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Optical quantum state generation with integrated frequency comb sources

Par

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Abstract

The exploitation of quantum mechanics has the potential to solve important challenges and introduce novel technologies enabling, among others, powerful computation (which can solve certain problems exponentially faster than classical computers), perfectly secure communications, as well as sensing and imaging with precision and resolutions not achievable by classical means.

In order to make use of the unique properties of quantum physics, experimental platforms that provide access to, and allow the control of quantum states, need to be developed. In particular, quantum systems composed of multiple parties are required, where each party is in a coherent superposition of states, and where multiples of such parties are in a coherent superposition with each other (e.g. being entangled). Additionally, in order to make use of such entangled quantum systems, it is required that each individual party can be coherently controlled and measured. The usability and available quantum resources of the entangled system is related to its size, which is proportional to the number of entangled parties, as well as the dimensionality of the superposition each party is in. For this reason, there is a imminent demand to achieve, investigate, and improve the generation and control of large quantum systems, comprised of as many entangled parties as possible, where each party is in an ideally high-dimensional superposition.

Many quantum platforms are currently subject to extensive research, such as trapped ions, superconducting circuits, defect centers in solid-state crystals, mechanical oscillators, and optical quantum states (i.e. photons). While all platforms provide distinct advantages (as well as challenges), optical photon states are of particular interest, because they can interact with other quantum systems, and can be transmitted over long distances while preserving their quantum coherence without the need for extensive shielding. A large variety of sources for optical quantum states has been demonstrated, however most such implementations suffer from high complexity, ultimately limiting their scalability. Integrated photonics has recently become a leading platform for the compact, cost-efficient, and stable generation and processing of optical quantum states. However, to date, integrated quantum sources have only been able to generate photon pairs, which operated on single channels, and single polarization modes. Furthermore, in the few realizations of on-chip entanglement, they were limited to two entangled parties, where each party was in a two-dimensional superposition. These limitations of current integrated photon

sources arise in large part from the concepts currently exploited to generate and control these states. In particular, path-entangled sources are the most commonly used, but increasing the quantum state complexity requires a significant, and very challenging, increase in device complexity. There is therefore an urgent need to develop new concepts that can achieve the on-chip generation of large and complex photon states without increasing source complexity, while still enabling coherent quantum state control and detection.

In this work, we aim to address these limitations by making use of multiple frequency and temporal modes within a single spatial waveguide mode (therefore readily compatible with standard optical fibers) of an on-chip microring resonator. This approach is in contrast to more commonly used techniques relying on path entanglement, which exploits multiple spatial modes and generates entanglement by means of complex waveguide structures. Working within a single spatial mode, we achieve the first demonstration of pure heralded single photons and entangled photon pairs, which are emitted on multiple frequency modes from a single source. We then demonstrate the first generation of multi-photon entangled quantum states on a photonic chip. Finally, by making use of the spectral multi-mode nature of optical frequency combs, we achieve the generation of photon pairs in a coherent superposition of multiple frequency modes, representing the first generation of high-dimensional entangled states on a photonic chip.

Our work represents a significant step forward in the generation of complex quantum states on a photonic chip and in a single spatial mode compatible with standard optical fibers. In addition, we demonstrated coherent quantum state control and manipulation for all generated states by means of standard telecommunications components, which could in the future be also integrated on single, monolithic chip. Our results indicate that the exploitation of integrated optical frequency comb sources for the generation of optical quantum states can provide a scalable and practical platform for quantum technologies.

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List of Abbreviations

AOM	A cousto O ptic M odulator
CAR	C oincidence to A ccidental R atio
CW	C ontinuous W ave
dB	Decibel
EPR	E instein P odolsky R osen (paradox)
GHZ	G reenberger H orne Z eilinger (quantum state)
HOM	H ong O u M andel (interference)
JSA	J oint Spectral A mplitude
JSI	J oint Spectral I ntensity
LOQC	L inear O ptical Q uantum C omputation
OAM	O rbital Angular M omentum
OPO	O ptical P arametric O scillator
PM	P olarization M aintaining
Q-factor	Q uality-factor
RF	R adio F requency
SFWM	S pontaneous F our- W ave M ixing
SPDC	S pontaneous P arametric D own- C onversion
TDC	T ime to D igital C onverter
TE	T ransverse E lectric
TM	T ransverse M agnetic

Declaration of independent work, and overlap with publications

I, Christian Reimer, hereby declare that I have independently written the here-presented Ph.D. thesis, that the presented experimental results were not manipulated or artificially generated, and that no unreferenced sources were used.

The French Summary (Chapter 6) was written by myself in the English language (see Appendix B), and was then translated to French by Dr. Yoann Jestin.

Parts of the written text and figures overlap with journal and conference publications, a book chapter, and patents that I have authored:

- Sec. 2.4: **C. Reimer**, L. Caspani et al., Optics Express 22, 6535-6546 (2014)
- Sec. 2.5.1: M. Lahiri, **C. Reimer**, and A. Zeilinger “Quantum Interference and Imaging,” in “Frontiers in Modern Optics,” Pages 159-170 (2016)
- Sec. 2.5.2: **C. Reimer**,* M. Kues* et al., Science 351, 1176-1180 (2016)
- Sec. 2.6: **C. Reimer**,* M. Kues* et al., Science 351, 1176-1180 (2016)
- Sec. 2.7: M. Kues,* **C. Reimer*** et al., Nature 546, 622-626 (2017)
- Sec. 2.9: **C. Reimer**,* P. Roztocky* et al., Patent: PCT/CA2016/051060
- Sec. 2.10: M. Kues,* **C. Reimer*** et al., Nature 546, 622-626 (2017)
- Sec. 3.1.1: **C. Reimer**, L. Caspani et al., Optics Express 22, 6535-6546 (2014)
- Sec. 3.1.2: **C. Reimer**, M. Kues et al., Nature Communications 6, 8236 (2015)
- Sec. 3.2.1: **C. Reimer**,* M. Kues* et al., Science 351, 1176-1180 (2016)
- Sec. 3.2.2: M. Kues,* **C. Reimer*** et al., Nature 546, 622-626 (2017)
- Sec. 3.3.1: **C. Reimer**,* M. Kues* et al., Science 351, 1176-1180 (2016)
- Sec. 3.3.2: **C. Reimer**,* M. Kues* et al., Patent: PCT/CA2016/050421
- M. Kues, **C. Reimer** et al., Advanced Photonics Congress, IPR, IW3A.3 (2016)

*Equally-contributing, joint first authors



.....
Christian Reimer, Montreal, August 30th 2017

1. Introduction

Chapter Abstract:

This chapter gives an introduction into optical quantum state generation and provides an overview of the current state-of-the-art with a focus on integrated sources. Additionally, the work objectives are described and the outline of the main document is presented.

Chapter Content:

- 1.1 General introduction
- 1.2 Optical quantum states within a single spatial mode
- 1.3 Objectives and thesis structure

1.1 General introduction

Quantum optics

The quantization of the electromagnetic field was introduced over a hundred years ago (1), leading to the concept of ‘photons’ as particles of light (2). Today, the field of quantum optics has become one of the most active avenues for fundamental research, as well as for practical applications (3). Based on the available, yet quickly evolving technical capabilities for the generation (4), manipulation (5) and detection of single photons (6), quantum optics enables today a large variety of fundamental experiments. Indeed, many foundational tests of quantum mechanics, such as the Einstein Podolsky Rosen paradox (7, 8), tests of non-locality (9) including loophole-free Bell inequality violations (10, 11), as well as the first realizations of experimental measurement-based quantum computation (12) have been implemented using photons. In addition, the possibility to transmit photons by means of free-space optical links or using fiber networks has given birth to the field of quantum communications (13), which enables the secure exchange of information.

One of the most pronounced properties of light is that it can interact with other systems (in general, light can be emitted, absorbed or scattered, imprinting information of the system on the light), and therefore suggests a means to observe and investigate their properties. Indeed, imaging (14), microscopy (15), and spectroscopy (16) applications can all be improved by making use of the quantum properties of light. Additionally, photons can interact with, control, and entangle other quantum systems such as trapped ions (17), defect centers in solid-state crystals (18), and mechanical quantum systems (19), making quantum optics an indispensable tool for quantum science.

Basic quantum light sources

In general, optical quantum sources can be described as belonging to one of three main categories: sources of squeezed states, sources of single photons, sources of photon pairs.

Squeezed states are optical quantum states where the amplitude and phase noise of the state are at the limit given by the Heisenberg uncertainty limit (20). In such states, the amplitude or phase noise can be ‘squeezed’ several decibels (dB) below the classical limit at the expense of the complementary variable. Among several applications, such squeezed states can be exploited for e.g. continuous-variable quantum computation (21), however they are very sensitive to losses, drastically reducing the amount of ‘squeezing’.

Single photon sources ideally only emit a single photon at a time, and can be realized (22) e.g. using quantum dots, defect centers, single atoms, and single molecules. Current challenges, subject to extensive research, include the deterministic generation of on demand photons (23), as well as the generation of pure, indistinguishable single-mode photons (24).

Photon pair sources ideally only emit two photons at the same time, and can be realized by means of nonlinear optical interactions (22). As in single photon sources, photon purity (25) and deterministic generation (26) pose significant challenges. Indeed, sources of pure photon pairs can be used as so-called heralded single photon sources, where one of the two photons is detected, in turn deterministically predicting the presence of the other photon of the pair. Additionally, and due to the conservation of energy, momentum, and spin in the nonlinear generation process (27), photon pair sources can directly generate a large variety of different entangled states (with the two photons in the pair being entangled in e.g. energy (28), polarization (4), orbital angular momentum (29), etc.), and are therefore commonly used for applications relying on entanglement as a quantum resource.

Advanced quantum light sources for more complex states

While entangled photon pairs can already find direct applications for example in quantum communications (13), their usability as a quantum resource is somewhat limited. In order to increase the quantum state complexity, two main approaches can be used: increasing the number of entangled photons (12), increasing the dimensionality of the states associated to their superposition (29), or both at the same time in high-dimensional multi-photon states (30).

The generation of multi-photon entangled states can be achieved by interfering multiple photon pairs by means of e.g. beam splitters (31). Using this approach, different multi-photon entangled states such as GHZ states (31) or cluster states (12) have been achieved. Indeed, the concatenation of multiple two-photon sources has enabled the preparation of states consisting of as many as ten entangled photons (32). However, an exclusive use of the multi-photon approach has two main draw-backs. First, the source complexity increases significantly with every added photon pair (e.g. all two-photon sources have to be coherently excited, requiring the phase stabilization of many different spatial paths) (32). And second, the detection rate decreases with an increasing number of entangled photons (32). In particular, even with state-of-the-art detectors, detection rates dropped from 1.2 million per second for two photon states to 4 per hour for ten photon states (32), placing a practical limitation on the achievable photon number.

On the other hand, photons are ideally suited to carry high-dimensional superpositions and entanglement (29), as photons can be entangled in many different degrees of freedom, such as spatial modes (e.g. orbital angular momentum (OAM) (29) and momentum-position (33)) or frequency-temporal modes (34). In particular, spatial modes have been widely exploited for high-dimensional entanglement, achieving experimental confirmation of high-dimensional Bell inequality violations (29), as well as for implementations of high-dimensional quantum communications protocols (35). Additionally, it has been shown that high-dimensional states are advantageous for quantum computation protocols (36).

Integrated quantum optics and sources

The most advanced sources of optical quantum states have in common that they mainly rely on free-space realizations, exploiting multiple spatial paths, as well OAM and polarization states of light. However, losses and implementation complexities (such as achieving perfect mode-overlap and phase stability in free-space setups) present significant challenges for reaching complex quantum circuits. In particular, the realization of multi-photon and high-dimensional quantum states is required to achieve powerful quantum information processing. There is therefore a need to produce stable and scalable platforms to generate and control such complex multi-photon and high-dimensional quantum states (30).

With the goal to decrease the size of quantum sources, in recent years there has been an increased focus towards compact optical fiber-based sources (37), as well as integrated sources based on compact waveguide structures (38). Advances in the miniaturization of nonlinear crystals has led to the realization of compact and fiber-coupled sources based on traditional nonlinear crystals (39). In addition to coupling crystal-based sources to optical fibers, the intrinsic nonlinearity of optical fibers can be directly used to generate photon pairs (37). However, the relatively low generation rates together with high Raman photon noise, typically necessitates tailored engineering (40) or the cryogenic cooling of fiber-based quantum sources (41). One of the largest advances in compact and scalable quantum devices has stemmed from the use of on-chip photonics technology (42).

Integrated photonics makes use of compact, mainly semiconductor-based photonic chips, where optical waveguide structures can be realized using micro- and nano-fabrication techniques (43). Integrated photonics was first used for the realization of linear optical devices, where single-mode optical waveguides were fabricated in silicon photonic chips (44). Due to the small mode

area of the optical fields, high optical field intensities can be reached, enabling the exploitation of nonlinear optical interactions (45). After the realization of second-order electro-optical phase modulation, and consequently the realization of phase and intensity modulators mainly in lithium niobate waveguides (46), also third-order nonlinear interactions have been demonstrated in silicon-based devices, such as parametric gain (47), frequency conversion (48), optical modulation (49), mode-locked laser sources (50), and optical frequency combs (51).

Advancing further from linear and nonlinear applications, integrated photonics is becoming an important platform also for quantum optics. Indeed, the manipulation (52), generation (53), and detection (54) of quantum states have recently been achieved. In particular, making use of directional couplers and phase shifters, devices capable of manipulating and controlling optical quantum states have been demonstrated (42). This includes, e.g., the realization of boson sampling (55), the implementation of linear quantum computation (52) including the Shor factorization algorithm (56), control of multi-photon entangled quantum states (57). However, these advanced implementations have relied on external free-space sources of optical quantum states (52, 56, 57). Nevertheless, single photon (58) and entangled two-photon quantum states (59–61) have also been generated in integrated structures. Yet, not all possible types of entanglement can be generated or exploited in integrated structures. For example, the mode dispersion and phase-matching condition in fibers and integrated waveguides usually prohibits the generation or exploitation of momentum-position entanglement, and orbital angular momentum is usually also not guided in such structures. Integrated sources of entangled photon pairs have mainly relied on polarization (60, 62, 63) and path (61, 64–66) entanglement.

While polarization entanglement has been demonstrated in integrated structures, the entangled states have only been generated (so far) at the output of the chip (60, 62, 63). Due to polarization dispersion, entanglement is only maintained in very specialized waveguides, e.g. achieved using direct laser writing in glass (67), which is why state manipulation of on-chip generated polarization entangled states has been performed using free-space devices. Path entanglement is currently one of the most exploited forms of entanglement in integrated structures (66), as it can be generated by concatenating multiple photon sources through optical waveguide couplers (61). Furthermore, path-entangled states can also be characterized with on-chip devices by means of beam splitters and phase shifters (52). However, for the path-entanglement concept, the device complexity (and consequently, footprint) scales with the state complexity (52), placing

significant technological challenges to achieve higher source complexities, as required, e.g., to achieve high-dimensional entanglement.

For powerful future applications, multi-photon and/or high-dimensional quantum states will be required (36). Polarization entanglement is intrinsically two-dimensional, and therefore incompatible for the generation of high-dimensional states. In addition, the realization of polarization-entangled multi-photon states requires elements such as directional couplers and polarization beam splitters, with equal dispersion for both polarization modes. Such devices have until now only been achieved in laser-written glass waveguides (67), but not yet in integrated on-chip devices due to strong polarization mode dispersion.

Hampered by these limitations, neither multi-photon nor high-dimensional entangled states have been generated in integrated formats. Furthermore, there is no clear path to produce, in the near future, large quantum states using path or polarization entanglement. New concepts for on-chip quantum state generation have to therefore be developed, with the goal to reach (at the same time) high-dimensional and multi-partite quantum states with favourable scaling in terms of source complexity. The optimum in terms of source complexity and footprint are sources that can generate complex quantum states within a single waveguide mode (68). Such quantum states will have the added benefit of being compatible with the fiber telecommunications infrastructure, certainly a significant advantage for communications applications.

Energy-time entangled states are well suited for their generation within a single spatial mode, and are therefore ideal candidates to produce useful states of non-classical light with low source complexity. Indeed, the first realization of energy-time entanglement in an on-chip source has been recently demonstrated (59). However, while such sources were more compact compared to those generating path-entanglement, realizations of energy-time entanglement in on-chip devices prior to this work did not provide more complex quantum states compared to polarization or path-entangled schemes. In particular, early attempts only exploited single sets of temporal and frequency modes achieving either two-dimensional (69), or continuous variable states where individual quantum modes cannot be addressed individually (59).

This thesis focuses on the realization of new concepts for the on-chip generation of complex quantum states, which can be generated and controlled within a single spatial mode.

1.2 Optical quantum states within a single spatial mode

Best suited for single spatial mode operation are two different types of optical quantum states: squeezed states (20) (amplitude-phase) and energy-time entangled states (70). Both allow transmission via optical fibers and have recently gained increasing attention. Applications such as quantum communications or sensing beyond the classical limit make use of quantum correlations between different frequency and temporal modes as well as their relative phase. Towards the goal of increasing the number of temporal and frequency modes accessible for quantum state preparation, the generation of optical quantum states has recently begun to exploit optical frequency combs (71, 72). Frequency combs are multi-frequency light sources characterized by the presence of multiple, spectrally-equidistant, phase-locked frequency modes (73). In the classical domain, frequency combs have revolutionized sensing and metrology, due to intrinsic precision and stability (73). In recent years, also the quantum correlations within optical frequency combs have been investigated (71, 72, 74–76). Indeed, it has been shown that complex quantum correlations between frequency comb modes can be achieved in synchronously pumped (72), as well as bi-chromatically excited optical parametric oscillators (OPO) (74). However, these investigations of quantum frequency combs were limited so far to squeezed states, where only moderate amounts of squeezing (< 10 dB) were achieved due to the typically high optical losses. In addition, the spectral modes in those realizations are separated by only a few MHz, making them inaccessible (with most optical filtering techniques). Quantum state manipulation is therefore not fully achievable in such systems.

