methods. These include ad hoc econometric estimation, or, more frequently, a search of the empirical literature to find plausible values for the free parameters. In the demonstration version of PEP-1-1, the free parameters are assigned arbitrary values which, in the experience of the authors, are reasonable ballpark figures.

The parametrization strategy involves, first, determining which parameters are to be calibrated, and which are to be determined otherwise (in some cases, modelers have a choice of which of a given pair of parameters is to be calibrated). Secondly, it involves specifying adequate formulae to compute calibrated parameters from SAM values, together with the order in which the formulae are to be applied in the calibration procedure, which is sometimes critical.

The following pages describe the calibration procedure in the demonstration version of PEP-1-1.

F2. Assignments from SAM data

Figure F1 below reproduces the file *SAM-rec.prn*, which contains the SAM data that is read into the GAMS program. The gray parts of the SAM are supposed to be empty. Superimposed cartoon balloons describe the content of the non-empty cells of the SAM.

F2.1 NOTATION

In what follows, values retrieved from the SAM are denoted *SAM*{ }; inside the curly parentheses is a mathematical expression which corresponds to the description of a cell or group of cells in the SAM found in Figure F1. The initial values, or base values, of variables are identified by a superscript *O*. In the GAMS code, this is done by adding the letter « O » to the variable name. In the GAMS language, these initial values are parameters.

In order to economize on the number of symbols, some parameters or variables are given temporary values. Such temporary values are topped by a horizontal stroke. Some of these temporary values are revised more than once, so that some of the equations below are of the form $\bar{x} = f(\bar{x})$ (for example, F094 and F095); this way of writing assignments is often used in programming, but in mathematics, the two $\bar{x}$ should be represented by different symbols, because the left-hand side $\bar{x}$ is different from the right-hand side $\bar{x}$.

F2.2 ASSIGNMENT OF TEMPORARY BASE VALUES TO VOLUME VARIABLES

Once the SAM has been read into the GAMS program, the following base values of volume variables are assigned temporary values. These assignments are provisional, because SAM data are not volumes (quantities); they are nominal values.

F001. $\overline{C_{i,h}^{O}} = SAM\left\{ PC_i \; C_{i,h} \right\}$

F002. $\overline{CG_{i}^{O}} = SAM\left\{ PC_i \; CG_i \right\}$

F003. $\overline{DS_{j,i}^{O}} = SAM\left\{ PL_i \; DS_{j,i} \right\}$

F004. $\overline{DD_{i}^{O}} = \sum_j \overline{DS_{j,i}^{O}}$

F005. $\overline{DI_{i,j}^{O}} = SAM\left\{ PC_i \; DI_{i,j} \right\}$

F006. $\overline{EX_{j,x}^{O}} = SAM\left\{ PE_x \; EX_{j,x} \right\}$

F007. $\overline{EXD_{x}^{O}} = SAM\left\{ PE_x^{FOB} \; EXD_x \right\}$

F008. $\overline{INV_{i}^{O}} = SAM\left\{ PC_i \; INV_i \right\}$

F009. $\overline{VSTK_{i}^{O}} = SAM\left\{ PC_i \; VSTK_i \right\}$

F010. $\overline{IM_{m}^{O}} = SAM\left\{ e \; PWM_m \; IM_m \right\}$

F011. $\overline{KD_{k,j}^{O}} = SAM\left\{ R_{k,j} KD_{k,j} \right\}$

F012. $\overline{LD_{l,j}^{O}} = SAM\left\{ W_l LD_{l,j} \right\}$

F2.3 ASSIGNMENT OF BASE VALUES TO NOMINAL VARIABLES

Next, base values are assigned to the following nominal variables (variables defined in terms of value).

F013. $SF_f^O = SAM\left\{ SF_f \right\}$

F014. $SG^O = SAM\left\{ SG \right\}$

F015. $SH_h^O = SAM\left\{ SH_h \right\}$

F016. $SROW^O = SAM\left\{ SROW \right\}$

F017. $TDF_f^O = SAM\left\{ TDF_f \right\}$

F018. $TDH_h^O = SAM\left\{ TDH_h \right\}$

F019. $TIC_i^O = SAM\left\{ TIC_i \right\}$

F020. $TIK_{k,j}^O = SAM\left\{ TIK_{k,j} \right\}$

F021. $TIM_m^O = SAM\left\{ TIM_m \right\}$

F022. $TIP_j^O = SAM\left\{ TIP_j \right\}$

F023. $TIX_x^O = SAM\left\{ TIX_x \right\}$

F024. $TIW_{l,j}^O = SAM\left\{ TIW_{l,j} \right\}$

F025. $TR_{ag,agj}^O = SAM\left\{ TR_{ag,agj} \right\}$

It should be noted that $TR_{agd,ROW}$ and $TR_{agng,GVT}$ are indexed transfers, and their base value is revised below (F4.4) given the base value of the consumer price index. However, in the present calibration procedure of PEP-1-1, the base value of the consumer price index is 1 and the revised value of the indexed transfers is equal to their temporary value.

F2.4 ASSIGNMENT OF TEMPORARY VALUES TO PARAMETERS

Finally, some parameters are assigned temporary values from SAM data.

F026. $\overline{\lambda_{ag,k}^{RK}} = SAM\left\{\lambda_{ag,k}^{RK}\left(\sum_j R_{k,j} KD_{k,j}\right)\right\}$

F027. $\overline{\lambda_{h,l}^{WL}} = SAM\left\{\lambda_{h,l}^{WL}\left(W_l \sum_j LD_{l,j}\right)\right\}$

F028. $\overline{tmrg_{i,m}} = SAM\left\{PC_i tmrg_{i,m}\left(DD_m + IM_m\right)\right\}$

F029. $\overline{tmrg_{i,nm}} = SAM\left\{PC_i\ tmrg_{i,nm}\ DD_{nm}\right\}$

F030. $\overline{tmrg_{i,x}^{X}} = SAM\left\{PC_i\ tmrg_{i,x}^{X}\ EXD_x\right\}$

F3. Free parameters

In the demonstration version of PEP-1-1, free parameters are assigned the following values.

F3.1 PRICE ELASTICITY OF INDEXED VALUES

The price elasticity of indexed values should be set equal to one when verifying model homogeneity.