Another approach towards realizing frequency combs relies on the use of integrated photonics structures, for which classical on-chip frequency comb generation is subject to extensive research (51). Such microresonator-based frequency combs are able to generate very broadband coherent light emission with spectral mode separation in the tens to hundreds of GHz, enabling easy access to individual frequency modes. In particular, advanced device optimization has led to mode-locking (50), soliton generation (77) as well as self-referenced stabilization of integrated frequency combs (78). Nevertheless, all investigations of these on-chip combs prior to this work have focused on their classical properties and integrated frequency comb sources have not been exploited for quantum state generation.

In the context of the work presented in this thesis, we proposed to use integrated optical frequency combs as quantum sources, as they have the potential to address many limitations of current integrated quantum sources. First, the device fabrication and optimization is already at an

advanced stage due to the rich history of classical comb realizations (79). Additionally, frequency comb sources are based on a single microring resonator, and are therefore much less complicated structures than, e.g., path entangled sources. Furthermore, the use of on-chip combs goes well above the advantage of simply miniaturizing quantum frequency combs, which have shown the generation of very complex squeezed quantum states. Indeed, the larger frequency mode spacing in integrated comb sources provides easy access to individual comb lines by means of optical filters. This also enables the investigation of quantum states other than squeezed states, and could lead to the implementation of quantum operations in the frequency domain. Integrated frequency combs are therefore ideal candidates for the generation of scalable and complex quantum states in compact sources, where the quantum resource does not scale with source complexity, and all quantum states are emitted within a single spatial mode.

This work represents the first systematic investigation of the quantum properties of photons generated in integrated optical frequency combs [1,4-6,9-10]. Instead of investigating the generation of squeezed states however, we focus on the generation of photon pairs, and particularly of energy-time entangled states. This type of entanglement is of particular interest as it can generate high-dimensional states. Additionally, and in contrast to squeezed states, photon states are more robust to losses, and discrete forms of energy-time entanglement exist which will enable the direct implementation of quantum gates.

The three different types of energy-time entanglement in terms of their temporal and frequency modes are summarized in Figure 1.1, namely continuous, time-bin and frequency-bin entanglement.

If a stable CW laser is used to generate a spectrally-broadband photon pair, the photons are generated in a coherent superposition of a continuum of temporal and frequency modes, forming continuous-variable entangled states (59), see Fig. 1.1 a). However, since it is very difficult (or even impossible) to address the individual quantum modes of continuous variable states, their implementation into applications is very challenging (80). For this reason it is often preferred to work with discrete quantum states, especially for the implementation of quantum gates (68, 81).

The discrete form of energy-time entanglement (where discrete temporal and/or frequency modes are present) can be categorized into time-bin (82) and frequency-bin (34) entanglement, depending on which variable (temporal or frequency mode) can be controlled, see Fig. 1.1 b,c).

In particular, if the photons are in a superposition of two (or more) distinct temporal modes, which do not overlap and can be manipulated independently (82), the state is referred to as time-

bin entangled. Since temporal modes are linked to the frequency domain through Fourier transformation, the presence of at least two temporal modes also dictates the presence of at least two frequency modes. However, these frequency modes usually overlap within the spectral bandwidth of the photon and cannot be selected or controlled independently, see Fig. 1.1 b).

If the two (or more) frequency modes do not overlap and are spectrally separated far enough so that they can be selected and manipulated by means of optical filters (usually requiring a spectral separation in the GHz range), the quantum state is referred to as frequency-bin entangled (34). For such states, the temporal modes overlap and cannot be addressed independently, see Fig. 1.1.c).

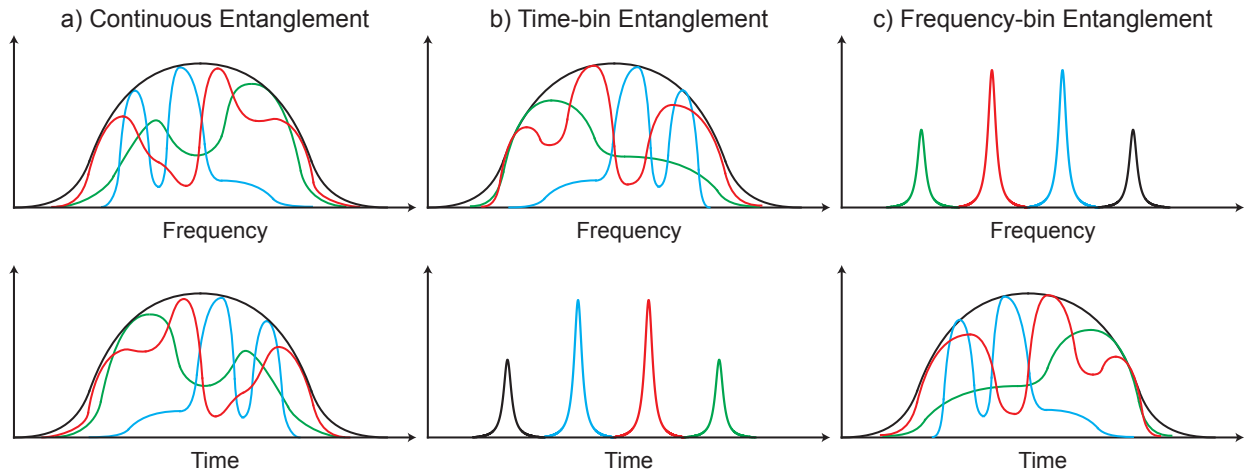


Fig. 1.1 Schematic of the different types of energy-time entanglement: Schematic of the temporal and frequency modes of **a)** continuous-variable, **b)** discrete time-bin, and **c)** discrete frequency-bin entanglement. Note that this figure is a very simplified visualization of otherwise complex mode structures. The main point is that in the continuous case all temporal and frequency modes overlap and cannot be independently addressed, while in the time-bin and frequency-bin discrete forms, either the temporal or frequency modes can be manipulated independently, since they do not overlap.

In addition to their potential on-chip generation, quantum state manipulation of discrete energy-time entanglement can in principal be implemented using on-chip devices, as it requires interferometers, frequency filters, modulators and phase shifters, all of which can be achieved by means of integrated photonics (44). For the above-stated reasons, the work presented in this thesis focuses on the realization of discrete energy-time entanglement using integrated microring resonator-based optical frequency combs.

1.3 Objectives and thesis structure

The objectives of this work focus on the investigation of optical quantum state generation within on-chip optical frequency comb sources. Particular focus is given on achieving the generation of quantum states within a single spatial mode, as this will allow a reduced source complexity, compatibility with single-mode fiber technology, as well as the generation of multi-photon and high-dimensional quantum states.

1. Generation of pure heralded single photons

This objective focuses on the realization of photon pair generation within microring resonators. The goal is to achieve the generation of pure photon pairs, which can be used for the realization of heralded single photons. The multi-frequency nature of the sources will be used for the simultaneous generation of photons pairs in different frequency modes. This could find applications in wavelength division multiplexing, and a single device could effectively be used as multiple sources of pure photons, distributed over different frequency channels.

2. Generation of entangled photon pairs

This objective focuses on the realization of entangled photon pairs within microring resonators. The goal is to achieve the generation of high quality two-photon entangled quantum states within a single spatial mode. To achieve this goal, a focus will be placed on time-bin as well as frequency-bin entanglement. For the time-bin approach, the frequency modes will be used for wavelength division multiplexing of entangled photons. In the frequency-bin approach, the individual frequency modes will be used as a quantum resource directly.

3. Generation of multi-partite entangled quantum states

This objective focuses on increasing the complexity of the generated photon states. The goal is to achieve the generation of multi-partite quantum states. In particular we investigate product states of Bell states, as well as multi-mode entangled states which provide correlations simultaneously between different sets of four frequency modes.

The presented thesis is structured along the three project objectives. In particular, the results section (Chapter 3) explicitly follows these objectives, where first photon pairs, then two-photon entanglement, and finally multi-partite entanglement are discussed. Chapter 3 is kept as short and concise as possible, only presenting the main results but not the experimental techniques used for the realizations.

The methods and techniques used are instead described in Chapter 2. The second chapter is structured along the development of the methodology for the results section. First the methods to generate and detect photon pairs are reviewed. Then quantum interference and quantum state characterization are summarized in a general context, followed by a detailed introduction into time-bin and frequency-bin entanglement. In addition to reviewing already existing techniques, new results are also presented at the end of Chapter 2. In particular, a novel method to stabilize unbalanced fiber interferometers, as well as a new technique to perform deterministic frequency mixing are presented. While these are new results developed during my Ph.D., they are presented in Chapter 2, as they do not overlap with the three main objectives, but are rather required to perform the main measurements.

Finally, Chapter 4 presents a short conclusion to the presented work, discusses the findings and gives an outline to future work and the potential impact of the results.

The references used in this work are cited in two different ways: my own publications are listed in angled brackets [1-11], and published literature (where I am not a co-author) is referenced in round brackets (1-147). The full Bibliography is listed in Chapter 5.

2. Methods and Techniques

Chapter Abstract:

This chapter describes the methods and techniques that were used to collect and analyse the experimental results shown in Chapter 3. First, photon pair generation using nonlinear optics and the properties of optical resonators are reviewed. Second, the techniques used to excite the microring resonators, as well as to detect and characterize emitted photons are discussed. Then, the fundamentals of quantum interference and state tomography are summarized. Finally, the two types of entanglement (time-bin and frequency-bin) used in this work are introduced, followed by the methods developed to characterize these types of entangled states.

Chapter Content:

- 2.1 Photon pair generation by means of nonlinear optical processes
- 2.2 Nonlinear integrated optical microring resonators
- 2.3 Optical excitation of microring resonators
 - 2.3.1 Continuous wave excitation
 - 2.3.2 Pulsed excitation
- 2.4 Correlated photon detection measurements
 - 2.4.1 Second-order photon coherence
- 2.5 Quantum interference measurements
 - 2.5.1 Single photon or ‘classical’ interference
 - 2.5.2 Two-photon interference
- 2.6 Quantum state tomography measurements
- 2.7 Bell inequalities
- 2.8 Entanglement in integrated microring resonators
 - 2.8.1 Time-bin entanglement
 - 2.8.2 Frequency-bin entanglement
- 2.9 Novel technique for fiber interferometer stabilization
- 2.10 Novel technique for frequency-domain projection measurements

2.1 Photon pair generation by means of nonlinear optical processes

One of the most common techniques to generate optical quantum states is based on exploiting nonlinear optical interactions. While individual quantum emitters such as molecules (83), quantum dots (84) and defect centers in crystals (18) can be used to generate single photons (which can be combined to form multi-photon states), nonlinear optical sources are commonly used to directly generate photon pairs. These sources are therefore ideal to generate entangled quantum states, where the sources can be designed to emit photon pairs, entangled in one or more degrees of freedom (85). The generation process is based on parametric optical interactions in nonlinear optical crystals or glasses (27).

When light propagates through a medium, the electric field induces dipole moments in the crystal by slightly displacing the electron cloud with respect to the positively charged cores. For low field intensities, these dipole moments behave fully linearly with respect to the input field (i.e. the electron displacement follows a linear potential), which gives rise to typical material properties such as refractive index and dispersion (27). For strong optical fields however, the spatial confinement of the electrons within the solid crystals does not allow the electrons and atom cores to linearly follow the incident oscillating field, meaning the displacement does not relate to a linear potential anymore. To describe the nonlinear response, the potential can be approximated with a Taylor series, which characterizes deviations from the linear optical response in quadratic, cubic and higher-order terms. These individual terms can then be linked to physical processes, and lead to the classification of second- and third-order nonlinear optical interactions (27).

An important criterion for the type of nonlinear interactions possible in a given crystal is their symmetry. It can be shown that second-order nonlinear terms cancel out in centro-symmetric crystals (27), such as in silicon or amorphous glasses. Second-order nonlinear processes can therefore only be achieved in non-centro-symmetric materials, such as Lithium Niobate or beta Barium Borate. While only few crystals can provide second-order nonlinearities, the nonlinear interactions are usually more efficient compared to third-order nonlinear crystals (27).

Among the many different nonlinear interactions that can be achieved in second- and third-order nonlinear media, mainly two nonlinear optical interactions are used for the generation of optical quantum states (27): Spontaneous Parametric Down Conversion (SPDC) and Spontaneous Four Wave Mixing (SFWM).

SPDC is a second-order nonlinear process, which is often referred to as the analog-inverse of second-harmonic generation (27). In this process, a photon with high energy (short wavelength) can spontaneously decay into two photons, referred to as signal and idler, see Fig. 2.1a). As this process conserves energy and momentum of the initial photon, the two photons are correlated, and can be entangled in many degrees of freedom. In particular, the sum of the energies (frequencies) of the photons has to be equal to the energy (frequency) of the incident photon, while the sum of the signal and idler momentum has to match the vector momentum of the incident photon (27).

SFWM is a third-order nonlinear process, which is in many ways very similar to SDPC, with the difference that not only one, but two incident photons decay into a signal and idler photon, see Fig. 2.1b). Again, the energy and momentum is fully preserved in the process, leading to strong correlations between the two generated photons.

In addition to the conservation of energy and momentum, the decay processes of SDPC and SFWM are essentially instantaneous (within the coherence-time of the initial photon), meaning that the signal and idler photons are generated at the same time. This strong correlation can lead to energy-time entanglement between the two photons. Furthermore, the generation process is spatially local, meaning that the signal and idler photons are essentially created at the same position within the nonlinear crystal. This property, together with momentum conservation can be used to generate entanglement in position and momentum. This type of entanglement is commonly exploited for ghost imaging experiments (86) and can be used to demonstrate the famous Einstein Podolsky Rosen (EPR) Paradox (8).

One critical situation that has to be fulfilled in order to enable parametric frequency conversion is the so-called phase-matching condition. Phase matching can be explained by the requirement that both energy and momentum have to be preserved in the nonlinear process (27). The momentum conservation simplifies to that the refractive index of all propagating waves has to be identical for perfect energy conversion. However, this is not achievable in dispersive materials, as here the refractive index changes with wavelength. Many different approaches have been achieved to achieve phase matching in dispersive media (27), such as periodic poling and birefringent phase matching.

In third-order nonlinear media, the refractive index becomes dependent on the optical power (27), meaning the refractive index increases with increasing optical power. This provides the possibility to achieve phase matching in third-order nonlinear media where the refractive index mismatch can be compensated by choosing an appropriate optical pump power. However, this

effect is very small and only small differences in the refractive index can be compensated for. Additionally, the refractive index can in most materials only be increased (27), which poses the requirement of anomalous dispersion (meaning the refractive index increases with increasing wavelength) (27).

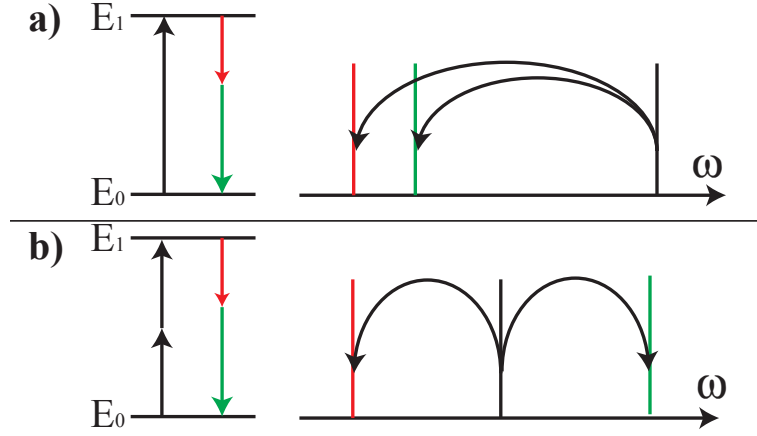


Fig. 2.1

Energy conservation in SPDC and SFWM:

Energy diagram and frequency-domain representation of photon pair generation in SPDC a) and SFWM b). The incident photon(s) (indicated in black) decay into two new photons (signal and idler, shown in red and green), where the total energy of the process is preserved (note that these are virtual energy levels, and not real ones as for example in atoms).

2.2 Nonlinear integrated optical microring resonators

In this work, we focus on the generation of optical quantum states using integrated nonlinear devices. While second-order nonlinear crystals provide much higher nonlinearities, their complex crystalline structure makes it very challenging to integrate these crystals on silicon-based chips (87). On the other hand, instead of growing second-order nonlinear crystals onto a chip, it is possible to exploit the intrinsic third-order nonlinearity of silicon or silicon based glasses (45). This significantly reduces fabrication complexity, however it comes at the cost of lower nonlinearities. In order to achieve large effects with a material that does not provide much intrinsic nonlinearity, higher optical pump powers are required. Additionally, one can exploit the field enhancement within resonant structures such as resonators (48) or photonic crystal waveguides (88).

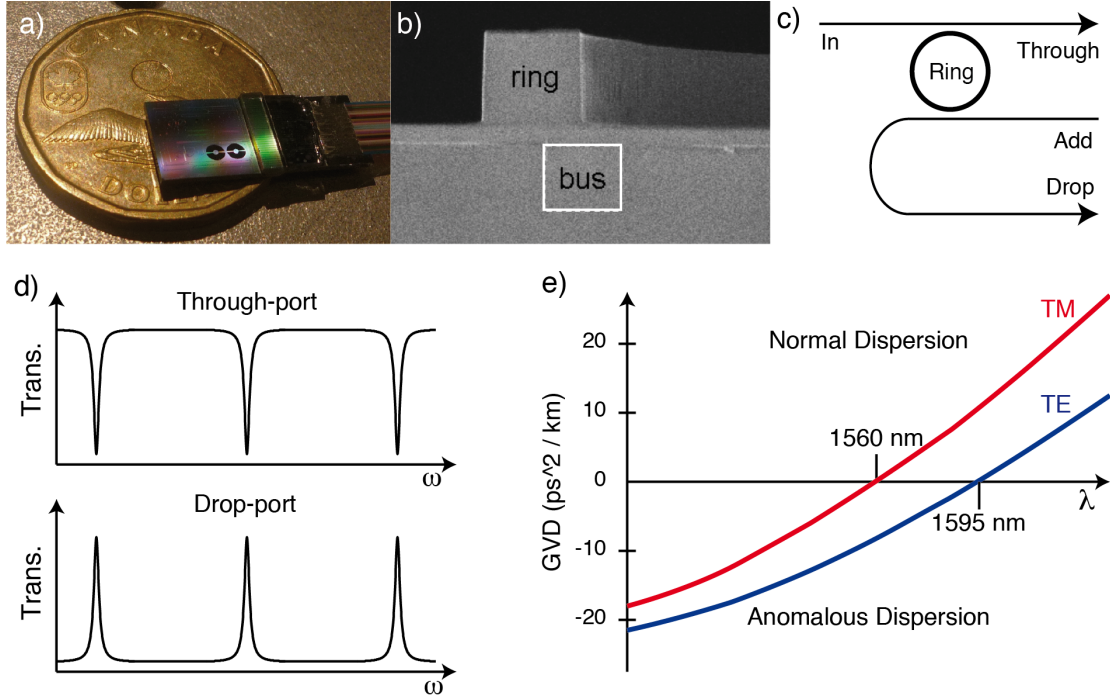


Fig. 2.2 Properties of a microring resonator:

a) Picture of the used chip, connected to optical fibers. b) Scanning electron microscope image of the microring resonator (cross section) before it was cladded in glass. c) Schematic of the used four-port microring resonator. d) Typical output at the ‘through’ and ‘drop’ port of the resonator for a broadband input at the ‘in’ port. e) Waveguide dispersions (Group Velocity Dispersion, GVD) within the microring resonator for both polarization modes; the zero dispersion wavelengths are at 1,560 nm and 1,595 nm for the TM and TE modes, respectively.