F031. $\eta = 1$

F3.2 CES AND CET ELASTICITIES

F032. $\sigma_j^{KD} = 0.8$

F033. $\sigma_j^{LD} = 0.8$

F034. $\sigma_m^{M} = 2$

F035. $\sigma_j^{VA} = 1.5$

F036. $\sigma_{j,x}^{X} = 2$

F037. $\sigma_j^{XT} = 2$

F3.3 ELASTICITY OF INTERNATIONAL DEMAND FOR EXPORTED COMMODITY X

F038. $\sigma_x^{XD} = 2$

F3.4 LES PARAMETERS

F039. $Frisch_h = -1.5$

F040. $\overline{\sigma_{AGR,h}^{Y}} = 0.7$

F041. $\overline{\sigma_{FOOD,h}^{Y}} = 1.1$

F042. $\overline{\sigma_{OTHIND,h}^{Y}} = 1.1$

F043. $\overline{\sigma_{SER,h}^{Y}} = 1.05$

F044. $\overline{\sigma_{ADM,h}^{Y}} = 1.05$

F3.4 INTERCEPTS OF TRANSFERS, DIRECT TAXES AND SAVINGS

In household savings and transfers-to-government functions, and in income-tax functions, one can choose to assign a value to the intercept and calibrate the slope accordingly, or the other way around. This type of modelling can be useful to take into account known marginal savings or taxation rates or to deal with negative average saving rates in cases where savings are negative for some household groups. When no specific information is available, one can simply set the intercepts to zero and calibrate an average rate: this is what we do here.

F045. $sh0_h = 0$

F046. $tr0_h = 0$

F047. $ttdf0_f = 0$

F048. $ttdh0_h = 0$

It should be noted that these intercepts are indexed, and their value is revised below (F4.4) given the base value of the consumer price index. However, in the present calibration procedure of PEP-1-1, the base value of the consumer price index is 1 and the revised value of the indexed intercepts is equal to their temporary value.

F3.5 EXOGENOUS PRICES

The base value of some prices is arbitrary, insofar as it is constrained only by the price × quantity product. In such cases, the arbitrary value assigned to the price implicitely determines the measurement unit of the quantity. The most convenient arbitrary price is obviously 1.

F049. $PL_i^O = 1$

F050. $PE_x^O = 1$

F051. $e^{O} = 1$

F052. $PWM_{m}^{O} = 1$

F053. $W_{l}^{O} = 1$

F054. $R_{k}^{O} = 1$

F055. $R_{k,j}^{O} = R_{k}^{O}$

F4. Calibration

F4.1 NOMINAL VARIABLES

The base values of nominal variables determined in equations 10-15 are computed in the following sequence:

F056. $YHK_{h}^{O} = \sum_{k} \overline{\lambda_{h,k}^{RK}}$

F057. $YHL_{h}^{O} = \sum_{l} \overline{\lambda_{h,l}^{WL}}$

F058. $YHTR_{h}^{O} = \sum_{ag} TR_{h,ag}^{O}$

F059. $YH_{h}^{O} = YHL_{h}^{O} + YHK_{h}^{O} + YHTR_{h}^{O}$

F060. $YDH_{h}^{O} = YH_{h}^{O} - TDH_{h}^{O} - TR_{gvt,h}^{O}$

F061. $CTH_{h}^{O} = YDH_{h}^{O} - SH_{h}^{O} - \sum_{agng} TR_{agng,h}^{O}$

The base values of nominal variables determined in equations 17-20 are computed in the following sequence:

F062. $YFK_{f}^{O} = \sum_{k} \overline{\lambda_{f,k}^{RK}}$

F063. $YFTR_{f}^{O} = \sum_{ag} TR_{f,ag}^{O}$

F064. $YF_{f}^{O} = YFK_{f}^{O} + YFTR_{f}^{O}$

F065. $YDF_{f}^{O} = YF_{f}^{O} - TDF_{f}^{O}$

The base values of nominal variables determined in equations 23-34 are computed in the following sequence:

F066. $YGK^{O} = \sum_{k} \overline{\lambda_{gvt,k}^{RK}}$

F067. $TDHT^{O} = \sum_{h} TDH_{h}^{O}$

F068. $TDFT^{O} = \sum_{f} TDF_{f}^{O}$

F069. $TICT^{O} = \sum_{i} TIC_{i}^{O}$

F070. $TIMT^{O} = \sum_{m} TIM_{m}^{O}$

F071. $TIXT^{O} = \sum_{x} TIX_{x}^{O}$

F072. $TIWT^{O} = \sum_{l,j} TIW_{l,j}^{O}$

F073. $TIKT^{O} = \sum_{k,j} TIK_{k,j}^{O}$

F074. $TIPT^{O} = \sum_{j} TIP_{j}^{O}$

F075. $TPRODN^{O} = TIWT^{O} + TIKT^{O} + TIPT^{O}$

F076. $TPRCTS^{O} = TICT^{O} + TIMT^{O} + TIXT^{O}$

F077. $YGTR^{O} = \sum_{agng} TR_{gvt,agng}^{O}$

F078. $YG^{O} = YGK^{O} + TDHT^{O} + TDFT^{O} + TPRODN^{O} + TPRCTS^{O} + YGTR^{O}$

From equations 45 and 47:

F079. $YROW^{O} = \sum_{m} IM_{m}^{O} + \sum_{k} \overline{\lambda_{row,k}^{RK}} + \sum_{agd} TR_{row,agd}^{O}$

F080. $CAB^{O} = -SROW^{O}$

Equation 92 yields

F081. $IT^{O} = \sum_{h} SH_{h}^{O} + \sum_{f} SF_{f}^{O} + SG^{O} + SROW^{O}$

F4.2 REVISED PARAMETER ASSIGNMENTS

Revised values are assigned to some parameters which had been assigned temporary values.

F082. $\lambda_{ag,k}^{RK} = \dfrac{\overline{\lambda_{ag,k}^{RK}}}{\sum_{j} KD_{k,j}^{O}}$

F083. $\lambda_{h,l}^{WL} = \dfrac{\overline{\lambda_{h,l}^{WL}}}{\sum_{j} LD_{l,j}^{O}}$

F4.3 PARAMETERS

F4.3.1 Miscelaneous

F084. $\lambda_{agng,h}^{TR} = \dfrac{TR_{agng,h}^{O}}{YDH_{h}^{O}}$

F085. $\lambda_{ag,f}^{TR} = \dfrac{TR_{ag,f}^{O}}{YDF_{f}^{O}}$

F086. $sh1_{h} = \dfrac{SH_{h}^{O} - \overline{sh0_{h}}}{YDH_{h}^{O}}$

F087. $tr1_{h} = \dfrac{TR_{GVT,h}^{O} - \overline{tr0_{h}}}{YH_{h}^{O}}$

F4.3.2 Investment and government expenditure shares

F088. $\gamma_{i}^{GVT} = \dfrac{\overline{CG_{i}^{O}}}{\sum_{ij} CG_{ij}^{O}}$

F089. $\gamma_{i}^{INV} = \dfrac{\overline{INV_{i}^{O}}}{\sum_{ij} INV_{ij}^{O}}$