In this work we focus on the generation of optical quantum states in integrated microring resonators fabricated from a silicon oxynitride glass (79), which provides a third-order nonlinear interaction.

The used resonators were designed and fabricated by our external collaborates Prof. Brent E. Little and Prof. Sai T. Chu. The microring resonator (see Fig. 2.2 a) for a picture of the chip and Fig. 2.2 b) for a scanning electron microscope image) is formed by a waveguide formed by deuterated silicon oxynitride and a silicon dioxide cladding (89). The waveguide dimensions were chosen to have a 1.5 by 1.45 μm cross section, which supports two fundamental polarization modes, as well as two higher-order modes. The waveguide is closed to form a ring with a diameter of around 250 μm . In such a structure only wavelengths, which are an integer multiple of the optical resonator length can propagate and are strongly enhanced, while all other wavelengths are suppressed within the resonator. In order to couple light in and out of the microring resonator, it is connected to two bus waveguides, which are placed below the resonator, see Fig. 2.2. Such a resonator is referred to as a four-port resonator, where the ports are called ‘in’, ‘through’, ‘drop’ and ‘add’, see Fig. 2.2 c). For a broadband input field at the ‘in’-port, typical outputs at the ‘through’ and ‘drop’ ports are shown in Fig. 2.2 d). The separation between two resonances is called the free spectral range (FSR) and is determined by the optical length of the resonator, while the quality factor (Q-factor) is given by the central wavelength of a given resonance divided by the bandwidth of that resonance. Finally, the finesse of the resonator is defined as the ratio between the FSR and spectral bandwidth of the resonance. The finesse gives a measure of the quality of a resonator, where an increasing finesse provides higher field enhancement. As the finesse and Q factor are directly proportional for a given FSR, the Q factor is also often used as a measure for the quality of a resonator and the field enhancement. Furthermore, only periodic resonant structures such as Fabry-Perot or ring cavities have a finesse, but other types of resonators such as photonic crystal cavities only provide a single resonance, meaning that neither FSR nor finesse are defined.

In order to achieve SFWM in the microring resonator, the dispersion is designed to be close to zero at 1550 nm (meaning very little change in refractive index as a function of wavelength in the 1550nm wavelength region). At 1550 nm wavelength, both fundamental polarization modes have low and anomalous dispersion with zero dispersion wavelengths at 1,595 nm and 1,560 nm for the transverse electric (TE) and transverse magnetic (TM) polarization modes respectively, see Fig. 2.2 e). This very low and anomalous dispersion enables phase matching for SFWM over a broad frequency band at already low optical pump powers. The microring resonator is therefore ideally suited for the generation of photon pairs, as here low pump powers are required.

2.3 Optical excitation of microring resonators

In order to enable the generation of photon pairs via SFWM within integrated microring resonators, these cavities have to be optically excited with a laser field. In general, this can be achieved in mainly two different regimes: continuous wave (CW), and pulsed excitation. CW excitation is usually experimentally easier to achieve. But due to the typically spectrally very narrow resonances (here 100 to 1000 MHz), small thermal drifts within the resonator (resulting in shifts of the central wavelength of the resonance) lead to high instability and power fluctuations, reflect in poor excitation and loss of coupled optical power (90). These fluctuations can be compensated for, e.g., by using an active locking of the CW laser to the resonance. Apart from stability issues, the main advantage of CW excitation is that the laser bandwidth is narrower than that of the resonance, allowing to couple high optical powers into the resonator (e.g. most of the CW input power can be coupled, in the experiments presented here between 10 and 100 mW).

Pulsed excitation of spectrally narrow resonances on the other hand is technically more challenging due to the large spectral mismatch between the pump laser and the microring resonator bandwidth. Specifically, standard telecommunications mode-locked lasers operate with spectral bandwidths of 3 nm or more (equivalent to ~ 400 GHz), which are at least three orders of magnitude larger than the resonance bandwidth of typical resonators. This leads to very inefficient coupling into the devices, where most power is usually lost. Even more important than the limitations due to low coupling efficiencies, is the concrete possibility to destroy of the device. In particular, the high optical peak power of the pulses can damage the facets or bus waveguides. In addition, the samples used in this work were permanently connected to optical fibers using epoxy glue. While this fiber connection enables very stable and long-term operation, the epoxy has a much lower damage threshold than the silicon oxynitride waveguides (i.e., it may easily melt or burn at high energies). For this reason, only low optical peak powers can be coupled into the chip (damage threshold is estimated to be around 600 W peak power, extrapolated from two samples we have damaged), making it particularly challenging to couple sufficient optical powers into the microring resonator for pulsed excitation (using a 10 MHz repetition-rate mode-locked laser we managed to inject 10 to 100 mW peak power, corresponding to around 0.1 to 1 mW average power. Here we consider resonances featuring a ~ 300 MHz spectral bandwidth).

2.1.1 Continuous wave excitation

To achieve CW coupling, we used a tunable telecommunications laser with around 100 kHz spectral bandwidth and 5 mW output power, where the emission wavelength can be tuned with 0.1 pm resolution. When the laser is coupled into the resonator, part of the optical power within the resonance is absorbed and heats up the resonator, which changes the refractive index of the material and shifts the resonances to longer wavelengths. If the laser wavelength is slowly increased, the coupled power increases almost linearly, up to the point it reaches a maximum (90). If the laser is then tuned past this maximum, the power reduces, the temperature drops and the resonance shifts back to lower wavelengths (different from the excitation laser wavelength), leading to an immediate loss of coupling (90). In order to couple into the resonance again, the laser wavelength has to be tuned to lower wavelengths past the resonance, and the locking has to be repeated, slowly tuning into the resonance from the lower wavelength side. This thermal bi-stability is very well documented in literature (90), and dictates that the excitation laser has to be tuned from the low-wavelength side into the resonance.

For high optical excitation powers, the bi-stability leads to a relatively stable operation, as small temperature fluctuations (mainly due to optical absorption in the resonator, but also due to environmental changes) transferred to the resonator only impose small wavelength shifts. Unfortunately, the lower the optical pump power, the higher the sensitivity to external fluctuations. As the quantum experiments require relatively low excitation powers (10 to 100 mW), and measurement times over several hours, we considered an active stabilization, to enable long-term operation. For this, the CW laser wavelength was computer controlled, and the pump power was stabilized at 80% of its maximum power value, using a standard PID feedback loop, acting on the laser wavelength to keep the power stable. The experimental setup is shown in Fig. 2.3. In particular, the CW laser is amplified using an erbium doped fiber amplifier (EDFA), and then filtered using a high isolation bandpass filter (120 dB isolation). This filter is particularly important to suppress noise photons emitted by the EDFA via amplified spontaneous emission (ASE), which are at the same frequencies as the generated single photons. The polarization is then optimized to maximize coupling into the resonator. At the drop port of the microring resonator, we placed a high isolation notch filter (120 dB isolation), which separated the excitation field and the photons generated by SFWM. The coupled output power is detected using a power meter.

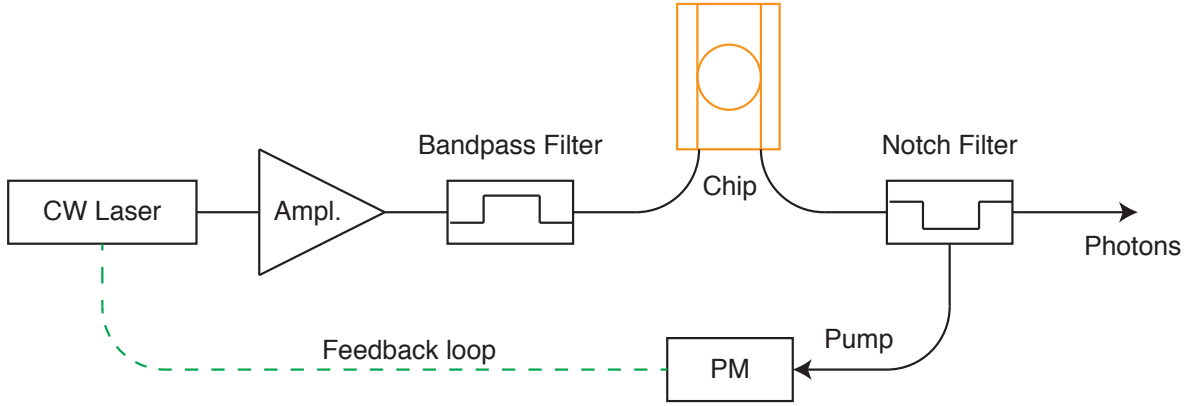


Fig. 2.3 Experimental setup for CW ring excitation:
A tunable CW laser is amplified (Ampl.), filtered to remove noise photons from the amplifier, and coupled into the microring resonator. The output of the chip is coupled into a notch filter, in order to separate the pump from the generated photons. The power after the resonator is measured with a power meter (PM). A feedback loop is used to stabilize the coupled power by adjusting the laser wavelength.

2.1.2 Pulsed excitation

To obtain pulsed excitation, we used a passively mode-locked fiber laser with 10 MHz repetition rate and 3 nm spectral bandwidth, as previously mentioned. In order to achieve efficient coupling into the microring resonator, the spectral bandwidth had to be narrowed significantly by filtering, before the light field was injected into the chip. As an example, using a pulse with a spectral bandwidth of 3 nm to excite a 1 pm wide resonance, would lead to less than 0.03% power coupling into the resonator. More significantly, if that input pulse is not spectrally filtered, all optical power is first injected into the access waveguide and most of it will not be coupled into the resonance (and will therefore be lost). The high optical peak power associated with a large bandwidth pulse can damage the facet of the chip, making it impossible to reach the required optical powers. Furthermore, due to nonlinear effects in the EDFA, large amplifications will lead to spectral pulse broadening. As a result, significant power will be shifted to frequency components located outside the resonance.

To couple 10 to 100 mW peak power into the resonator, while keeping the total peak power of the broadband pulse below 600 W at the input of the chip, we used a concatenation of multiple filtering and amplification stages: First, the laser was filtered to a bandwidth of 0.6 nm using a thin-film coated Gaussian transmission filter, followed by an EDFA, which was set to a power that

enabled maximal amplification without self-phase modulation-induced spectral broadening. The pulse was then filtered down to a bandwidth of 0.07 nm using a grating based filter, followed by a second EDFA amplification stage. After the second amplifier, a third, high isolation filter was used to suppress the amplifier noise, see Fig. 2.4. Since the pulse bandwidth is still significantly broader than the microring resonance, thermal drifts in the device can be neglected, and no active stabilization is required. After all filtering stages, 3-5 % of the average optical input power injected into the chip was coupled into the microring resonator, depending on the ring's resonance bandwidth.

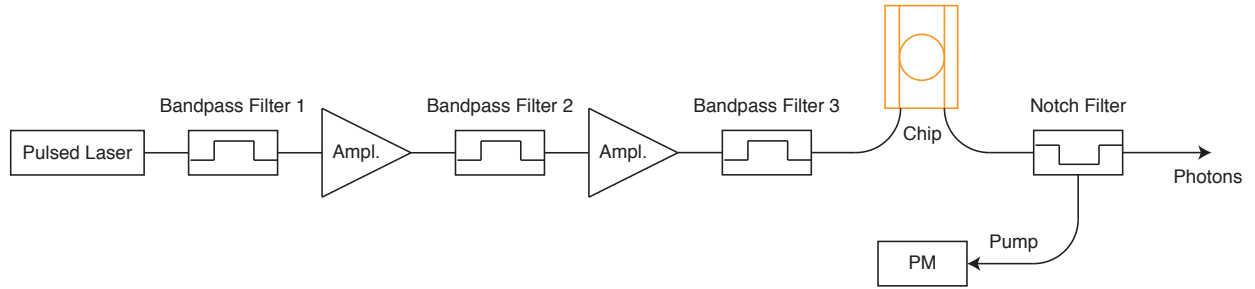


Fig. 2.4

Experimental setup for pulsed excitation of the micro cavity:

A passively mode-locked laser is filtered and re-amplified before it is coupled into the integrated microring resonator (Bandpass Filter 1 reduces the spectral bandwidth to 0.6nm, Bandpass Filter 2 further decreases it down to 0.07 nm, while Bandpass 3 acts as a high-isolation filter, thus removing the ASE noise of the amplifiers). At the output of the chip, the photons are separated from the pump field by means of a Notch Filter, and the coupled power is monitored using a power meter (PM).

2.4 Single photon detection measurements

The capability to detect single photons is at the foundation of all experiments performed in this work. Suitable detectors therefore represent a central part of the equipment for the experimental characterizations conducted in this thesis. The results presented in Chapter 3 were performed using two different types of single photon detectors, both sensitive at 1550 nm wavelength. For the experiments discussed in Sec. 3.1, we use single photon avalanche detectors (idQuantique, id210), which have a detection efficiency of $\sim 10\%$. In addition to the efficiency, important characteristics of the detectors are the dark counts per second, which are false detections triggered by noise, as well as the dead-time, which characterizes the duration the detectors are disabled after a photon (or dark count) was measured. The idQuantique detectors have a significant dark count rate of 4 kHz and a dead time of 10 μs . For the experiments presented in Sec. 3.2 and 3.3 we use superconducting nanowire detectors with 85% detection efficiency, 1 kHz dark count rate, and only 80 ns dead time (Quantum Opus, Opus1). Both systems generate an electronic pulse when a single photon is detected. This pulse is sent to precise timing electronics (a time-to-digital converter, TDC), which allocates a time-tag (i.e. the measurement time) to each detection event. We used a TDC with 1 ps timing resolution and up to four input channels, i.e. up to four single photon detectors can be used simultaneously (PicoQuant, HydraHarp400). While the resolution is 1 ps, the timing accuracy of the full system is much lower due to timing jitter of the detectors and their electronics. For the idQuantique system, we measured a temporal resolution of around 600 ps, while a system resolution of around 150 ps was achieved for the Quantum Opus detectors.

As a time-tag for each event is saved, it becomes possible to perform different types of post-processing. In each measurement involving two or more photons, the photons are sent to different single photon detectors by means of either frequency filters or beam splitters. The events for each used detector are collected on a different channel of the TDC. After collecting data for a long period of time (depending on the detection rates, but usually on the order of 1 to 60 minutes), the arrival times of multiple single photon detection events were collected and sorted to build up a histogram over different relative arrival times (between two photons, or a single photon event and a trigger). This histogram can then be used to characterize the temporal waveform, or to extract other information about single, two-photon or multi-photon quantum states.

In particular, if a CW excitation is used, the absolute arrival times were measured for photon detection events. The temporal difference between clicks in both channels was then calculated from

the measured time-tags, and a histogram was generated, where similar relative arrival times were counted together (meaning the number of measured events for each relative time difference is calculated). Such a histogram for photons generated from SFWM shows a clear coincidence peak at zero time-delay, as both photons are generated at the same time, see Fig. 2.5. In addition to the peak, a clear flat background is visible. This arises from noise photons (non-filtered ASE, Raman photons, or multi-photon events) as well as detector dark counts.

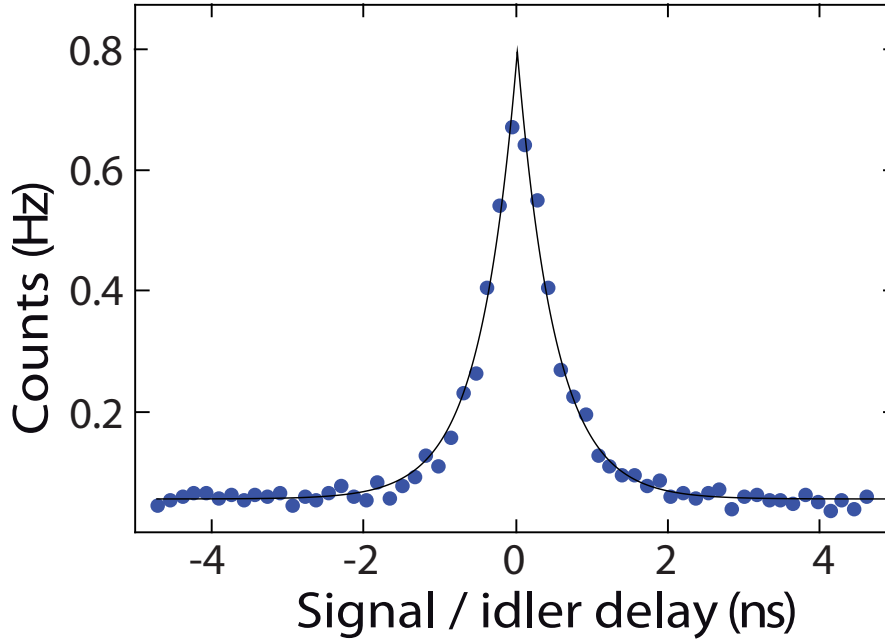


Fig. 2.5 Measured photon coincidence peak for a CW excitation of our microring:
A clear peak is measured at zero-time delay, which is placed on top of a flat background. The background is caused by noise photons (generation of multiple photon pairs, Raman scattering, or other noise sources e.g. from lamps in the laboratory) and detector dark counts. Figure published in [4].