F4.3.3 Margin rates, tax rates, prices and volumes

F090. $$ttdf1_{f} = \frac{TDF_{f}^{O} - \overline{ttdf0_{f}}}{YFK_{f}^{O}}$$

F091. $$ttdh1_{h} = \frac{TDH_{h}^{O} - \overline{ttdh0_{h}}}{YH_{h}^{O}}$$

F092. $$PC_{m}^{O} = \frac{\overline{DD_{m}^{O}} + \overline{IM_{m}^{O}} + \sum_{i} \overline{tmrg_{i,m}} + TIC_{m}^{O} + TIM_{m}^{O}}{\overline{DD_{m}^{O}} + \overline{IM_{m}^{O}}}$$

F093. $$PC_{nm}^{O} = \frac{\overline{DD_{nm}^{O}} + \sum_{i} \overline{tmrg_{i,nm}} + TIC_{nm}^{O}}{\overline{DD_{m}^{O}}}$$

F094. $$\overline{tmrg_{i,ij}} = \frac{\overline{tmrg_{i,ij}}}{PC_{i}^{O}}$$

F095. $$\overline{tmrg_{i,ij}^{X}} = \frac{\overline{tmrg_{i,ij}^{X}}}{PC_{i}^{O}}$$

F096. $$DD_{i}^{O} = \frac{\overline{DD_{i}^{O}}}{PL_{i}^{O}}$$

F097. $$IM_{m}^{O} = \frac{\overline{IM_{m}^{O}}}{e^{O}\ PWM_{m}^{O}}$$

F098. $$tmrg_{i,nm} = \frac{\overline{tmrg_{i,nm}}}{DD_{nm}^{O}}$$

F099. $$tmrg_{i,m} = \frac{\overline{tmrg_{i,m}}}{DD_{nm}^{O} + IM_{m}^{O}}$$

F100. $$ttic_{nm}^{O} = \frac{TIC_{nm}^{O}}{\left(PL_{nm}^{O} + \sum_{i} PC_{i}^{O} tmrg_{i,nm}\right) DD_{nm}^{O}}$$

F101. $$ttic_{m}^{O} = \frac{TIC_{m}^{O}}{\left(PL_{m}^{O} + \sum_{i} PC_{i}^{O} tmrg_{i,m}\right) DD_{m}^{O} + \left(e^{O} PWM_{m}^{O} + \sum_{i} PC_{i}^{O} tmrg_{i,m}\right) IM_{m}^{O}}$$

F102. $PD_{i}^{O} = \left(1 + ttic_{i}\right)\left(PL_{i}^{O} + \sum_{ij} PC_{ij}^{O} \; tmrg_{ij,i}\right)$

F103. $ttim_{m} = \frac{TIM_{m}^{O}}{e^{O} PWM_{m}^{O} IM_{m}^{O}}$

From equation 82:

F104. $PM_{m}^{O} = \left(1 + ttic_{m}\right)\left(\left(1 + ttim_{m}\right) e^{O} \; PWM_{m}^{O} + \sum_{i} PC_{i}^{O} tmrg_{i,m}\right)$

F105. $EX_{j,x}^{O} = \frac{\overline{EX_{j,x}^{O}}}{PE_{x}^{O}}$

F106. $tmrg_{x}^{X} = \frac{\overline{tmrg_{x}^{X}}}{\sum_{j} EX_{j,x}^{O}}$

F107. $ttix_{x} = \frac{TIX_{x}^{O}}{\overline{EXD_{x}^{O}} - TIX_{x}^{O}}$

From equation 80, compute the FOB price of exports:

F108. $PE_{x}^{FOB\_O} = PE_{x}^{O} \left(1 + \sum_{i} tmrg_{i,x}^{X}\right)\left(1 + ttix_{x}\right)$

Next, in order that the base value of EXD_{x} be consistent with equation 64, set

F109. $PWX_{x}^{O} = \frac{PE_{x}^{FOB\_O}}{e^{O}}$

And then revise the temporary values of the following:

F110. $EXD_{x}^{O} = \frac{\overline{EXD_{x}^{O}}}{e^{O} \; PWX_{x}^{O}}$

From equations 62 and 79:

F111. $XS_{j,nx}^{O} = DS_{j,nx}^{O}$

F112. $P_{j,nx}^{O} = PL_{nx}^{O}$

F113. $DS_i^O = \frac{\overline{DS_i^O}}{PL_i^O}$

The following assignment may be seen as a normalization rule which implicitely defines the measurement unit of $XS_{j,x}$, the base value of its price given PL_x^O and PE_x^O, and the parameter $B_{j,x}^X$ given the aggregator function of equation 61.

F114. $XS_{j,x}^O = DS_{j,x}^O + EX_{j,x}^O$

From equation 78:

F115. $P_{j,x}^O = \frac{PL_x^O DS_{j,x}^O + PE_x^O EX_{j,x}^O}{XS_{j,x}^O}$

The following assignment may be seen as a normalization rule which implicitely defines the measurement unit of XST_j, the base value of its price given the $P_{j,x}^O$, and the parameter $B_{j,x}^{XT}$ given the aggregator function of equation 59.

F116. $XST_j^O = \sum_i XS_{j,i}^O$

From equation 77:

F117. $PT_j^O = \frac{\sum_i P_{j,i}^O XS_{j,i}^O}{XST_j^O}$

The following assignment may be seen as a normalization rule which implicitely defines the measurement unit of Q_m, the base value of its price given PM_m^O and PD_m^O, and the parameter B_m^M given the aggregator function of equation 65.

F118. $Q_m^O = IM_m^O + DD_m^O$

From equation 66:

F119. $Q_{nm}^O = DD_{nm}^O$

From equation 58:

F120. $MRGN_i^O = \sum_{ij} tmrg_{i,ij} DD_{ij}^O + \sum_m tmrg_{i,m}\ IM_m^O + \sum_{j,x} tmrg_{i,x}^X\ EX_{j,x}^O$

Conversion of other temporary base values of volume variables into true volumes:

F121. $C_i^O = \frac{\overline{C_i^O}}{PC_i^O}$

F122. $CG_i^O = \frac{\overline{CG_i^O}}{PC_i^O}$

F123. $DI_{i,j}^O = \frac{\overline{DI_{i,j}^O}}{PC_i^O}$

F124. $INV_i^O = \frac{\overline{INV_i^O}}{PC_i^O}$

F125. $VSTK_i^O = \frac{\overline{VSTK_i^O}}{PC_i^O}$

From equation 54:

F126. $GFCF^O = IT^O - \sum_i PC_i^O \; VSTK_i^O$

The following assignment may be seen as a normalization rule which implicitely defines the measurement unit of CI_j (the aggregate intermediate demand of industry *j*), and, hence, the base value of its price given the PC_i^O, and also, given equation 9, implicitely scales the $aij_{i,j}$ input-output coefficients so that their sum is equal to 1.