For the pulsed excitation scheme, a trigger from the mode-locked laser is used for additional synchronization of the detection electronics. As photons from SFWM can only be generated while the pulse is inside the microring resonator, we first produce histograms between the photon detection times of each channel (i.e. photon detection) with respect to the pulsed laser, see Fig. 2.6 a). From this histogram, only detection events which are within the temporal windows indicated in Fig. 2.6 a) were selected, since all other events have to come from detection dark counts and not from photon detections. Following this post-selection procedure, a new histogram can be calculated for detection events on different channels, see Fig. 2.6 b). Similar to the CW excitation case, a clear peak at zero time-delay is measured. But instead of having a flat background, smaller peaks are

measured at multiples of the laser repetition rate, while no photon events are recorded between the pulses. The pulsed case has the advantage that we can eliminate most of dark detection noise, as this noise is temporally not correlated.

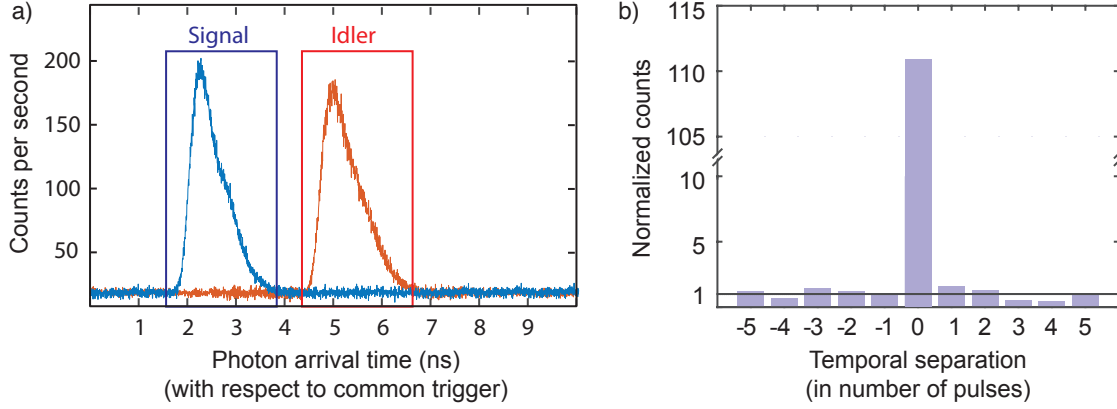


Fig. 2.6 Measured photon coincidence peak with pulsed excitation of the microring:
a) Measured histogram where single channel (signal and idler) detections are shown with respect to a common trigger from the pump laser. Photon events within the indicated windows are selected to calculate a new histogram, shown in b). A clear peak is measured at zero-time delay, with a background only measured at multiples of the laser repetition rate. The counts are normalized such that the average background is at one, so that the maximum of the central peak corresponds to the coincidence to accidental ratio (CAR). Note that the y-axis is broken and jumps from 10 to 105, since otherwise the background would not be visible. Figure 2.6 b) published in [10].

Having generated histograms with photon coincidences, one can extract the so-called Coincidence-to-Accidental Ratio (CAR). The CAR is a signal-to-noise metric, which is defined as the coincidence counts within the full-width-at-half-maximum (FWHM) of the coincidence peak, divided by the amount of counts within the same time-window placed outside of the peak (91). For the CW pumped source, this accidental window can be arbitrarily chosen as long as it is far away from the central peak. In the pulsed case, this window has to be placed centered over an adjacent smaller peak.

2.4.1 Second-order photon coherence

Higher-order optical coherence measurements are commonly used to identify the statistical properties of light, which can be categorized into three main types: thermal, coherent, and quantum (92, 93). The second-order coherence can be measured using a Hanbury Brown and Twiss type

setup (94), which consists of a beam-splitter followed by two time-resolving detectors. The second-order coherence can then be extracted from coincidence measurements, as long as the temporal resolution of the detectors is higher than the first-order temporal coherence of the incident light (95). If single photon detectors are used (with detection times in the hundreds of picoseconds) often spectral filtering has to be performed to narrow the input light bandwidth, and thereby increase its first-order temporal coherence time so that it is longer than the detector resolution. Such filtering does not impact the photon statistics, neither do the additionally added losses (96).

The typical expected second-order coherence functions for thermal, coherent and quantum light sources are summarized in Fig. 2.7. For a true single-photon emitter, the probability to emit two photons at the same time is zero, leading to a dip at $t = 0$, which approaches zero. The width of this peak is defined by the temporal first-order coherence of the emitted photons. Instead, for thermal light, typical photon bunching is observed. This can be intuitively understood by the fact that a photon which was spontaneously generated within a source can also seed the generation of another photon, increasing the probability that two photons are detected at the same time. The ratio between spontaneous and stimulated emission in fully thermal sources leads to a bunching peak of 2 at $t = 0$ in the second-order coherence measurement (96). Finally, for a coherent field, the temporal photon distribution is fully random, leading to second-order coherence which is equal at all times, i.e. without a peak or dip. In a real measurement, also the response function of the used detectors has to be considered, where the measured results are given by the convolution of the ideal curve with that response function. Therefore, only if the temporal resolution is much higher than the coherence time of the source, a peak or dip can be measured. If the detectors have a response time slower than the photon lifetime, the peak or dip washes out and a flat line is measured, falsely indicating coherent light independent of the actual photon statistics (97).

It has been shown that the statistics of light generated from SFWM is thermal in nature (96), where a source emitting perfect single-mode and pure photons has a second-order coherence of 2 at $t = 0$. However, if the photons are generated over a large spectral bandwidth (which is larger than the bandwidth of the excitation field), the photons are emitted into multiple spectral modes. Indeed, it has been shown that the maximum of the second-order coherence function is directly related to the number of spectral modes of the photons with $g^{(2)}(t=0) = 1 + 1/(\text{Nr. of Modes})$ (96). The second-order coherence measurement can therefore be used to determine the purity of the emitted photons (96), where pure photons are characterized by exhibiting only a single frequency mode.

As SFWM always generates two photons at the same time, this process can be used to generate so-called heralded single photons, where the signal photon is detected, heralding the presence of the idler photon. For a perfect two-photon source, this idler photon is considered to be a single photon and should show quantum characteristics. However, if the two-photon source is achieved by exploiting a nonlinear process, photon bunching can still occur. The ratio between the presence of a true single photon and multiple photons can be measured with the heralded second-order coherence function, which effectively compares the ratio between true heralded single and multi photon emission. If a dip in the heralded second-order coherence measurement below 0.5 is measured (92, 98), the source operates in the quantum regime, and the detection of one photon heralds the arrival of a single photon.

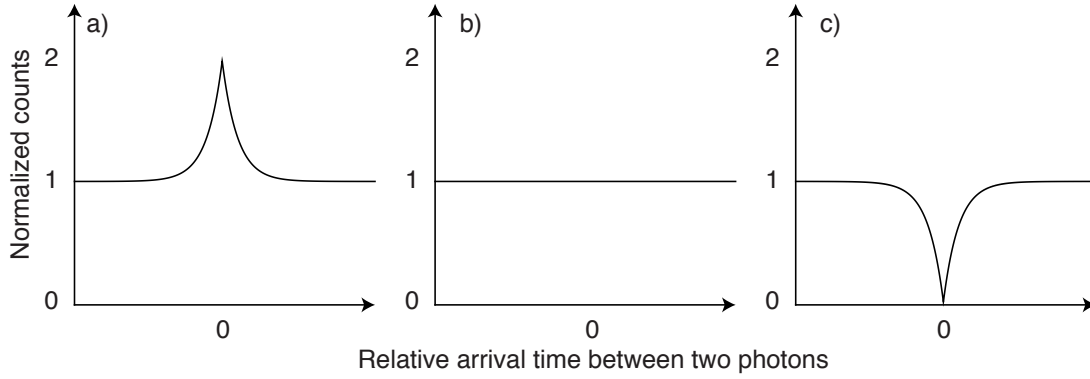


Fig. 2.7

Expected second-order coherence of different light sources:

a) Thermal light is characterized by a bunching peak at zero time delay, with a maximum of 2. b) Coherent light has fully random photon distribution, leading to a constant second order coherence. c) Single photon sources are characterized by a dip at zero time delay, where an ideal source has a dip going to zero. However, any source with a dip below 0.5 is operating in the quantum regime.

2.5 Quantum interference

Quantum interference is at the basis of almost all applications exploiting optical quantum states. Quantum interference can be used to confirm the presence of entanglement (28), to transmit data in a secure way (99), to enable measurements with precision exceeding classical limits (15), and to perform quantum computation (12). In this work, quantum interference will be used mainly as a tool to characterize the optical quantum states, generated in the microring resonators.

2.5.1 Single photon or “classical” interference

In quantum mechanics, interference originates from the lack of information (100). This can be illustrated with the example of a ‘classical’ optical interferometer (a ‘non-classical’ interferometer is described in Sec. 2.8.1). The interference phenomena in e.g. a Mach-Zehnder interferometer (see Fig. 2.8) can be described as single-photon interference, which arises from the absence of photon path information (or indistinguishability of photon paths). Starting with a source that emits a single photon, the photon passes through the interferometer and is then detected by a detector. After the first beam splitter, the photon can take two alternative paths (1 and 2 in Fig. 2.8), to arrive at the detector.

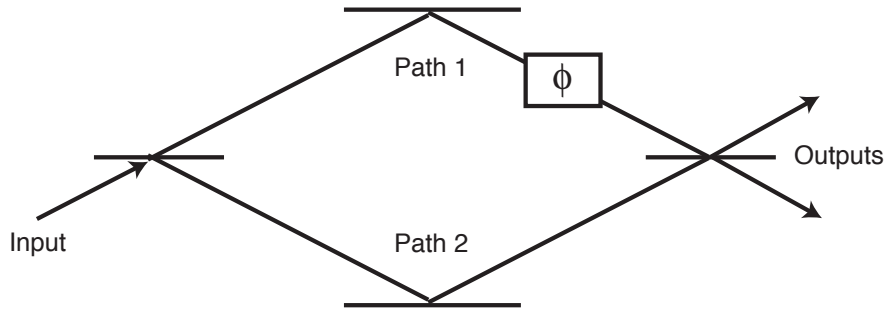


Fig. 2.8 Schematic of a typical Mach-Zehnder interferometer:

An optical input is split into two paths 1 and 2 by a beam splitter. The phase difference between these paths is described by ϕ . Both paths are then re-combined using a second beam splitter, and photons can be detected at one of the output ports (or both).

The probability of detecting the photon when it travels via path 1 is given by $P_1 = |a_1|^2$, with a_1 being the corresponding probability amplitude (which is in general a complex number). P_1 is directly proportional to the detection rate of the detector when path 2 is blocked. Similarly, the probability of detecting the photon when it travels via path 2 is given by $P_2 = |a_2|^2$, with the

corresponding probability amplitude a_2 . When both paths are open, interference occurs and the total probability of the photon being detected is given by

$$P = |a_1 + a_2|^2 = P_1 + P_2 + 2\sqrt{P_1 P_2} \cos \phi$$

where $\phi = \arg \{a_1^* a_2\}$ with ‘arg’ being the argument of a complex number (this corresponds to the phase difference between the two optical paths). According to quantum mechanics, interference occurs if and only if there is absolutely no possibility of determining which path the photon took to arrive at the detector (100). In other words, interference occurs if and only if there is no path information anywhere in the universe. In contrast, when information is available, no interference occurs, and the probability of detecting the photon is given by

$$P = |a_1|^2 + |a_2|^2 = P_1 + P_2$$

It is important to note that this information does not need to be extracted by a measurement; the presence of the path information alone is enough to destroy the interference (101). This was for example demonstrated in a single photon interference experiment, where the interfering photon was created by a two-photon source (101). The experiment was built in such a way that in a certain condition, the detection of the second photon carried information of the path the other photon took in the interferometers. The single photon interference disappeared in this configuration, even though the other photon was never detected. This experiment (101), and a lot of other work (102–104) show that the (non-) presence of information is the most important factor that enables or destroys interference.

An interesting situation arises if we allow the possibility of the path information to be ‘partially available’. In order to understand this case we can introduce the visibility V , which quantifies the quality of interference:

$$V = \frac{P_{Max} - P_{Min}}{P_{Max} + P_{Min}}$$

Here P_{Max} and P_{Min} are the maximum and minimum measured values of P , respectively. It can be seen that for $P_1 = P_2$ (and complete unavailability of path information under ideal conditions) the visibility becomes unity ($V=1$). Similarly, the complete availability of path information implies zero visibility.

It is also possible to have a case where the visibility is less than unity but more than zero. In such a case, if $P_1 = P_2 = P_0$, the total probability of detecting a photon is given by

$$P = 2P_0(1 + V \cos \phi)$$

In such a situation, we say that the information is partially available, meaning it is in principle possible to determine with a certain probability which path the photon took. The quantum mechanical interpretation of interference thus allows us to consider the visibility as a measure of the unavailability of information (101–104).

In conclusion, interference arises from the unavailability of information, and the visibility of interference measurements provides a measure for this unavailability of information.

2.5.2 Two photon interference

Single photon interference results in a phase dependent probability to measure a single photon at the output port of an interferometer, as in the above discussed example. Single photon interference is also often referred to as classical interference, as it can be perfectly described using classical wave optics, where a wave propagates through two paths and interferes with itself. Instead, two-photon interference (often also referred to as ‘quantum’ interference) does not have such a classical analogy, and describes the phase dependent probability to measure two photon coincidences.

A famous example of two photon interference, which can be used to illustrate the concept, is the so-called Hong-Ou-Mandel (HOM) effect, see Fig. 2.9, where two photons enter two different ports of a beam splitter (105). If the two photons are identical (meaning they are fully indistinguishable, i.e. no information on which photon entered which port of the beam splitter is available), the photons will always exit together at the same output port, and the probability that the two photons exit different ports becomes zero. With a detector at each output port, the probability of measuring photon coincidences goes to zero for perfectly indistinguishable photons. If a temporal delay between the photons is introduced, the photons become temporally distinguishable, can thus exit different output ports and coincidences can be detected.

Furthermore, HOM interference results in a coherent superposition of either two photons exiting port 1 or port 2 of the beam splitter. The output of two identical photons entering two different ports of a beam splitters is therefore a path-entangled state:

$$|\psi_{ent}\rangle = \frac{1}{\sqrt{2}}(|1\rangle_s|1\rangle_i + |2\rangle_s|2\rangle_i) = \frac{1}{\sqrt{2}}(|1,1\rangle + |2,2\rangle)$$

HOM interference can therefore be used to generate entanglement, and is at the basis of linear optical quantum computation (LOQC) (81).

Reversing the HOM effect, one can immediately conclude that if a path-entangled state enters a beam splitter, two indistinguishable photons will exit at different output ports, and the probability of detecting two photons at the same port is zero (105). While the HOM effect requires indistinguishable photons, the inverse effect requires that the input state is entangled, meaning that the two incident photons are in a coherent superposition. This analogy of reversed HOM holds perfectly for indistinguishable photons, however, the effect of quantum interference with entangled input states also holds for photons, which are distinguishable in some degrees of freedom. The analogy can therefore not be applied for all states, but it shows that there is a relationship between HOM interference and quantum interference with entangled photons.

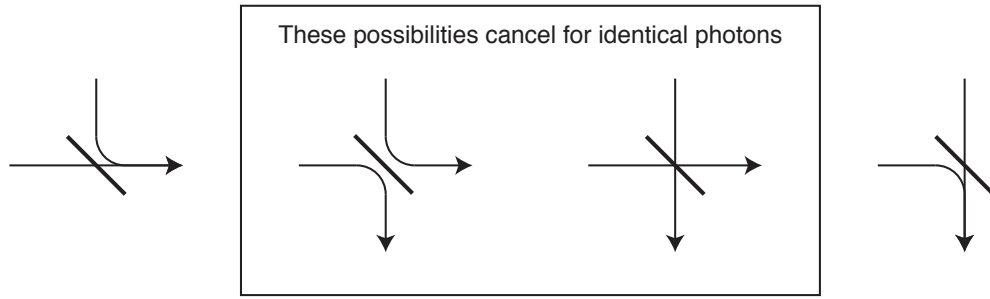


Fig. 2.9

Two-photon Hong Ou Mandel interference:

Two photons enter two different input ports of a beam splitter. If the photons are perfectly identical, the two photons bunch and always exit one of the ports together, leading to a zero probability to detect coincidences between two detectors placed at the two output ports of the beam splitter.

Let us consider a beam splitter with input ports 1 and 2 and output ports a and b (we now introduce different labels for input and output ports for clarity). Measuring a photon at port a or b corresponds to a projection on

$$|\psi_{P,a}\rangle = \frac{1}{\sqrt{2}}(|1\rangle + i|2\rangle)$$

$$|\psi_{P,b}\rangle = \frac{1}{\sqrt{2}}(i|1\rangle + |2\rangle)$$

respectively, where the imaginary components result from a π phase shift upon reflection at the beam splitter. For an entangled input state, the probability to measure photon coincidences at port combinations of a and b can be calculated.

$$|\psi_{ent}\rangle = \frac{1}{\sqrt{2}}(|1,1\rangle + e^{i\varphi}|2,2\rangle)$$

$$\langle\psi_{P,a}, \psi_{P,b}| = \frac{1}{\sqrt{2}}(-i\langle 1,1| + \langle 1,2| + \langle 2,1| - i\langle 2,2|)$$

$$P_{a,b} = |\langle \psi_{P,a}, \psi_{P,b} | \psi_{ent} \rangle|^2 = \frac{1}{4} (1 + \cos \varphi)$$

$$P_{b,a} = |\langle \psi_{P,b}, \psi_{P,a} | \psi_{ent} \rangle|^2 = \frac{1}{4} (1 + \cos \varphi)$$

$$P_{a,a} = |\langle \psi_{P,a}, \psi_{P,a} | \psi_{ent} \rangle|^2 = \frac{1}{4} (1 - \cos \varphi)$$

$$P_{b,b} = |\langle \psi_{P,b}, \psi_{P,b} | \psi_{ent} \rangle|^2 = \frac{1}{4} (1 - \cos \varphi)$$

It can be clearly seen that the sum of all possible outcomes is unity, but that for a given projection, clear interference in the photon coincidences is seen, which is the effect of two-photon interference. Similarly to the example of single photon interference, it is also possible to consider the visibility of HOM interference as a measure of photon distinguishability (106). As perfectly indistinguishable photons, in combination with HOM interference, generates a maximally entangled state, partially distinguishable photons will result in non-maximally or even non-entangled photon states. Vice versa, the visibility of entanglement-based two-photon interference can be used to confirm and quantify the presence of entanglement in the input state, see Sec. 2.7.

2.6 Quantum state tomography

Quantum state tomography is an experimental method that allows the extraction of the density matrix of a quantum state, which fully describes the quantum state (107). In this below listed particular example, we assume a particular entangled two-photon state in two dimensions, which can be written as $|\Psi\rangle = \frac{1}{\sqrt{2}}(|1,1\rangle + |2,2\rangle)$, where 1 and 2 are two orthogonal modes. For this wave function the density matrix ρ is then given by:

$$\rho = |\Psi\rangle\langle\Psi| = \frac{1}{2}(|1,1\rangle + |2,2\rangle)(\langle 1,1| + \langle 2,2|)$$

Using a set of orthogonal basis vectors $\{|1,1\rangle, |1,2\rangle, |2,1\rangle, |2,2\rangle\}$ this results in the ideal theoretical two-photon density matrix:

$$\rho_{th} = \begin{pmatrix} 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 \end{pmatrix}.$$

It is possible to measure this density matrix through projection measurements of the signal and idler qubits, resulting in a minimum of 16 required values for the full description of the two-photon state (107). Each of these projection measurements can be described through a wave vector $|\Psi_v\rangle$. The projection measurements can be almost arbitrarily chosen, as long as the below described B matrix can be inverted. This approach is not limited to two photons or two-dimensional quantum states. Indeed, it can be used for any quantum state, showing the power of this approach (108). The photon counts collected in these projection measurements are mathematically given by:

$$n_v = C \langle \Psi_v | \rho | \Psi_v \rangle,$$

where C is a measurement constant dependent on the integration time.

The challenge now is to reconstruct the density matrix from the measured n_v values, which can be achieved mathematically if the B matrix, reported below, is invertible. In particular, the measured density matrix can then be reconstructed using the following relations, where M and B have no direct physical meaning but are part of the mathematical reconstruction process:

$$\begin{aligned} \rho &= C^{-1} \sum_v M_v n_v, \\ M_v &= \sum_x \Gamma_x (B^{-1})_{x,v}, \\ B_{x,y} &= \langle \Psi_x | \Gamma_y | \Psi_x \rangle, \text{ and} \end{aligned}$$

$$C = \sum_k n_k, \text{ for } \text{Tr}\{M_k\} = 1.$$

In addition to the measured values, a set of linearly-independent and normalized matrices Γ is required. Any set of such matrices can be used as long as they fulfill $\text{Tr}\{\Gamma_x \Gamma_y\} = \delta_{x,y}$.

A fast way to construct this set of matrices is to use the generators of the SU(2) (two photons each in 2 dimensions), SU(3) (two photons each in 3 dimensions), or SU(4) (four photons each in 2 dimensions, or two photons each in four dimensions) special unitary groups (plus the identity matrix), and construct the Γ matrices through the Kronecker product of all combinations. The generator matrices of the SU(2), SU(3) and SU(4) groups are listed in Appendix A.

A density matrix associated to a physical system has to be Hermitian and positive definite. However, the matrix extracted from measurements usually does not comply with this requirement. To retrieve a physically-meaningful density matrix, it is possible to perform a maximum-likelihood estimation, which is a method used to find the physically-realistic density matrix closest to the measured one (107).

Once the density matrix ρ_{exp} is found, it is possible to extract the fidelity of the measured state. The fidelity describes the overlap between the ideal theoretical and measured density matrix (107), and is given by $F = \left(\text{Tr}([\sqrt{\rho_{th}} \rho_{exp} \sqrt{\rho_{th}}]^{1/2}) \right)^2$. A fidelity of one corresponds to a perfect overlap with the ideal entangled state.

2.7 Bell inequalities

The violation of a Bell inequality confirms the violation of classical inequalities, meaning that a system cannot be described using classical physics (i.e. with local hidden variables).

A Bell test measures the strength of correlations in different basis sets. While classical systems can have perfect correlations in specific Basis sets, the over-all correlation strength cannot exceed a specific level, as soon as correlation measurements are performed in different (and usually rotated) basis sets. As an example, let us consider a source that generates photons in an incoherent mixture of either two horizontally or vertically polarized photons, described by a density matrix $\rho_{mix.} = 0.5 * (|H, H\rangle\langle H, H| + |V, V\rangle\langle V, V|)$. Such a source can provide very high correlations in certain polarization settings (i.e. H and V), however, the light is entirely uncorrelated in diagonal or circular polarization basis settings. This is in stark contrast to e.g. polarization entangled sources, where two photons are in a coherent superposition of polarization modes, described by a density matrix $\rho_{ent.} = 0.5 * (|H, H\rangle + |V, V\rangle) * (\langle H, H| + \langle V, V|)$, where perfect correlations can be achieved in any polarization basis (horizontal, vertical, circular or a combination of them).

A Bell inequality classifies the over-all correlation strength and assigns a measurable quantity, the $I_{D=2}$ parameter (for two-dimensional systems), which cannot be larger than 2 in classical systems, but has a maximum of $2\sqrt{2} \approx 2.83$ for quantum systems (109, 110). Bell inequalities are therefore often used to confirm the quantum properties and a system.

There are different ways to measure the Bell parameter I_D . One way to extract the Bell parameter by means of quantum interference measurements. If the correct projections are chosen (111), it becomes possible to fit a specific function and extract the visibility of the quantum interference. For the two-photon Bell State, this projection is $|\Psi_{Proj.}\rangle = \frac{1}{2}(|H\rangle_i + e^{i\theta}|V\rangle_i)(|H\rangle_s + e^{i\theta}|V\rangle_s)$, which results in quantum interference with twice the period compared to the quantum interference if one phase (or polarizer) is kept constant. It now becomes possible to define a visibility V of the quantum interferences and fit a function $C(\theta) = 1 + V \cdot \cos(2\theta)$. If this function can be fitted to the measured quantum interference, the visibility becomes directly proportional (using a linear noise model) to the Bell factor, $I_2 = 2\sqrt{2} * V$. Measuring a visibility higher than $V > 1/\sqrt{2} \approx 0.72$ therefore directly violates a Bell inequality (under the only assumption of an equal distribution of noise).

The approach of measuring a Bell inequality by means of quantum interference can be generalized to higher dimensions (110). In particular, we consider a quantum system consisting of two photons, where each photon can be in a D -dimensional superposition, i.e. quDit state. For such a D -dimensional bipartite system, a Bell parameter I_D can be defined, where $I_D > 2$ denotes the violation of an inequality and thereby confirms entanglement and the exclusion of local realism (109, 110). The value of I_D can be extracted from the visibility associated to the quantum interference measurements (110), which can be obtained by projecting the states onto the projection vector:

$$|\Psi_{\text{Proj},D}\rangle = \frac{1}{D} \left(\sum_{k=1}^D e^{ik\theta} |\bar{k}\rangle_s \right) \left(\sum_{k=1}^D e^{ik\theta} |\bar{k}\rangle_i \right),$$

where $|\bar{k}\rangle$ represent any type of photon mode, e.g spatial, temporal, polarization, etc. Assuming a maximally entangled state of the form $|\Psi_D\rangle = \frac{1}{\sqrt{D}} \sum_{k=1}^D |k\rangle_s |k\rangle_i$, the probability to measure photon coincidences can be calculated.

$$|\langle \Psi_{\text{Proj},D=2}(\theta) | \Psi_{D=2} \rangle|^2,$$

$$|\langle \Psi_{\text{Proj},D=3}(\theta) | \Psi_{D=3} \rangle|^2, \text{ and}$$

$$|\langle \Psi_{\text{Proj},D=4}(\theta) | \Psi_{D=4} \rangle|^2.$$

The expected quantum interference signal then takes the form:

$$C_{D=2}(\theta) = 1 + \varepsilon_2 \cdot \cos(2\theta),$$

$$C_{D=3}(\theta) = 3 + 2\varepsilon_3 \cdot [2\cos(2\theta) + \cos(4\theta)],$$

$$C_{D=4}(\theta) = 4 + 2\varepsilon_4 \cdot [3\cos(2\theta) + 2\cos(4\theta) + \cos(6\theta)],$$

where ε_D ($D = 2, 3$ or 4) emerges from a symmetric noise model (110), and describes the deviation from a pure, maximally-entangled quantum state. This coefficient is related to the visibility:

$$V_2 = \varepsilon_2,$$

$$V_3 = \frac{3\varepsilon_3}{2+\varepsilon_3}, \text{ and}$$

$$V_4 = \frac{4\varepsilon_4}{2+2\varepsilon_4}.$$

The Bell parameter can again be directly linked to the measured visibilities:

$$I_2 = 2\sqrt{2} * V_2$$

$$I_3 = \frac{2(6\sqrt{3} - 5)}{3(6\sqrt{3} - 9)} * V_3$$

$$I_4 = \frac{3 + \sqrt{2} + \sqrt{10 - \sqrt{2}}}{3} * V_4$$

Additionally, the Bell inequalities can also be expressed directly in terms of visibilities (*III*):

$$\frac{1}{\sqrt{2}} \approx 0.7071 < V_2,$$

$$\frac{3(6\sqrt{3}-9)}{6\sqrt{3}-5} \approx 0.7746 < V_3, \text{ and}$$

$$\frac{6}{3+\sqrt{2}+\sqrt{10-\sqrt{2}}} \approx 0.8170 < V_4.$$

The measurement of two photon interference can therefore be used to violate a Bell inequality and confirm the presence of entanglement (*III*), if the correct projection vectors are chosen. The experimental challenge (among other) lies in realizing the required projection measurements for the two photons.

2.8 Entanglement in integrated microring resonators

As already introduced in Sec. 1.1, integrated sources have managed to produce different types of entangled quantum states such as path entanglement (61, 64–66), polarization entanglement (60, 62, 63), as well as continuous variable energy-time entanglement (59). Most sources have mainly relied on simple waveguides (53), photonic crystals (26), as well as microring resonators (91). While microring resonators have been used for the generation of photon pairs (91), and recently also the generation of continuous energy-time entanglement (59), up to the starting point of this work, none of the work has studied the generation of photons in multiple frequency modes. Additionally, only one publication had achieved entanglement within microring resonators, which exploited continuous variable energy-time entanglement (59).

As discussed in Sec. 1.1, there is particular interest in the generation of discrete variable entangled quantum states, as they provide easy manipulation and access to quantum modes. For this reason, this work focuses on the realization of discrete energy-time entanglement. In particular, both discrete versions of energy-time entanglement, called time-bin (112) and frequency-bin (113) entanglement are ideally suited for their generation and exploitation in integrated waveguide structures, as they can be generated in a single spatial mode.

2.8.1 Time-bin entanglement

Time-bin entanglement is very commonly used for applications in quantum communications (82), but it can also be exploited for quantum computation protocols (68). Time-bin entanglement can be generated by exciting a nonlinear source with coherent double pulses (112). Such pulses can for example be generated using an unbalanced interferometer, where the path length difference is much larger than the coherence length of the input pulse (therefore no classical, single photon interference can be measured with such an interferometer for the input pulses, as they temporally are fully distinguishable). For a single input pulse, such an interferometer will generate two pulses at the output, which are delayed in time (defined by the path length difference of the interferometer) and which have a fixed phase relationship (defined by the phase of the interferometer). If a nonlinear source is excited with such a double pulse, and the power is chosen in such a way that the probability can be neglected that each of the two pulses generates a photon pair, then a photon pair generated from such a double pulse is generated in a coherent superposition of two temporal

modes: $|\psi_{time-bin}\rangle = \frac{1}{\sqrt{2}}(|S, S\rangle + |L, L\rangle)$, where S and L refer to a short and long time bin, respectively.

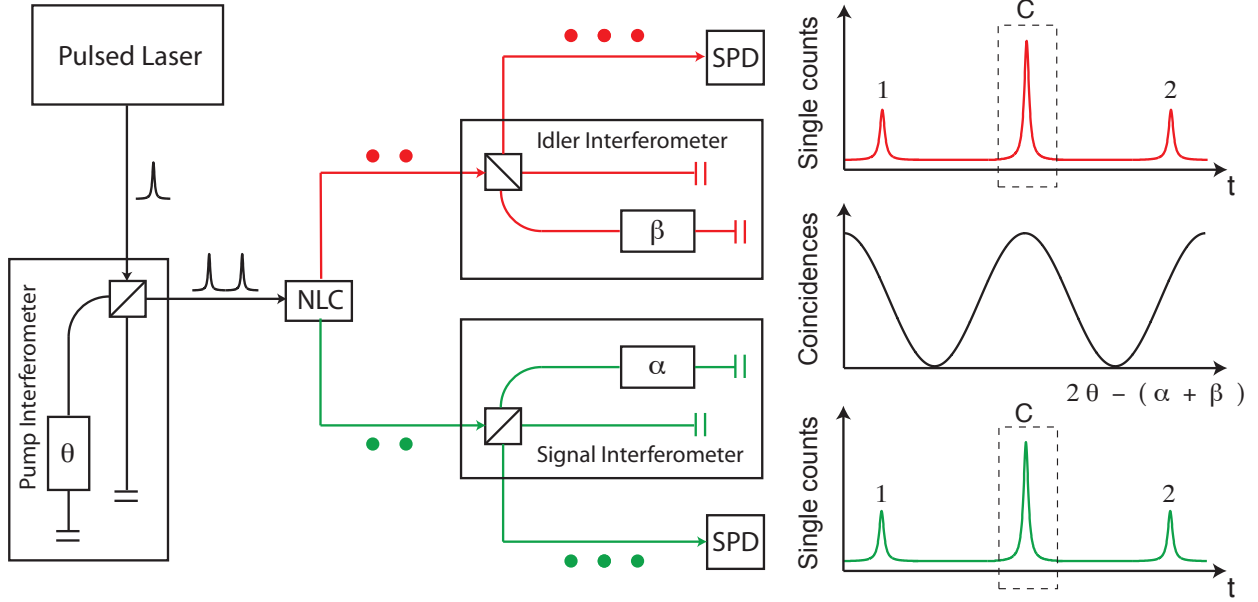


Fig. 2.10 **Generation and characterization of time-bin entangled states:**

Typical setup for the generation and characterization of time-bin entangled photon pairs. A pulsed laser is passed through an unbalanced interferometer to generate double pulses with a relative phase θ . These pulses then excite the generation of photon pairs in a nonlinear crystal (NLC). The photons (signal, green; idler, red) then pass through two more unbalanced interferometers (adding phases α and β , respectively) and are detected with single photon detectors (SPDs). The arrival time of the photons after the interferometer is used to select the projection, where the first and last bins (1 and 2) correspond to projections on the short and long time-bin respectively, while the central peak (C) corresponds to a projection on a superposition of both time-bins. If the central bins are selected for coincidence detection, quantum interference is measured as a function of $\cos(2\theta - \alpha - \beta)$.

In order to perform time-bin entanglement measurements, additional stable interferometers are required to implement the projection measurements, and are discussed in Sec. 2.9. In particular, interferometers with a temporal delay equal to the temporal difference between the short and long time bin are required. If the time-bin entangled state is placed at the input of such an unbalanced interferometer, a photon at the output of the interferometer can be measured in three different time slots, named ‘1’, ‘C’ (for center), and ‘2’. Where ‘1’ and ‘2’ project the state on $|S\rangle$ and $|L\rangle$ respectively, the central slot C projects the state on $|\psi_{P,C}\rangle = \frac{1}{\sqrt{2}}(|S\rangle + e^{i\varphi}|L\rangle)$, where φ is the phase of the interferometer.

A common setup for the generation and characterization of time-bin entangled photon pairs is shown in Fig. 2.10, including the typical histograms measured of photons arriving with respect to a common trigger signal. By post selecting signal and idler photons in the time slot ‘C’, and adjusting the phases of the interferometers, it becomes possible to perform Bell inequality measurements. In order to perform quantum state tomography, photon coincidences between all combinations of the three different arrival times have to be considered.

While stable interferometers are sufficient to perform all required projection measurements for the characterization of time-bin entangled states, the implementation of gate operations would require e.g. fast switches and more complex waveguide structures (68, 114). Such investigations are outside the scope of this work.

As both the generation and the characterization require unbalanced interferometers, the availability of very stable interferometers is paramount for experimental investigation of time-bin entanglement. In addition to the stability, it is important that the interferometer phases are known and can be controlled with high precision. For this reason, we developed a new stabilization scheme for fiber interferometers, which is discussed in Sec. 2.9.

2.8.2 Frequency-bin entanglement

Compared to time-bin entanglement, frequency-bin entanglement is very rarely used for applications. This arises from two main issues related to their generation and detection. In particular, frequency-bin entanglement was first realized by filtering a broadband photon pair into discrete frequency modes (113), see Fig. 2.11. While each of these modes still represent a continuous state, the correlations within these modes were neglected and treated as frequency-bins (113). The problem with this approach is that most of the photons are lost during filtering (i.e. the photon flux decreases), reducing the generation efficiency. Additionally, most of the entanglement resource is neglected as entanglement within the frequency modes is not used.

After successful generation, the states have to be controlled, and projection measurements have to be performed to characterize the state. In order to carry out phase gate operations, a free-space spatial light modulator together with a grating can be used to adjust the amplitude and phase of each of the frequency bins (113), see Fig. 2.11. This is an efficient way to manipulate the frequency-bin entangled states, as such spatial light modulators can implement very versatile operations on many frequency modes simultaneously.