F127. $CI_j^O = \sum_i DI_{i,j}^O$

From equation 57:

F128. $DIT_i^O = \sum_j DI_{i,j}^O$

Applying the definition of total government expenditures on goods and services yields

F129. $G^O = \sum_i PC_i^O \; CG_i^O$

The price of industry *j*'s aggregate intermediate input, given the PC_i^O , is given by

F130. $PCI_j^O = \frac{\sum_i PC_i^O \; DI_{i,j}^O}{CI_j^O}$

F131. $ttiw_{l,j} = \frac{\overline{TIW_{l,j}^O}}{\overline{LD_{l,j}^O}}$

From equations 73 and 75:

F132. $WTI_{l,j}^O = W_l^O \left(1 + ttiw_{l,j}\right)$

F133. $ttik_{k,j} = \frac{\overline{TIK_{k,j}^O}}{\overline{KD_{k,j}^O}}$

F134. $RTI_{k,j}^O = R_{k,j}^O \left(1 + ttik_{k,j}\right)$

Conversion of temporary base values of labor volume variables into true volumes, given wage rates W_l^O :

F135. $LD_{l,j}^O = \frac{\overline{LD_{l,j}^O}}{W_l^O}$

The following assignment may be seen as a normalization rule which implicitely defines the measurement unit of LDC_j (the aggregate labor demand of industry *j*), and, hence, the base value of aggregate wage rate WC_j given the $WTI_{l,j}^O$, and the parameter B_j^{LD} given the aggregator function of equation 5.

F136. $LDC_j^O = \sum_l LD_{l,j}^O$

From equation 72:

F137. $WC_j^O = \frac{\sum_l WTI_{l,j}^O LD_{l,j}^O}{LDC_j^O}$

Conversion of temporary base values of capital volume variables into true volumes, given rental rates $R_{k,j}^O$:

F138. $KD_{k,j}^O = \frac{\overline{KD_{k,j}^O}}{R_{k,j}^O}$

The following assignment may be seen as a normalization rule which implicitely defines the measurement unit of KDC_j (the aggregate capital demand of industry j), and, hence, the base value of aggregate rental rate RC_j given the $RTI_{k,j}^O$, and the parameter B_j^{KD} given the aggregator function of equation 7.

F139. $KDC_j^O = \sum_k KD_{k,j}^O$

From equation 91:

F140. $KS_k^O = \sum_j KD_{k,j}^O$

From equation 74, the base value of aggregate rental rate RC_j given the $RTI_{k,j}^O$ is computed to be consistent with the normalization rule applied above:

F141. $RC_j^O = \dfrac{\sum_k RTI_{k,j}^O KD_{k,j}^O}{KDC_j^O}$

The following assignment may be seen as a normalization rule which implicitely defines the measurement unit of VA_j (the aggregate value added of industry j), and, hence, the base value of its price PVA_j given WC_j^O and RC_j^O, and the parameter B_j^{VA} given the aggregator function of equation 3.

F142. $VA_j^O = LDC_j^O + KDC_j^O$

Then, from equation 71:

F143. $PVA_j^O = \dfrac{WC_j^O LDC_j^O + RC_j^O KDC_j^O}{VA_j^O}$

F144. $ttip_j = \dfrac{TIP_j^O}{PVA_j^O VA_j^O + \sum_i PC_i^O DIO_{i,j}^O}$

F145. $PP_j^O = \dfrac{PT_j^O}{\left(1 + ttip_j\right)}$

The base values of price indexes are computed from equations 85-88. It is easily verified that they are all equal to 1.

F146. $$PIXGDP^{O} = \sqrt{\frac{\sum_{j} PVA_{j}^{O} VA_{j}^{O}}{\sum_{j} PVA_{j}^{O} VA_{j}^{O}} \frac{\sum_{j} PVA_{j}^{O} VA_{j}^{O}}{\sum_{j} PVA_{j}^{O} VA_{j}^{O}}}$$

F147. $$PIXCON^{O} = \frac{\sum_{i} PC_{i}^{O} \sum_{h} C_{i,h}^{O}}{\sum_{i} PC_{i}^{O} \sum_{h} C_{i,h}^{O}}$$

F148. $$PIXGVT^{O} = \prod_{i} \left(\frac{PC_{i}^{O}}{PC_{i}^{O}} \right)^{\gamma_{i}^{GVT}}$$

F149. $$PIXINV^{O} = \prod_{i} \left(\frac{PC_{i}^{O}}{PC_{i}^{O}} \right)^{\gamma_{i}^{INV}}$$

F4.4 INDEXED TRANSFERS AND INTERCEPTS

Indexed transfers and intercepts are now revised in accordance with the base value of the consumer price index. Given that the base value of the consumer price index is 1, the revised values are equal to the original ones: these assignments are executed merely as a matter of caution, so that the calibration remains consistent in the case of changes in the calibration procedure.

F150. $$TR_{agd,ROW}^{O} = \frac{\overline{TR_{agd,ROW}^{O}}}{\left(PIXCON^{O}\right)^{\eta}}$$

F151. $$TR_{agng,GVT}^{O} = \frac{\overline{TR_{agng,GVT}^{O}}}{\left(PIXCON^{O}\right)^{\eta}}$$

F152. $$ttdf0_{f} = \frac{\overline{ttdf0_{f}}}{\left(PIXCON^{O}\right)^{\eta}}$$

F153. $$ttdh0_{h} = \frac{\overline{ttdh0_{h}}}{\left(PIXCON^{O}\right)^{\eta}}$$

F154. $$sh0_{h} = \frac{\overline{sh0_{h}}}{\left(PIXCON^{O}\right)^{\eta}}$$

F155. $tr0_h = \dfrac{\overline{tr0_h}}{\left(PIXCON^O\right)^{\eta}}$

F4.5 LEONTIEF INPUT COEFFICIENTS

Leontief input coefficients are calibrated as volume ratios:

F156. $io_j = \dfrac{CI_j^O}{XST_j^O}$

F157. $v_j = \dfrac{VA_j^O}{XST_j^O}$

F158. $aij_{i,j} = \dfrac{DI_{i,j}^O}{CI_j^O}$

F4.6 CET PARAMETERS

F4.6.1 CET between commodities

In accordance with the algebra of the CET transformation function (Appendix C5),

F159. $\rho_j^{XT} = \dfrac{1+\sigma_j^{XT}}{\sigma_j^{XT}}$

Next comes the calibration of the $\beta_{j,i}^{XT}$ .First, note that these are defined only up to a factor of proportionality. Indeed, let

F160. $\beta_{j,i}^{XT*} = \mu\beta_{j,i}^{XT}$

and

F161. $\left[\sum_i \beta_{j,i}^{XT*} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}} = \left[\sum_i \mu\beta_{j,i}^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}} = \mu^{\frac{1}{\rho_j^{XT}}}\left[\sum_i \beta_{j,i}^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}}$