Having achieved state generation and control, one of the biggest challenges is to perform frequency-bin entanglement measurements. i.e. to carry out projection measurements. While in the temporal domain this can be done using an interferometer, in the frequency domain the spectral modes have to be mixed, i.e. the photon frequency has to be changed. Frequency projection measurements were in the past achieved by using nonlinear frequency mixing inside a nonlinear crystal (113, 115), which converts two photons back into a single photon, see Fig. 2.11. This process however is very inefficient and cannot be used for deterministic measurements, as only one in over a million photon pairs gets successfully back-converted (113, 115).

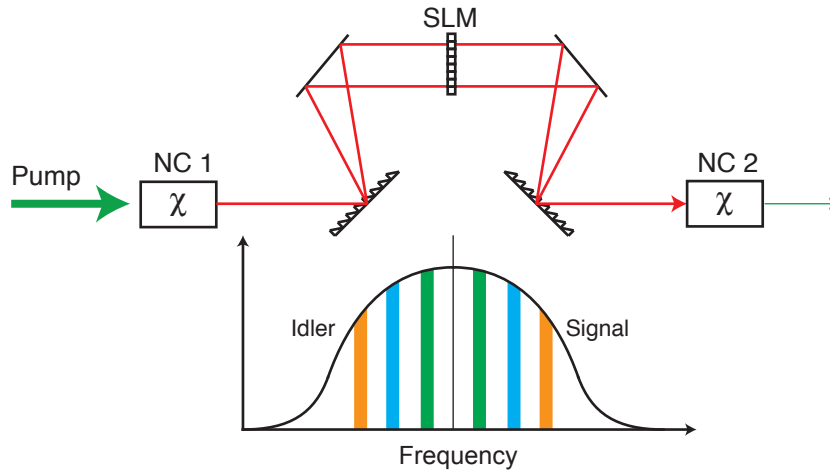


Fig. 2.11 Generation and characterization of frequency-bin entangled states: Typical setup for the generation, control and detection of frequency-bin entangled quantum states. A nonlinear crystal (NC 1) generated broadband photon pairs (red) from a strong pump (green). These are cut into frequency bins (indicated in the bottom inset with different colors), and phase shifts are added using a spatial light modulator (SLM). To perform state characterization, the photon pair is converted into a single photon (small green arrow) using a second nonlinear crystal (NC 2), and this single photon is then detected (detection not shown in figure).

Taking the generation, manipulation and detection of frequency-bin entanglement into account, one can immediately see that the current approaches to exploit this type of entanglement are very challenging and inefficient, which is why frequency-bin entanglement is currently very rarely used. In order to efficiently exploit this type of entanglement, it is therefore required to re-visit the generation, manipulation and detection approach, which is a main focus of this work.

Indeed, integrated microring resonators are ideally suited to generate frequency-entangled states. In particular, if the phase-matching condition is such that photon pairs can be generated over multiple resonances symmetric to the pump, photon pairs are intrinsically created into a frequency-bin entangled states, see Fig. 2.12. The advantage in the generation is threefold: First, the free-

spectral range of the microring resonators can be chosen in the GHz range, making it possible to select individual frequency modes with standard telecommunications filters. Second, the photons are directly generated into the frequency modes of the resonators, so no photons are lost due to external filtering. And third, the photons can be generated into pure states, if a pulsed pump configuration is chosen, since in such a configuration the excitation bandwidth can be matched to the photon bandwidth. In other words, true frequency-bin entangled states can be produced, where each frequency mode is a pure state.

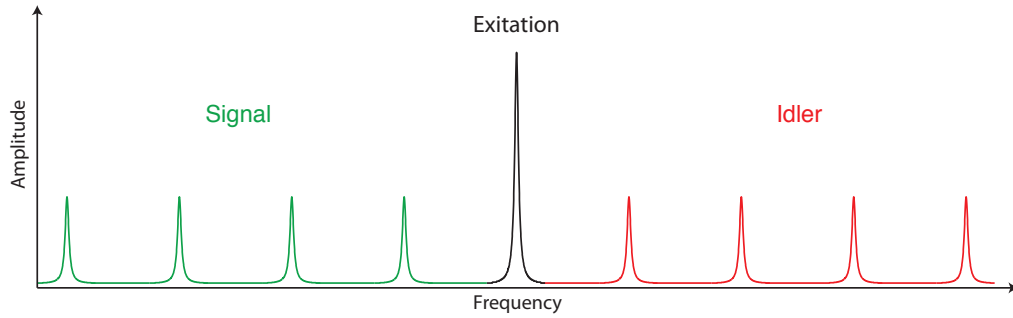


Fig. 2.12 Frequency-bin entanglement generation in microring resonator: A single photon pair can be generated in a superposition of multiple frequency modes, if the phase matching condition is broad enough to enable the nonlinear frequency conversion over a broad spectral range. In this case, the photons are directly generated in a frequency-bin entangled quantum state.

Even though the generation of frequency-bin entangled photon pairs can be almost naturally achieved using our nonlinear integrated source, the issue to efficiently measure these states remains a challenge. To address this problem, we developed a new coherent control scheme, based on both fiber-coupled spatial light modulators and electro-optical phase modulation. In particular, driven by the fiber telecommunications industry, fully packaged fiber coupled frequency dependent amplitude and phase filters are now commercially available with low losses and high performance (we used a Finisar Waveshaper). Additionally, advances in optical phase modulation allow to routinely achieve bandwidths in the tens of GHz (we used EO-Space modulators, optimized to 40 GHz). We therefore proposed to exploit deterministic frequency conversion in electro-optical phase modulators in combination with amplitude and phase control to perform state manipulation and characterization. This new scheme to perform deterministic projection measurements for frequency-bin entangled states is described in Sec. 2.10.

2.9 Novel technique for fiber interferometer stabilization

As discussed in Sec. 2.8.1, unbalanced interferometers are ideally suited to generate and characterize time-bin entangled quantum states. However, to perform practical measurements, it is required that the interferometer phase is stable, known and controllable.

In order to achieve high stability, we chose to use fiber-based Michelson interferometers, formed by a fiber 50:50 coupler, a piezo fiber stretcher element (used to adjust the phase of the interferometer) and two Faraday mirrors, which compensate for the polarization changes in the birefringent single mode fibers. We chose a temporal unbalance of ~ 11 ns, which was much larger than the temporal coherence of the single photons (~ 1 ns), therefore eliminating single photon interference effects.

The interferometers were then actively stabilized using a stable CW laser at a wavelength different from the quantum signal. For an input CW laser field, the output power signal cannot give details to the exact phase of the interferometer, as the same output power can correspond to different phase settings. In addition, it is not possible to stabilize on all phases, as at the maxima or minima a change in power does not provide information about the direction of the phase drift. In order to remove this issue and allow stabilization on an arbitrary phase, a modulation (dither) on the interferometer arm or the stabilizing laser can be carried out (116). However, this dither is associated with an additional source of noise. In free-space interferometers, it is possible to use for example diverging beams together with multiple detectors or a CCD camera to extract the absolute phase of the interferometer and in turn stabilize on it (117). This has the advantage that it allows continuous stabilization of the interferometer at any phase without the need to introduce noise by dithering, which is of particular interest for quantum applications. However, the required angle misalignment of the interferometer results in a shift in the phase over the cross section of the beam, which has to be compensated adding complexity. While these methods led to the stabilization on arbitrary phase, they typically require a free-space setup and are not transferable to fiber-based interferometers. This is unfortunate, as the latter offer better intrinsic stability and furthermore require no precise alignment, forming a robust and versatile platform.

In order to enable stabilization on all phase settings, we developed a new scheme that can be applied to fiber-based interferometers [P4]. In particular, we achieved a simple method to stabilize a fiber Michelson interferometer on an arbitrary phase with a single laser and without the need of dithering. For this, we utilize polarization multiplexing of frequency shifted and un-shifted

beams, allowing to measure and stabilize on any target phase of the interferometer. The experimental setup is illustrated in Fig. 2.13.

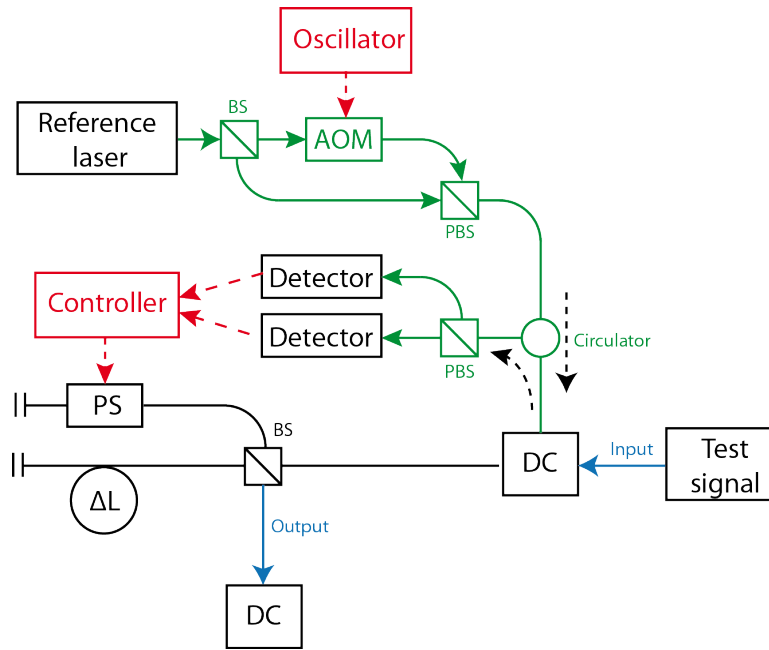


Fig. 2.13 Experimental setup for active stabilization of a fiber interferometer:

The input CW laser is launched onto a 50:50 coupler (BS), where one signal is shifted in frequency by an acousto-optic modulator, driven by an RF oscillator. Both paths are then combined using a polarizing beam splitter (PBS). Polarization maintaining fibers are shown in green, while standard single mode fibers are in black. The stabilization signal is overlapped with the test signal (in this case the single photons, or the excitation field) using a frequency dependent directional coupler (DC). The signal from the two detectors is used to calculate the absolute phase of the interferometer, and a feedback signal is subsequently calculated by a microcontroller and sent to the phase shifter (PS) to perform active stabilization on a target phase.

To reduce fast fluctuations of the interferometer, it is mounted on a steel plate to remove vibrations. The stabilization is performed using a stable CW laser (NKT Photonics Koheras, 100 Hz linewidth) operating at 1550 nm wavelength. The output of the laser is split using a fiber coupler, where one of the outputs is shifted in frequency by an Acousto Optic Modulator (AOM, Gooch & Housego). In order to compensate for losses inside the AOM, a 90:10 fiber coupler was chosen. Both beams are then recombined using a fiber polarizing beam splitter, placing the frequency shifted beam on the slow-axis, while the un-shifted beam is placed on the fast axis of a standard polarization maintaining (PM) fiber. Both beams pass a PM circulator and enter the single mode interferometer.

Upon reflection on the Faraday mirrors, the polarization is rotated by 90 degrees, so that the light initially on the fast axis re-enters the circulator on the slow axis and vice versa. After the circulator, both beams are separated by a standard polarizing beam splitter and are detected with two photo detectors.

The measured power at both detectors is given by:

$$P_1 = \frac{P_{1,\max} + P_{1,\min}}{2} + \frac{P_{1,\max} - P_{1,\min}}{2} \cos(\varphi)$$

$$P_2 = \frac{P_{2,\max} + P_{2,\min}}{2} + \frac{P_{2,\max} - P_{2,\min}}{2} \cos(\varphi - \theta)$$

Where $P_{\max}$ and $P_{\min}$ are the maximum and minimum measured optical powers, set by the input power of the CW laser and losses inside the setup, and θ is the phase difference between both signals, determined by the frequency shift induced by the AOM, see Fig. 2.14. In particular, the interference output is different for the shifted frequency, since the path-length difference in the interferometer is a different multiple for the input frequency.

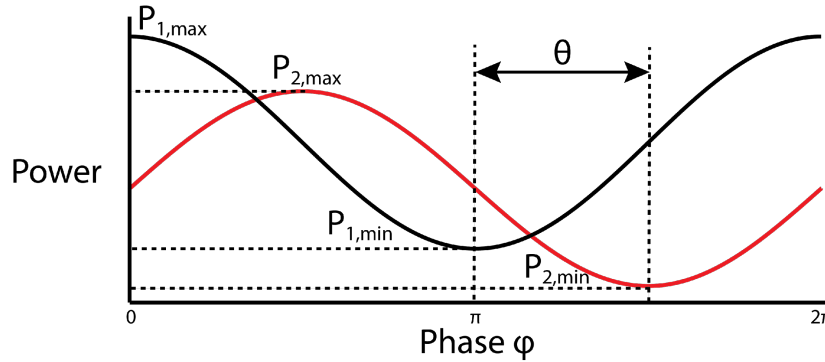


Fig. 2.14 Measured output signal of the interferometer stabilization scheme:

The measured power of the two photo detectors as a function of the fiber interferometer phase. The offset between the two curves can be adjusted by changing the acousto-optic modulator frequency. The maximum and minimum are defined by the total power and noise of the detectors, which can be different for both signals.

The interferometer phase φ (which we want to stabilize) is then given by:

$$\varphi = \arctan\left(\frac{1}{\sin\theta} \frac{P_{1,\max}}{P_{2,\max}} \frac{2P_2 - P_{2,\max}}{2P_1 - P_{1,\max}} - \cot\theta\right)$$

It can be seen that stabilization on arbitrary phase is not possible for $\theta = n\pi$ while the equation simplifies for $\theta = n\pi + \pi/2$ to:

$$\varphi = \arctan\left(\frac{P_{1,\max}}{P_{2,\max}} \frac{2P_2 - P_{2,\max}}{2P_1 - P_{1,\max}}\right)$$

As the frequency of the AOM can be adjusted, it becomes possible to tune the frequency to achieve the optimum condition. The desired $\pi/2$ phase shift is dependent on the length difference ΔL of the interferometer, given by (resulting in the desired AOM frequency):

$$\Delta\omega_{\pi/2} = \frac{c}{4 n_g \Delta L}$$

Where n_g is the group index of the fiber and c the vacuum speed of light. The AOM modulation frequency was adjusted to achieve a $\theta = n\pi + \pi/2$ phase shift (by applying a 198 MHz frequency shift). Then, the error signal was computed as the difference between the actual and target phase of the interferometer. Finally, the interferometer was stabilized on the error signal using a standard PID algorithm implemented using an Arduino microcontroller, allowing stabilization at 10 kHz speed in steps of 2.5 milli rad. The crosstalk between the two signals was measured to be at 18 dB corresponding to the polarization crosstalk of standard PM panda fiber. As the crosstalk is very small, it is neglected in the process to extract the interferometer phase. The stability of the interferometer is summarized in Fig. 2.15.

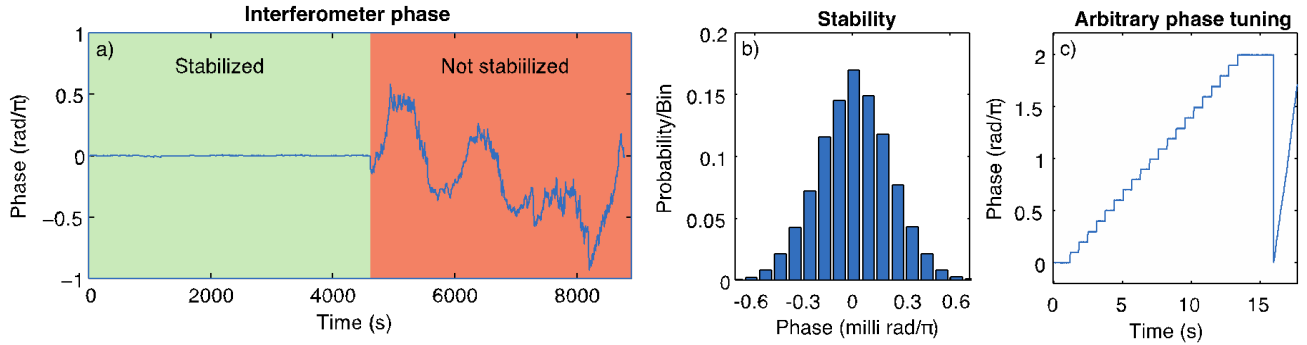


Fig. 2.15 Interferometer stabilization performance:

- a) Long time phase measurement with and without active stabilization.
- b) Phase stability, leading to a error of $\sigma < 0.2$ milli rad/ π for a one hour measurement.
- c) To show that the interferometer can be stabilized on arbitrary phase, the phase was swept from 0 to 2π in 20 steps, each for half a second.

In order to stabilize multiple interferometers, PM fiber couplers were used after the frequency shifter and un-shifted signals were combined, and before the PM circulators. This allows the stabilization of multiple interferometers to the same reference.

2.10 Novel technique for frequency-domain projection measurements

For measuring frequency-bin entanglement, the photon frequency has to be shifted deterministically, and different frequency modes have to be overlapped (see Sec. 2.8.2). This is similar to a beam splitter overlapping spatial modes and an unbalanced interferometer mixing temporal modes. Such a frequency shift can be achieved using electro-optic phase modulation, driven by a single radio-frequency tone (118, 119). For the case of a single optical frequency mode input, phase modulation creates side-bands, the amplitudes and spectral spacing of which can be controlled via the modulation voltage and RF frequency, respectively. If an integer multiple of this side-band spacing matches the frequency spacing of the input quantum state, the coherent mixing of optical frequency modes can be achieved.

The mixing process is governed by a linear, unitary, and time-dependent operator, which has the following form in the time domain:

$$H_t = e^{iV_m\kappa \sin(2\pi\nu_m t + \phi_m)}.$$

Here, i is the imaginary unit, t represents time, V_m is the voltage amplitude of the RF tone, κ is the electro-optical coefficient of the phase modulator, ν_m is the RF frequency, and ϕ_m is the initial phase of the optical carrier.

Due to its intrinsic time-periodicity, this operator can be expressed in the form of an infinite trigonometric series (n and m being integer numbers),

$$H_t = e^{iV_m\kappa \sin(2\pi\nu_m t + \phi_m)} = \sum_{n=-\infty}^{\infty} a_n e^{i2n\pi\nu_m t}.$$

The side-band coefficients a_n are determined by the Jacobi-Anger expansion:

$$a_n = J_n(V_m\kappa) e^{in\phi_m}$$

where $J_n(V_m\kappa)$ is the Bessel function of the first kind and order n , evaluated at $V_m\kappa$. The Bessel coefficients determine the side-band amplitudes, and thus for a given input optical signal, the phase-modulated output is described by a series of harmonic functions weighted by these coefficients. Each frequency mode is split into a series of modulation products (frequency modes), spectrally spaced by multiples of the driving frequency ν_m .