Now let

F162. $B_j^{XT*} = \mu^{-\frac{1}{\rho_j^{XT}}} B_j^{XT}$

and it follows that

F163. $$XST_j = B_j^{XT*}\left[\sum_i \beta_{j,i}^{XT*} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}} = \mu^{-\frac{1}{\rho_j^{XT}}} B_j^{XT} \mu^{\frac{1}{\rho_j^{XT}}}\left[\sum_i \beta_{j,i}^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}}$$

F164. $$XST_j = B_j^{XT*}\left[\sum_i \beta_{j,i}^{XT*} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}} = B_j^{XT}\left[\sum_i \beta_{j,i}^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}}$$

With that in mind, develop equation 60 to obtain relative commodity supply:

F165. $$\frac{XS_{j,i}}{XS_{j,ij}} = \left[\frac{P_{j,i}}{\beta_{j,i}^{XT}}\frac{\beta_{j,ij}^{XT}}{P_{j,ij}}\right]^{\sigma_j^{XT}}$$

F166. $$\frac{XS_{j,i}}{XS_{j,ij}} = \left[\frac{P_{j,i}}{\beta_{j,i}^{XT}}\frac{\beta_{j,ij}^{XT}}{P_{j,ij}}\right]^{\frac{1}{\rho_j^{XT}-1}}$$

F167. $$\frac{XS_{j,i}}{XS_{j,ij}} = \left[\frac{\beta_{j,i}^{XT}}{P_{j,i}}\frac{P_{j,ij}}{\beta_{j,ij}^{XT}}\right]^{\frac{1}{1-\rho_j^{XT}}}$$

F168. $$\left(\frac{XS_{j,i}}{XS_{j,ij}}\right)^{1-\rho_j^{XT}} = \frac{\beta_{j,i}^{XT}}{P_{j,i}}\frac{P_{j,ij}}{\beta_{j,ij}^{XT}}$$

F169. $$\beta_{j,ij}^{XT} P_{j,i} XS_{j,i}^{1-\rho_j^{XT}} = \beta_{j,i}^{XT} P_{j,ij} XS_{j,ij}^{1-\rho_j^{XT}}$$

F170. $$\sum_{ij}\beta_{j,ij}^{XT} P_{j,i} XS_{j,i}^{1-\rho_j^{XT}} = \sum_{ij}\beta_{j,i}^{XT} P_{j,ij} XS_{j,ij}^{1-\rho_j^{XT}}$$

F171. $$P_{j,i} XS_{j,i}^{1-\rho_j^{XT}} \sum_{ij}\beta_{j,ij}^{XT} = \beta_{j,i}^{XT} \sum_{ij} P_{j,ij} XS_{j,ij}^{1-\rho_j^{XT}}$$

We have shown that the $\beta_{j,i}^{XT}$ are defined only up to a factor of proportionality. Therefore, we can, without loss of generality, impose the normalization rule $\sum_{ij}\beta_{j,ij}^{XT} = 1$. It then follows that

F172. $$P_{j,i} XS_{j,i}^{1-\rho_j^{XT}} = \beta_{j,i}^{XT} \sum_{ij} P_{j,ij} XS_{j,ij}^{1-\rho_j^{XT}}$$

Whence

F173. $$\beta_{j,i}^{XT} = \frac{P_{j,i} XS_{j,i}^{1-\rho_j^{XT}}}{\sum_{ij} P_{j,ij} XS_{j,ij}^{1-\rho_j^{XT}}}$$

Then, from equation 59

F174. $$B_j^{XT} = \frac{XST_j^O}{\left[\sum_i \beta_{j,i}^{XT} \left(XS_{j,i}^O\right)^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}}}$$

F4.6.2 CET between exports and local production

In accordance with the algebra of the CET transformation function (Appendix C6),

F175. $$\rho_{j,x}^X = \frac{1+\sigma_{j,x}^X}{\sigma_{j,x}^X}$$

Following the same calibration strategy as for the CET between commodities, develop equation 63:

F176. $$\frac{EX_{j,x}}{DS_{j,x}} = \left[\frac{1-\beta_{j,x}^X}{\beta_{j,x}^X}\frac{PE_x}{PL_x}\right]^{\sigma_{j,x}^X}$$

F177. $$\frac{EX_{j,x}}{DS_{j,x}} = \left[\frac{1-\beta_{j,x}^X}{\beta_{j,x}^X}\frac{PE_x}{PL_x}\right]^{\frac{1}{\rho_{j,x}^X-1}}$$

F178. $$\frac{EX_{j,x}}{DS_{j,x}} = \left[\frac{\beta_{j,x}^X}{1-\beta_{j,x}^X}\frac{PL_x}{PE_x}\right]^{\frac{1}{1-\rho_{j,x}^X}}$$

F179. $$\left(\frac{EX_{j,x}}{DS_{j,x}}\right)^{1-\rho_{j,x}^X} = \frac{\beta_{j,x}^X}{1-\beta_{j,x}^X}\frac{PL_x}{PE_x}$$

F180. $$\beta_{j,x}^X PL_x DS_{j,x}^{1-\rho_{j,x}^X} = \left(1-\beta_{j,x}^X\right) PE_x EX_{j,x}^{1-\rho_{j,x}^X}$$

F181. $$\beta_{j,x}^{X} PL_x DS_{j,x}^{1-\rho_{j,x}^{X}} = PE_x EX_{j,x}^{1-\rho_{j,x}^{X}} - \beta_{j,x}^{X} PE_x EX_{j,x}^{1-\rho_{j,x}^{X}}$$

F182. $$\beta_{j,x}^{X}\left(PL_x DS_{j,x}^{1-\rho_{j,x}^{X}} + PE_x EX_{j,x}^{1-\rho_{j,x}^{X}} \right) = PE_x EX_{j,x}^{1-\rho_{j,x}^{X}}$$

F183. $$\beta_{j,x}^{X} = \frac{PE_x EX_{j,x}^{1-\rho_{j,x}^{X}}}{PL_x DS_{j,x}^{1-\rho_{j,x}^{X}} + PE_x EX_{j,x}^{1-\rho_{j,x}^{X}}}$$

And from equation 61

F184. $$B_{j,x}^{X} = \frac{XS_{j,x}^{O}}{\left[\beta_{j,x}^{X}\left(EX_{j,x}^{O}\right)^{\rho_{j,x}^{X}} + \left(1-\beta_{j,x}^{X}\right)\left(DS_{j,x}^{O}\right)^{\rho_{j,x}^{X}} \right]^{\frac{1}{\rho_{j,x}^{X}}}}$$

F4.7 CES PARAMETERS

F4.7.1 Composite good

In accordance with the algebra of CES aggregator functions (Appendix C7),

F185. $$\rho_m^M = \frac{1-\sigma_m^M}{\sigma_m^M}$$

Develop equation 67:

F186. $$\frac{IM_m}{DD_m} = \left[\frac{\beta_m^M}{1-\beta_m^M} \frac{PD_m}{PM_m} \right]^{\sigma_m^M}$$