The advantage of this method is that the achievable modulation bandwidth is not only dependent on the driving frequency (as is the case for amplitude modulation), but also on the voltage V_m . This means that a low-frequency RF tone (low ν_m) can generate modulation products at high frequencies, provided that a sufficiently high value of V_m is supplied.

In our experiments, the frequency spacing of the quantum state modes is 200 GHz and a modulation frequency of $\nu_m \approx 33.3$ GHz was chosen, dictated by the available RF sources and amplifiers. Using a sine-wave RF signal, we achieved, for the $D=2$ and $D=3$ case, 11% power transfer to each of the ± 200 GHz modes, while 8% of the power remained (un-modulated) in the fundamental frequency, leading to the mixing of a resonance mode with its two first-neighbours. For $D=4$, we achieved a 4% power transfer to the ± 100 GHz, as well as ± 300 GHz modulation sidebands, leading to a mixing of four resonance modes in a vacuum mode, see Fig. 2.16. The rest of the power was lost to other modulation sidebands, not required for the targeted mixing process.

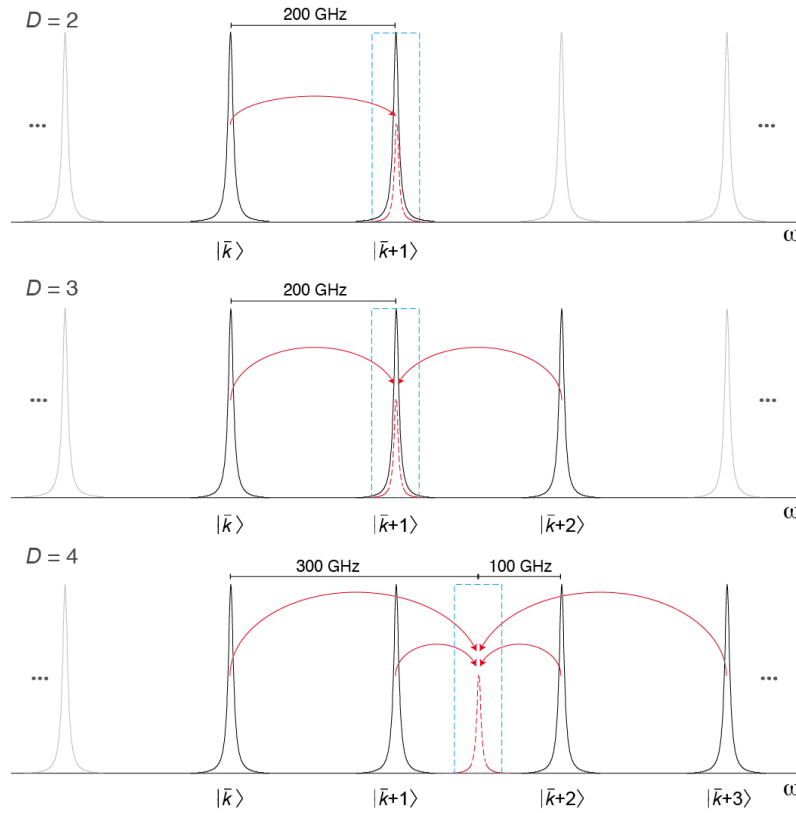


Fig. 2.16 Coherent mixing of multiple frequency modes:

D modes (here, $D=2, 3$ or 4) are spectrally selected (solid black line) and mixed (red arrows) by means of an electro-optic phase modulator. The frequency mode where all components overlap (red dashed line) is then selected via a narrow spectral filter (blue dashed window). For $D=2$ and 3 , a frequency shift of 200 GHz (equal to the FSR) is implemented, whereas for $D=4$ two different frequency shifts of 100 GHz (equal to $1/2$ FSR) and 300 GHz (equal to $3/2$ FSR) are enforced. In all cases, this is achieved through sideband generation. Note that for $D=4$, and in contrast to $D=2$ and 3 , the final frequency mode does not overlap with any microcavity resonance. Figure published in [9].

Quantum mechanically, an operator that describes this process and which transforms the input into the output state is defined as (120, 121):

$$A_{\text{Mod}}^{\{s,i\}} = \sum_{\bar{k}=1}^D \sum_{\bar{l}=1}^D c_{\{s,i\},\bar{k}-\bar{l}}^{\text{Mod}} |\bar{l}\rangle \langle \bar{k}|,$$

for the signal (s) and idler (i) photons. The mixing coefficients are related to the side-band modulation terms by $c_{\{s,i\},x}^{\text{Mod}} = a_{x \cdot p}$ where p is a natural number such that $p\nu_m$ matches the frequency spacing of the quantum state, here the FSR of the ring resonator.

The phase modulator can therefore be used to achieve projection measurements of the form

$$|\Psi_{\text{Proj},D}\rangle = \frac{1}{D} (\sum_{k=1}^D |\bar{k}\rangle),$$

where $\bar{k}$ are the chosen frequency modes that are mixed. However, it is not possible to adjust the phases in front of the individual frequency modes, as it would be required to perform Bell test measurements or state tomography. Nevertheless, such a phase control can be achieved using a frequency dependent phase shifter, which is a common telecommunications component (see Sec. 3.2.2).

3. Results

Chapter Abstract:

This chapter presents the main results concerning quantum state generation via integrated frequency comb sources, making use of their multi-mode nature. The results are structured by complexity of the generated quantum states, starting with single photons and photon pairs, continuing with two-photon entanglement, and finally multi-partite states.

Chapter Content:

- 3.1 Frequency comb of correlated photon pairs
 - 3.1.1 Generation of multiplexed heralded single photons
 - 3.1.2 Generation of cross-polarized photon pairs
- 3.2 Frequency comb of entangled photon pairs
 - 3.2.1 Generation of time-bin entangled photon pairs
 - 3.2.2 Generation of frequency-bin entangled photon pairs
- 3.3 Frequency comb of multi-partite entangled quantum states
 - 3.3.1 Generation of four-photon product states
 - 3.3.2 Generation of multi-mode quantum states

3.1 Frequency comb of correlated photon pairs

Correlated photons form the most basic quantum states that can be generated through spontaneous nonlinear optical processes. These states form the foundation for more complex entangled quantum states. Nevertheless, even without the presence of entanglement, the generation of two correlated photons can already be exploited for quantum applications. Mainly the photons' correlation in arrival time allows to herald the presence of a single photon by detecting the other. Such a heralded single photon source can for example be implemented into quantum communication protocols, which rely on imprinting information on single photons.

As a starting point to investigate the potential of optical quantum state generation using integrated frequency combs, we focused on photon pair generation in integrated microring resonators and their full characterization.

3.1.1 Generation of multiplexed heralded single photons

Integrated sources have been successfully used for the generation of heralded single photons (53, 122–124). However, all sources demonstrated to-date emitted photons in a single frequency channel, limiting their application potential. In particular, for their implementation into quantum communication protocols, frequency multiplexed sources are highly desired, as they would enable the use of wavelength division multiplexing to increase the data rates or enable multi-user quantum networks (125, 126).

To demonstrate the first multiplexed integrated source of heralded single photons, we chose to use an integrated microring resonator with 200 GHz free spectral range (FSR), 1.3 million Q-factor and very low and anomalous dispersion, see Sec. 2.2. This resonator was previously successfully used for classical frequency comb generation (127). Having an OPO threshold at around 120 mW continuous wave (CW) optical input power (127), we expected that the resonator emits photon pairs well below this input power if excited with a CW laser, see Sec. 2.2 and Sec. 2.3.1. To show the frequency-multiplexed nature of our photon source, five different signal/idler wavelength pairs symmetrically located around the excitation wavelength (see Fig. 3.1) were selected via telecommunication filters. The individual frequency channels were then sent to single photon detectors for coincidence detection, see Sec. 2.4. Clear coincidence peaks were measured on all five symmetric channel pairs (Fig. 3.2), while no coincidences were measured between non-symmetric channel combinations.

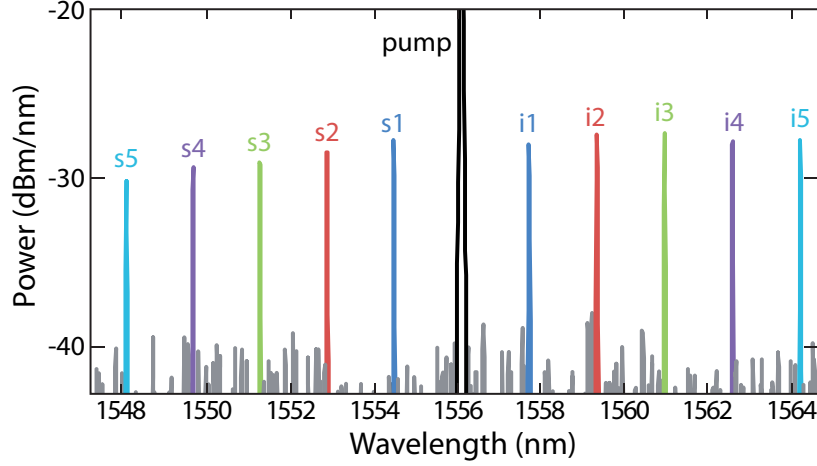


Fig. 3.1

Resonance structure of integrated optical frequency comb:

Optical spectrum of the above-threshold integrated optical frequency comb. Here we show five different signal/idler (s5-s1 and i1-i5, respectively) couples. The black curve represents the pump, while the grey traces are due to the noise of the optical spectrum analyzer. Figure published in Ref. [1].

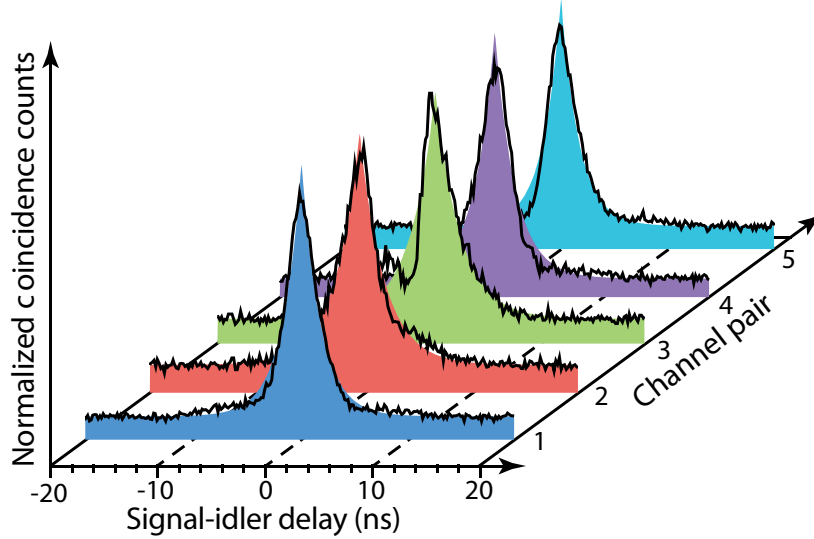


Fig. 3.2

Photon pair generation from an integrated frequency comb source:

Measured coincidence peaks for five channel pairs centered around the pump wavelength. Clear coincidence peaks with a measured bandwidth of 110 MHz (corresponding to 2.9 ns) are visible for all channel pairs. The solid-shaded curves are the fits of the experimental data (black curves). Figure published in Ref. [1].

For a CW pump power of 30 mW at the ring input, CAR values between 10 and 14 were obtained at single count rates between 8.5 and 10 kHz per channel, resulting in coincidence rates between 26.4 and 48.4 Hz per channel (raw values, without any background subtraction). Considering the

losses of the detection system (but not the on chip and coupling losses) and the dark count rate of the detectors, this corresponds to a pair production rate per channel between 286 and 346 kHz (at the output port of the microring resonator), and pair production probabilities per channel between 2.4 and 2.9×10^{-12} (measured as the probability that two pump photons at the input fiber of the microring resonator produce a signal/idler pair at the output fiber). With increasing pump power the CAR decreases and would eventually reach a minimum of one at the OPO threshold (see Fig. 3.3). This behaviour is expected since higher pump powers increase the generation of multiple photon pairs, in turn resulting in additional accidental counts. Even though our CAR values are not the highest reported for integrated structures (91) (partly a consequence of the losses in our detection systems), they are nonetheless suitable for many quantum communication (13).

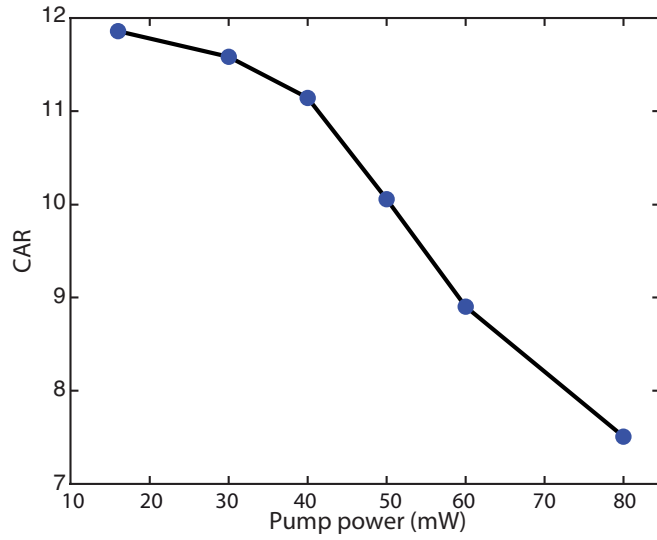


Fig. 3.3 Power scaling of the coincidence-to-accidental ratio (CAR): The CAR for channel pair s5-i5 is shown as a function of pump power. As expected, the CAR reduces at higher pump power. Figure published in Ref [1].

The signal/idler temporal correlation is characterized by the Glauber second-order (cross) correlation function (98);

$$g_{si}^{(2)} = 1 + \frac{2\pi\Delta\nu}{2R} \exp(-2\pi\Delta\nu|\tau|)$$

where $\Delta\nu$ is the resonator line width, R is the pair production rate, and τ is the relative signal-idler delay in their arrival times.

Fitting this function to the measured coincidence peak resulted in a measured spectral bandwidth of $\Delta\nu_{\text{fit}}=110$ MHz, consistent with the linewidth of the ring resonator (measured to be 120 MHz), after considering the time jitter of the single photon detectors (810 ps, resulting in a

corrected $\Delta\nu_{\text{jitter}} = 112$ MHz). This spectral bandwidth leads to a single photon coherence time of $\tau_{\text{coh}} = 1/(\pi\Delta\nu_{\text{fit}}) = 2.89$ ns, assuming a pure single mode photon.

In order to demonstrate heralded single photons, we measure the heralded second-order coherence of the photons, see Sec. 2.4.1. For a perfect heralded single photon source only one idler photon is expected to be present upon detection of a signal photon. This characteristic manifests itself by means of a dip in the second-order coherence function, see Sec. 2.4.1, where a value below 0.5 confirms the quantum nature of the photon. A clear dip of $g_h^{(2)}(0) = 0.114 \pm 0.008 < 0.50$ was measured (see Fig 3.4), confirming that the source operated in the single-photon regime.

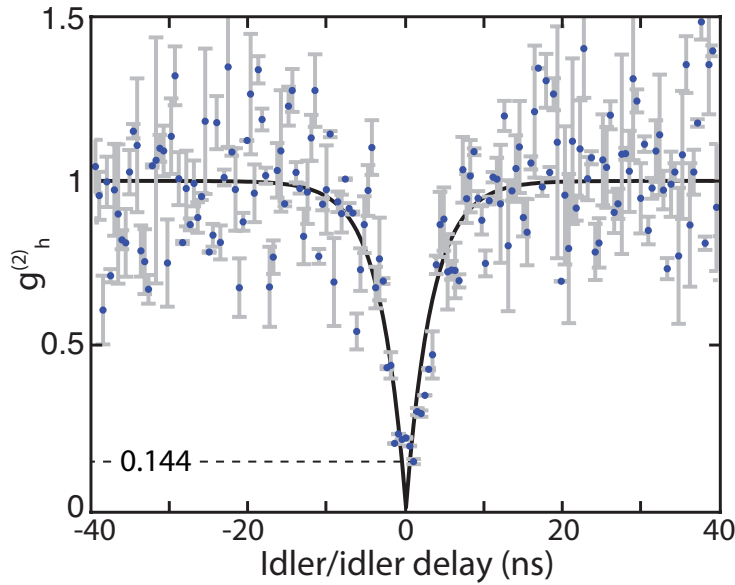


Fig. 3.4 **Heralded second-order coherence of the photon pair output:** The anti-bunching dip of 0.144 ± 0.008 demonstrates the single photon character of the photon pair emission, and confirms that our ring resonator can be used as a source of heralded single photons. Figure published in Ref. [1].

Having confirmed that the device can be used as a heralded single photon source, we estimated the heralded efficiency η_h , i.e. the probability of detecting an idler photon upon detection of the signal one. The heralding efficiency can be evaluated as the ratio between the coincidence counts and the signal counts, in a configuration where the idler detector is triggered by the presence of a signal photon. At a pump power of 30 mW (high CAR value, while maintaining a reasonable count rate), a heralding efficiency (including all losses) of $\eta_h = 10\%$ was obtained, limited primarily by the losses of the system and low detection efficiency of the detectors.

Finally, using pulsed excitation and increasing the pump power to enhance the probability to generate multiple photon pairs, we measured the second-order coherence of the single photon to extract the effective number of modes, see Sec. 2.4.1. We observed a bunching peak as high as 1.537, corresponding to $N = 1.86$ effective modes (see Fig. 3.5 a), in turn demonstrating the high purity of the source. However, we expected to reach a peak of 2 resulting in a single mode. The presence of multiple effective modes most likely arises from a non-perfect resonance, i.e. resonance shapes that cannot be described by a Lorentzian curve. Making use of a high-resolution optical spectrum analyzer, broadband classical light was sent through the resonator to resolve the resonator transmission. The transmission measurement showed a clear split resonance (see Inset in see Fig. 3.5 a), effectively forming a two sharp resonance features which can explain the 1.86 measured effective modes. Such split resonances can occur due to back-scattering within the high-Q resonance, leading to an interference effect between co- and counter-propagating light within the resonance. Repeating the measurements with a different resonator featured by 200 GHz FSR and 240,000 Q-factor, a perfect photon purity (see Fig. 3.5 b) as well as Lorentzian line shape (see Inset in Fig 3.5 b) were measured, showing that in this case our lower-Q microring resonator is better suited for the conducted quantum experiments.