F187. $$\frac{IM_m}{DD_m} = \left[\frac{\beta_m^M}{1-\beta_m^M} \frac{PD_m}{PM_m} \right]^{\frac{1}{\rho_m^M+1}}$$

F188. $$\left(\frac{IM_m}{DD_m} \right)^{\rho_m^M+1} = \frac{\beta_m^M}{1-\beta_m^M} \frac{PD_m}{PM_m}$$

F189. $$\beta_m^M PD_m DD_m^{\rho_m^M+1} = \left(1-\beta_m^M\right) PM_m IM_m^{\rho_m^M+1}$$

F190. $\beta_m^M \left(PD_m DD_m^{\rho_m^M+1} + PM_m IM_m^{\rho_m^M+1} \right) = PM_m IM_m^{\rho_m^M+1}$

F191. $\beta_m^M = \dfrac{PM_m IM_m^{\rho_m^M+1}}{PD_m DD_m^{\rho_m^M+1} + PM_m IM_m^{\rho_m^M+1}}$

From equation 65

F192.. $B_m^M = \dfrac{Q_m^O}{\left[\beta_m^M \left(IM_m^O\right)^{-\rho_m^M} + \left(1-\beta_m^M\right)\left(DD_m^O\right)^{-\rho_m^M} \right]^{-\frac{1}{\rho_m^M}}}$

F4.7.2 Composite capital

In accordance with the algebra of the CES production function (Appendix C2),

F193. $\rho_j^{KD} = \dfrac{1-\sigma_j^{KD}}{\sigma_j^{KD}}$

The calibration of the $\beta_{k,j}^{KD}$ follows the same strategy as that of the $\beta_{j,i}^{XT}$.

Develop equation 8 to obtain relative demand:

F194. $\dfrac{KD_{k,j}}{KD_{kj,j}} = \left[\dfrac{\beta_{k,j}^{KD}}{\beta_{kj,j}^{KD}} \dfrac{RTI_{kj,j}}{RTI_{k,j}} \right]^{\sigma_j^{KD}}$

F195. $\dfrac{KD_{k,j}}{KD_{kj,j}} = \left[\dfrac{\beta_{k,j}^{KD}}{\beta_{kj,j}^{KD}} \dfrac{RTI_{kj,j}}{RTI_{k,j}} \right]^{\frac{1}{\rho_j^{KD}+1}}$

F196. $\left(\dfrac{KD_{k,j}}{KD_{kj,j}} \right)^{\rho_j^{KD}+1} = \dfrac{\beta_{k,j}^{KD}}{\beta_{kj,j}^{KD}} \dfrac{RTI_{kj,j}}{RTI_{k,j}}$

F197. $\beta_{kj,j}^{KD} RTI_{k,j} KD_{k,j}^{\rho_j^{KD}+1} = \beta_{k,j}^{KD} RTI_{kj,j} KD_{kj,j}^{\rho_j^{KD}+1}$

F198. $\sum_{kj} \beta_{kj,j}^{KD} RTI_{k,j} KD_{k,j}^{\rho_j^{KD}+1} = \sum_{kj} \beta_{k,j}^{KD} RTI_{kj,j} KD_{kj,j}^{\rho_j^{KD}+1}$

F199. $RTI_{k,j} KD_{k,j}^{\rho_j^{KD}+1} \sum_{kj} \beta_{kj,j}^{KD} = \beta_{k,j}^{KD} \sum_{kj} RTI_{kj,j} KD_{kj,j}^{\rho_j^{KD}+1}$

We know that the $\beta_{k,j}^{KD}$, like the $\beta_{j,i}^{XT}$, are defined only up to a factor of proportionality. Therefore, we can, without loss of generality, impose the normalization rule $\sum_{kj} \beta_{kj,j}^{KD} = 1$. It then follows that

F200. $RTI_{k,j} KD_{k,j}^{\rho_j^{KD}+1} = \beta_{k,j}^{KD} \sum_{kj} RTI_{kj,j} KD_{kj,j}^{\rho_j^{KD}+1}$

Whence

F201. $$\beta_{k,j}^{KD} = \frac{RTI_{k,j}^{O} \left(KD_{k,j}^{O} \right)^{\rho_j^{KD}+1}}{\sum_{kj} RTI_{kj,j}^{O} \left(KD_{kj,j}^{O} \right)^{\rho_j^{KD}+1}}$$

Then, from equation 7

F202. $$B_j^{KD} = \frac{KDC_j^{O}}{\left[\sum_{k} \beta_{k,j}^{KD} \left(KD_{k,j}^{O} \right)^{-\rho_j^{KD}} \right]^{-\frac{1}{\rho_j^{KD}}}}$$

F4.7.3 Composite labor

In accordance with the algebra of the CES production function (Appendix C2),

F203. $$\rho_j^{LD} = \frac{1-\sigma_j^{LD}}{\sigma_j^{LD}}$$

The calibration of the $\beta_{l,j}^{LD}$ follows the same strategy as that of the $\beta_{j,i}^{XT}$.

Develop equation 5 to obtain relative demand:

F204. $$\frac{LD_{l,j}}{LD_{lj,j}} = \left[\frac{\beta_{l,j}^{LD}}{\beta_{lj,j}^{LD}} \frac{WTI_{lj,j}}{WTI_{l,j}} \right]^{\sigma_j^{LD}}$$

F205. $\frac{LD_{l,j}}{LD_{lj,j}} = \left[\frac{\beta_{l,j}^{LD}}{\beta_{lj,j}^{LD}} \frac{WTI_{lj,j}}{WTI_{l,j}} \right]^{\frac{1}{\rho_j^{LD}+1}}$

F206. $\left(\frac{LD_{l,j}}{LD_{lj,j}} \right)^{\rho_j^{LD}+1} = \frac{\beta_{l,j}^{LD}}{\beta_{lj,j}^{LD}} \frac{WTI_{lj,j}}{WTI_{l,j}}$

F207. $WTI_{l,j} \left(\frac{LD_{l,j}}{LD_{lj,j}} \right)^{\rho_j^{LD}+1} = WTI_{lj,j} \frac{\beta_{l,j}^{LD}}{\beta_{lj,j}^{LD}}$

F208. $\beta_{lj,j}^{LD} WTI_{l,j} LD_{l,j}^{\rho_j^{LD}+1} = \beta_{l,j}^{LD} WTI_{lj,j} LD_{lj,j}^{\rho_j^{LD}+1}$

F209. $\sum_{lj} \beta_{lj,j}^{LD} WTI_{l,j} LD_{l,j}^{\rho_j^{LD}+1} = \sum_{lj} \beta_{l,j}^{LD} WTI_{lj,j} LD_{lj,j}^{\rho_j^{LD}+1}$

F210. $WTI_{l,j} LD_{l,j}^{\rho_j^{LD}+1} \sum_{lj} \beta_{lj,j}^{LD} = \beta_{l,j}^{LD} \sum_{lj} WTI_{lj,j} LD_{lj,j}^{\rho_j^{LD}+1}$

We know that the $\beta_{l,j}^{LD}$, like the $\beta_{j,i}^{XT}$, are defined only up to a factor of proportionality. Therefore, we can, without loss of generality, impose the normalization rule $\sum_{lj} \beta_{lj,j}^{LD} = 1$. It then follows that

F211. $WTI_{l,j} LD_{l,j}^{\rho_j^{LD}+1} = \beta_{l,j}^{LD} \sum_{lj} WTI_{lj,j} LD_{lj,j}^{\rho_j^{LD}+1}$

Whence

F212. $\beta_{l,j}^{LD} == \frac{WTI_{l,j}^{O} \left(LD_{l,j}^{O} \right)^{\rho_j^{LD}+1}}{\sum_{lj} WTI_{lj,j}^{O} \left(LD_{lj,j}^{O} \right)^{\rho_j^{LD}+1}}$

Then, from equation 5

F213. $B_j^{LD} = \frac{LDC_j^O}{\left[\sum_{l} \beta_{l,j}^{LD} \left(LD_{l,j}^{O} \right)^{-\rho_j^{LD}} \right]^{-\frac{1}{\rho_j^{LD}}}}$

F4.7.4 Value added

In accordance with the algebra of CES aggregator functions (Appendix C1),

F214. $$\rho_j^{VA} = \frac{1-\sigma_j^{VA}}{\sigma_j^{VA}}$$

Develop equation 4:

F215. $$\frac{LDC_j}{KDC_j} = \left[\frac{\beta_j^{VA}}{1-\beta_j^{VA}}\frac{RC_j}{WC_j}\right]^{\sigma_j^{VA}}$$

F216. $$\frac{LDC_j}{KDC_j} = \left[\frac{\beta_j^{VA}}{1-\beta_j^{VA}}\frac{RC_j}{WC_j}\right]^{\frac{1}{\rho_j^{VA}+1}}$$

F217. $$\left(\frac{LDC_j}{KDC_j}\right)^{\rho_j^{VA}+1} = \frac{\beta_j^{VA}}{1-\beta_j^{VA}}\frac{RC_j}{WC_j}$$

F218. $$\beta_j^{VA} RC_j KDC_j^{\rho_j^{VA}+1} = \left(1-\beta_j^{VA}\right) WC_j LDC_j^{\rho_j^{VA}+1}$$

Whence

F219. $$\beta_j^{VA} = \frac{WC_j^O\left(LDC_j^O\right)^{\rho_j^{VA}+1}}{WC_j^O\left(LDC_j^O\right)^{\rho_j^{VA}+1} + RC_j^O\left(KDC_j^O\right)^{\rho_j^{VA}+1}}$$

From equation 3

F220. $$B_j^{VA} = \frac{VA_j^O}{\left[\beta_j^{VA}\left(LDC_j^O\right)^{-\rho_j^{VA}} + \left(1-\beta_j^{VA}\right)\left(KDC_j^O\right)^{-\rho_j^{VA}}\right]^{-\frac{1}{\rho_j^{VA}}}}$$

F4.8 LES PARAMETERS

We refer to the elasticity of consumption demand for commodity *i* with respect to household *h*'s total consumption budget CTH_h as the « income elasticity » of the consumption of commodity *i* by household *h*[19]. From equation 53, the income elasticity is given by

F221. $$\sigma_{i,h}^{Y} = \frac{\partial \ln C_{i,h}}{\partial \ln CTH_h} = \frac{CTH_h}{C_{i,h}} \frac{\partial C_{i,h}}{\partial CTH_h} = \frac{CTH_h}{C_{i,h}} \frac{\partial}{\partial CTH_h} \left[C_{i,h}^{MIN} + \frac{\gamma_{i,h}^{LES}}{PC_i} \left(CTH_h - \sum_{ij} C_{ij,h}^{MIN} PC_{ij} \right) \right]$$

F222. $$\sigma_{i,h}^{Y} = \frac{CTH_h}{C_{i,h}} \frac{\gamma_{i,h}^{LES}}{PC_i} \frac{\partial CTH_h}{\partial CTH_h} = \gamma_{i,h}^{LES} \frac{CTH_h}{PC_i \; C_{i,h}}$$

This implies

F223. $$\gamma_{i,h}^{LES} = \frac{\sigma_{i,h}^{Y} PC_i \; C_{i,h}}{CTH_h}$$

Now it is readily verified from equation 53 that the household budget constraint $\sum_i PC_i \; C_{i,h} = CTH_h$ implies $\sum_i \gamma_{i,h}^{LES} = 1$. Given equation F217, this requires the elasticities to fulfill the condition

F224. $$\frac{\sum_i \sigma_{i,h}^{Y} PC_i \; C_{i,h}}{CTH_h} = \sum_i \gamma_{i,h}^{LES} = 1$$

As the assigned values of income elasticities may not satisfy this condition *a priori*, the elasticities are adjusted proportionnally as follows:

F225. $$\sigma_{i,h}^{Y} = \frac{\overline{\sigma_{i,h}^{Y}} CTH_h}{\sum_{ij} \overline{\sigma_{ij,h}^{Y}} PC_{ij}^{O} \; C_{ij,h}^{O}}$$

Then, from equation F217

F226. $$\gamma_{i,h}^{LES} = \frac{\sigma_{i,h}^{Y} PC_i^{O} \; C_{i,h}^{O}}{CTH_h^{O}}$$

---

19 If income taxes, transfers and savings were a constant proportion of income, then so would total consumption expenditure, and the elasticity of consumption demand for commodity *i* with respect to household *h*'s total consumption budget would indeed be identical to its income-elasticity.

The Frisch parameter is (Appendix C4)

F227. $$Frisch = -\lambda\, CTH_h = -\frac{CTH_h}{CTH_h - \sum_i PC_i C_{i,h}^{MIN}}$$

So, from equation 53

F228. $$C_{i,h} PC_i = C_{i,h}^{MIN} PC_i + \gamma_{i,h}^{LES} \frac{CTH_h}{-Frisch} = C_{i,h}^{MIN} PC_i - \gamma_{i,h}^{LES} \frac{CTH_h}{Frisch}$$

F229. $$C_{i,h}^{MIN} PC_i = C_{i,h} PC_i + \gamma_{i,h}^{LES} \frac{CTH_h}{Frisch}$$

The calibration formula is then

F230. $$C_{i,h}^{MIN\_O} = C_{i,h}^{O} + \gamma_{i,h}^{LES} \frac{CTH_h^O}{PC_i^O\, Frisch}$$

F5. Calibration of gross domestic products

From equations 95-98,

F231. $$GDP^{BP\_O} = \sum_j PVA_j^O VA_j^O + TIPT^O$$

F232. $$GDP^{MP\_O} = GDP^{BP\_O} + TPRCTS^O$$

F233. $$GDP^{IB\_O} = \sum_{l,j} W_l^O LD_{l,j}^O + \sum_{k,j} R_{k,j}^O KD_{k,j}^O + TPRODN^O + TPRCTS^O$$

F234. $$GDP^{FD\_O} = \sum_i PC_i^O \left[\sum_h C_{i,h}^O + CG_i^O + INV_i^O + VSTK_i^O \right] + \sum_x PE_x^{FOB\_O} EX_x^O - \sum_m e^O * PWM_m^O IM_m^O$$

References for Appendix F

ANNABI, Nabil, Bernard DECALUWÉ and John COCKBURN (2006) *Functional Forms and Parametrization of CGE Models*, PEP-MPIA Working Paper 2006-04, Politiques économiques et pauvreté / Poverty and Economic Policies, Université Laval, Québec.

LEONTIEF, Wassily W. (1941), *The structure of American economy 1919-1929*, New York, Oxford University Press.

LEONTIEF, Wassily W. (1951), *The structure of American economy 1919-1939*, New York, Oxford University Press.

LEONTIEF, Wassily W. et al. (1953), *Studies in the structure of the American economy*, New York, Oxford University Press.

STONE, J.R.N. (1951), «Simple transaction models, information and computing», *Review of Economic Studies*, 19.

STONE, J.R.N. et UTTING, J.E.G. (1953), «The relationship between input-output analysis and national accounting», in NETHERLANDS ECONOMIC INSTITUTE (1953), *Input-Output Relations*, Proceedings of a conference on inter-industrial relations held at Driebergen, Holland, 1953; Leyden, H.E. Stenfort Kroese N.V.

Figure F1 – Structure of the SAM (part I)

TABLE SAM(*,*,*,*)

		L.L1	L.L2	K.CAP	K.LAND	AG.HRP	AG.HUP	AG.HRR	AG.HUR	AG.FIRM	AG.GVT	AG.TD	AG.TM	AG.TI	AG.L1	AG.L2	AG.CAP	AG.LAND
L.	L1																	
L.	L2																	
K.	CAP																	
K.	LAND																	
AG.	HRP	5915	5078	526	1132													
AG.	HUP	7300	4697	1054							139							
AG.	HRR	872	1242	526	3602													
AG.	HUR	1210	1010	6221	100					1900								
AG.	FIRM			4741	488			10			10							
AG.	GVT					40	100		122	81		2308	2500	4375	1843	137	46	
AG.	TD					44	428	146	390	1300								
AG.	TM																	
AG.	TI																	
AG.	L1																	
AG.	L2																	
AG.	CAP																	
AG.	LAND																	
AG.	ROW			42	911				10	370	30							
J.	AGR																	
J.	IND																	
J.	SER																	
J.	ADM																	
I.	AGR					6338	4496	2441	1755									
I.	FOOD					2504	3050	950	2400									
I.	OTHIND					1090	1601	1003	3043									
I.	SER					2635	3515	1620	2426									
I.	ADM										8255							
X.	AGR																	
X.	FOOD																	
X.	OTHIND																	
X.	SER																	
OTH.	INV							72	295	1598	1231							
OTH.	VSTK																	
OTH.	TOT	15297	12027	13110	6233	12651	13190	6242	10441	5249	9665	2308	2500	4375	1843	137	46	

$\lambda_{h,l}^{WL}\left(W_l\sum_j LD_{l,j}\right)$

$\lambda_{ag,k}^{RK}\left(\sum_j R_{k,j}KD_{k,j}\right)$

$TR_{agng,h}$

$TR_{ag,f}$

TDF_f

$TR_{agng,gvt}$

TMT

$TICT$

$TR_{gvt,h}$

TDH_h

$TDFT + TDHT$

$\sum_j TIW_{l,j}$

$\sum_j TIK_{k,j}$

$PC_i\, C_{i,h}$

$PC_i\, CG_i$

SH_h

SG

SF

Figure F1 – Structure of the SAM (part II)

+		AG.ROW	J.AGR	J.IND	J.SER	J.ADM	I.AGR	I.FOOD	I.OTHIND	I.SER	I.ADM	X.AGR	X.FOOD	X.OTHIND	X.SER	OTH.INV	OTH.VSTK	OTH.TOT
L.	L1		10002	2289		3006												15297
L.	L2		910		10147	970												12027
K.	CAP		2086	7015	4009													13110
K.	LAND		6133	100														6233
AG.	HRP																	12651
AG.	HUP																	13190
AG.	HRR																	6242
AG.	HUR																	10441
AG.	FIRM																	5249
AG.	GVT		-1693	-293								99						9665
AG.	TD																	2308
AG.	TM						500	200	1800									2500
AG.	TI						684	1400	1554	737								4375
AG.	L1		1500	343														1843
AG.	L2		137															137
AG.	CAP		46															46
AG.	LAND																	
AG.	ROW						2613	4928	6500	1691								
J.	AGR						17834	250		210		7417						[illegible]
J.	IND						400	12228	3470	20			241	1180				17539
J.	SER							150		18532					2653			21335
J.	ADM										8255							8255
I.	AGR		2715	1402	1327	93										2164	-600	22131
I.	FOOD		825	770	1600											6857	200	19156
I.	OTHIND		1102	2930	455	2100												13324
I.	SER		1948	2983	3797	2086	100					60		20				21190
I.	ADM																	8255
X.	AGR	7576																7576
X.	FOOD	241																241
X.	OTHIND	1200																1200
X.	SER	2653																2653
OTH.	INV	5425																8621
OTH.	VSTK															-400		-400
OTH.	TOT	17095	25711	17539	21335	8255	2213[illegible]	[illegible]	13324	21190	8255	7576	[illegible]	1200	2653	8621	[illegible]400	

$W_l\ LD_{l,j}$

$R_{k,j}\ KD_{k,j}$

$TR_{agd,row}$

TIM_m

TIC_i

TIX_x

TIP_j

$PE_x\ EX_{j,x}$

$TIW_{l,j}$

$e\ PWM_{i,m}\ IM_m$

$PC_i\ INV_i$

$PL_i\ DS_{j,i}$

$PC_i\ VSTK_i$

$TIK_{k,j}$

$PC_i\ DI_{i,j}$

$SROW$

$PE^{FOB}{}_x\ EXD_x$

$PC_i tmrg_{i,m}\left(DD_m + IM_m\right)$ for imported goods

$PC_i\ tmrg_{i,nm}\ DD_{nm}$ for non-imported goods

$PC_i\ tmrg^{X}_{i,x}\ EXD_x$

$\sum_i PC_i\ VSTK_i$