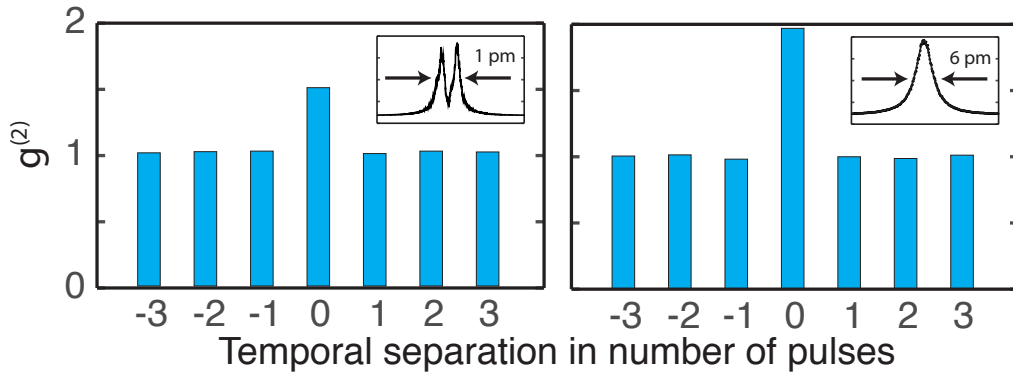


Fig. 3.5

Second-order coherence and mode measurement:

a) Using pulsed excitation, a clear photon bunching peak is measured at zero time delay with a maximum of 1.537 (corresponding to 1.86 effective modes) for a 1.3M Q-Factor microring resonator. Inset: High-resolution optical transmission measurement of the microring resonator, showing a clear broken resonance, which explains that close to two modes were measured. **b)** Second-order coherence measurement of a microring resonator with a Q-factor of 240k, showing a photon bunching peak with a maximum of 1.98 (corresponding to 1.02 effective modes). Inset: High-resolution optical transmission measurement of the microring resonator, exhibiting a perfect Lorentzian line shape.

In a next step, we investigated the spectral bandwidth of the quantum frequency comb. The resonator (200 GHz FSR, 240,000 Q-factor) was pumped using a spectrally-filtered mode-locked lasers, and the single photon count rate as a function of wavelength was characterized using a high-resolution tunable C-band wavelength filter, as well as a grating-based spectrum analyzer (see Fig. 3.6). We calculated the photon pair production rate per pulse from these measurements. It can be clearly seen that a very broad frequency comb of photons is emitted, covering the full S, C, and L International Telecommunication Union (ITU) bands (wavelengths ranging from 1470 to 1620 nm). The SFWM process generates a spectrum symmetric in frequency, while the spectral asymmetry in the measured photon counts can be explained by Raman scattering, which could be further reduced by cooling the chip. Due to the broad phase-matching condition, achieved through the close-to-zero waveguide dispersion, the emitted comb exhibits a very flat and broadband spectrum with close to uniform pair production rates, ranging from 0.02 to 0.04 pairs per pulse, over the full measured comb.

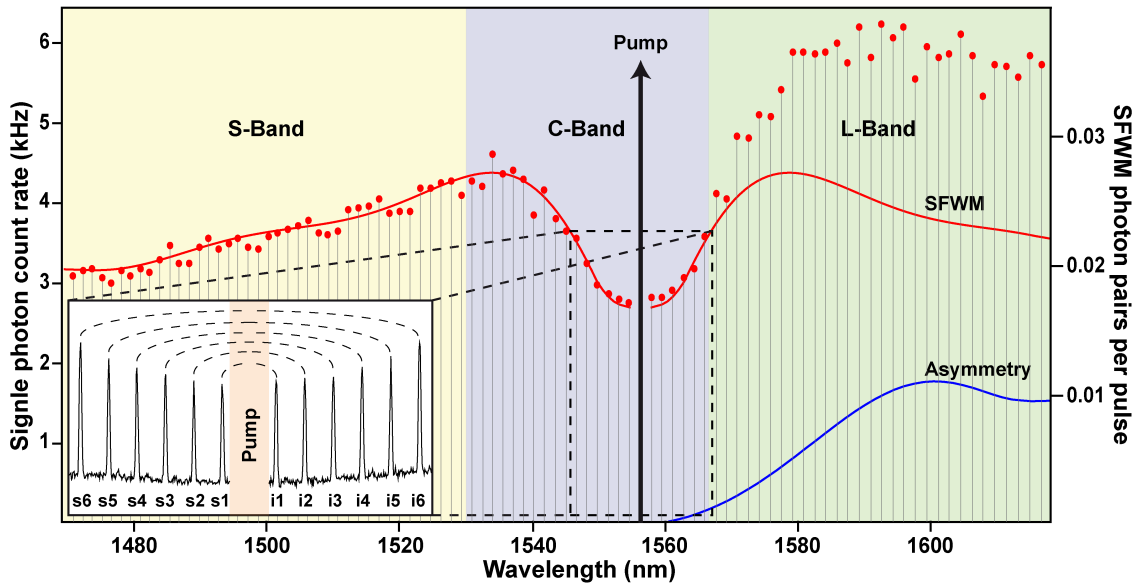


Fig. 3.6

Single photon spectrum of the integrated quantum frequency comb:

Single-photon spectrum (red circles) emitted by the microring resonator, measured using a grating-based spectrum analyzer and a high-resolution digital tunable filter in the C band (Inset). The S, C, and L telecommunications bands are indicated. The red curve shows the symmetric contribution generated through SFWM, whereas the blue curve shows the spectral asymmetry, which can be explained by Raman scattering. Figure published in Ref. [6].

3.1.2 Generation of cross-polarized photon pairs

Polarization is a commonly used degree of freedom for quantum information processing (4, 12). However, making use of polarization in integrated structures is very challenging due to the commonly disadvantageous polarization mode dispersion of waveguides, which are usually only designed for a single polarization. In contrast to other on-chip devices based on silicon or silicon nitride, the silicon oxynitride waveguides used here have similar dispersion properties for the two polarization modes, which is enabled by the almost square waveguide dimensions (1.5 x 1.45 μm) (79). Because of these waveguide properties, we were able to exploit both polarization modes for nonlinear optical interactions.

Based on the frequency and polarization of the interacting fields, SFWM and SPDC can be categorized into different types of interactions (27). In general, the fields generated through parametric frequency conversion can either have identical (degenerate) or different (non-degenerate) frequencies and present different combinations of polarization, according to the following definitions (128–130): Type-0, the pump and generated fields are co-polarized (as used in Sec. 3.1.1); Type-I, the generated fields are co-polarized but different from the pump polarization; and Type-II, the generated fields have orthogonal polarizations. Table 3.1 summarizes the different types of FWM in terms of linear polarization for the different pump and generated fields (horizontal and vertical), as well as their efficiency assuming equal and perfect phase-matching conditions (131).

Table 3.1 List of different FWM processes and their relative efficiencies in terms of pump power (P), propagation length (L) and nonlinear parameter (γ). H denotes horizontal and V vertical polarization. Table published in Ref. [4].

	P1	P2	S	I	Efficiency
Type-0	H	H	H	H	$\propto (\gamma \times L)^2 P_1 \times P_2$
	V	V	V	V	
Type-I	H	H	V	V	$\propto (\gamma \times L/3)^2 P_1 \times P_2$
	V	V	H	H	
Type-II	H	V	H	V	$\propto (\gamma \times L/3)^2 P_1 \times P_2$
	H	V	V	H	
	V	H	H	V	
	V	H	V	H	

All these three polarization combinations can be achieved in second-order nonlinear media (27), enabled by the crystal symmetry, and have been used, for example, to generate (entangled) photon

pairs. In contrast, integrated third-order devices have not been able to exploit all of these degrees of freedom. While Type-II SFWM could in principle be achieved by using two orthogonally polarized pump fields, the overlapping and dominant stimulated processes seeded by the presence of two excitation fields generally make Type-II SFWM experimentally undetectable. Achieving efficient Type-II SFWM on a chip therefore requires structures that not only operate on two orthogonal polarizations with specific dispersion properties, but also provide nonlinear enhancement, while suppressing competing stimulated processes. To achieve this, we developed a novel scheme to suppress stimulated FWM between the two excitation fields by choosing a resonator that has a frequency offset between the TE and TM resonances, while having almost identical FSRs of both polarization modes, allowing Type-II spontaneous FWM to take place at targeted resonances (see Fig. 3.7).

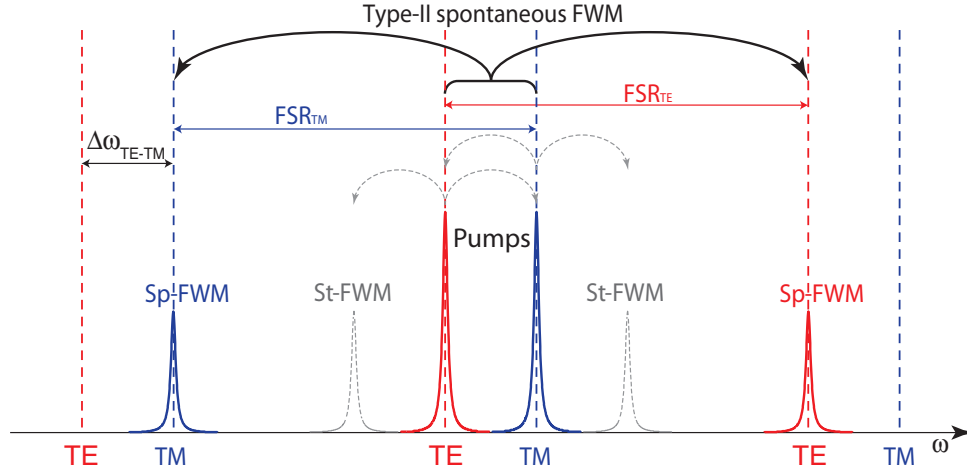


Fig. 3.7 Schematic of the to achieve Type-II SFWM on a chip: Stimulated FWM (St-FWM, dotted lines) is suppressed by an offset between the TE and TM resonances (dashed vertical lines), as the St-FWM gain does not overlap with any of the microring resonances. Type-II SFWM (continuous line) is allowed and enhanced by the resonator. Figure published in Ref. [4].

This offset is generated by the slightly different dispersion of the TE and TM modes, resulting in different effective resonator lengths and hence different resonant frequencies. Energy conservation dictates that the stimulated FWM bands have to be symmetric with respect to the two pump frequencies. Due to the frequency offset between the TE and TM resonances the spectral position of the stimulated FWM gain does not overlap with the ring resonances, thus efficiently suppressing this process inside the microring resonator. At the same time, the TE and TM mode dispersion has to be kept similar so that the difference in FSR between the two modes (120 MHz) is smaller than

the bandwidth of the resonances, to achieve energy conservation for Type-II SFWM processes. Furthermore, the mismatch between the FSRs with respect to the resonator bandwidth has to be minimized to obtain high efficiency in Type-II SFWM. Finally, the required phase-matching condition can be achieved by operating in a slightly anomalous dispersion regime for both modes. Characterizing the available resonators provided by our collaborator, we found a resonator that fulfilled all requirements, having a frequency offset between TE and TM modes of 70 GHz, while exhibiting almost equal FSRs of both polarization (200.39 and 200.51 GHz for TE and TM) modes, together with anomalous dispersion at 1556 nm wavelength for both modes.

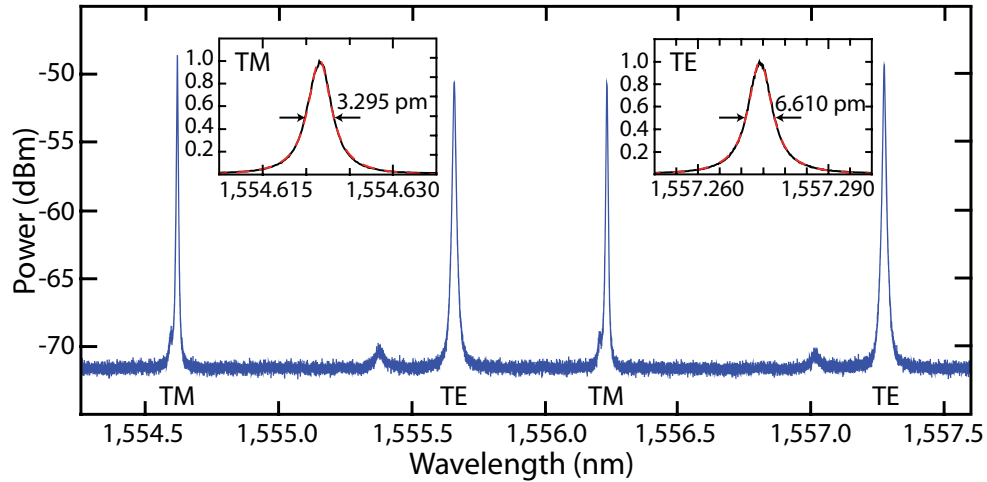


Fig. 3.8 Microring resonator characteristics for Type-II SFWM: Transmission spectrum measured with a high resolution OSA, showing two TE and two TM resonances, with a relative frequency offset of 70 GHz. The insets show the TE and TM resonances in a linear scale (black) with a Lorentzian fit (red dashed). The small peaks close to the TE resonance arise from a higher-order mode, which is present in the resonator but does not play a role in the nonlinear interaction. Figure published in Ref. [4].

This particular resonator had Q-factors of 235,000 and 470,000 for TE and TM modes, respectively (see Fig. 3.8). To demonstrate Type-II SFWM, photon coincidence measurements were performed. In particular, the generated photons were separated by a polarizing beam splitter and detected with single-photon detectors. A clear coincidence peak (see Fig. 3.9a) with a CAR of up to 12 without any background subtraction was measured (see Fig. 3.9 b). As photon pairs can only be generated via spontaneous nonlinear processes, the measured photon coincidences give a strong indication that the photon pairs are generated through Type-II SFWM, with stimulated processes being successfully suppressed. The power-scaling behaviour provides further insight into the process associated with the generation of the photon pairs. Only when one photon from each pump field is

used to create two daughter photons, it becomes possible to directly generate orthogonally polarized photon pairs. Therefore, the coincidence counts are expected to scale with the product of both pump powers. If the power of one pump field is kept constant, and the power of the second one is increased, a linear scaling behaviour is predicted for type-II SFWM, whereas if the power of both pump fields is simultaneously increased (with constant power ratio), a quadratic scaling is expected instead. As shown in Fig. 3.9 c), no coincidences (within the noise) were measured when the ring was either not pumped or pumped with the TE field alone. On the other hand, a clear linear scaling behaviour is visible with increasing TM pump power and constant TE power, while a quadratic (without linear contribution) scaling is observed with balanced pumps.

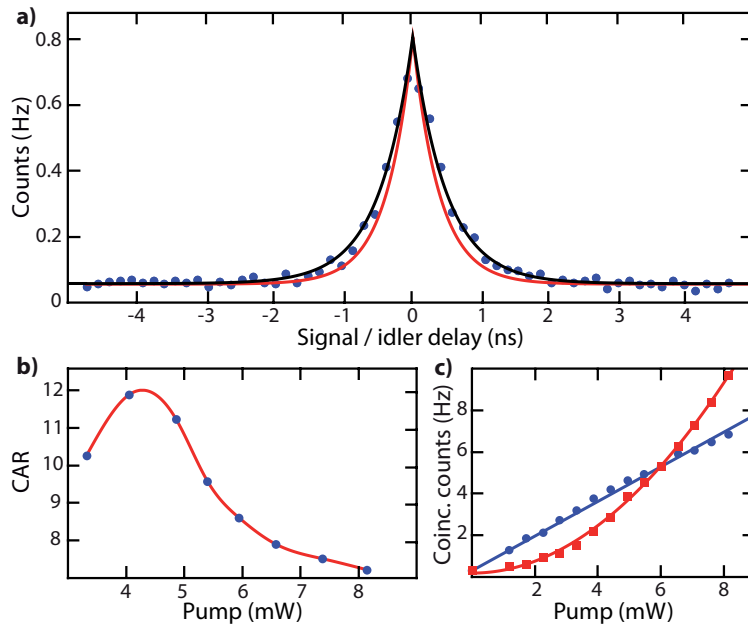


Fig. 3.9

Cross-polarized photon pair source characterisation:

a) Measured photon coincidence peak, showing the raw measured coincidence counts in Hz. The black curve corresponds to the optimum fit resulting in a measured photon bandwidth of 320 MHz, while the red curve corresponds to the fit with the expected photon bandwidth of 410 MHz. **b)** Coincidence-to-accidental ratio (CAR) as a function of balanced pump power, showing a CAR > 10 for balanced pump powers between 3 and 5 mW. **c)** Measured photon coincidence counts for balanced and unbalanced pump powers. In the unbalanced configuration (blue circles), the TE pump power is kept constant at 6 mW and the TM pump power is increased, showing a linear scaling behaviour. In the balanced configuration (red squares), TE and TM pump powers are identically increased, showing a clear quadratic scaling behaviour without any linear contribution. Figure published in Ref [4].

Due to the difference in linewidth between the TE and TM resonances, the spectral bandwidth of the emitted photons is determined by the narrower resonance. From the coincidence measurement (see Fig. 3.9 a) we extract a measured photon bandwidth of 320 MHz (black line), which is in good agreement with the resonator bandwidth of 410 MHz (red line). We note that the difference can be explained by the timing jitter of the detectors and electronics, resulting in a small temporal broadening of the measured peak. The measured CAR of up to 12 is limited by loss, dark counts and the quantum efficiency of the detectors (idQuantique), as well as by the photons generated through Type-0 SFWM of the individual pumps, see Section 3.3.2.

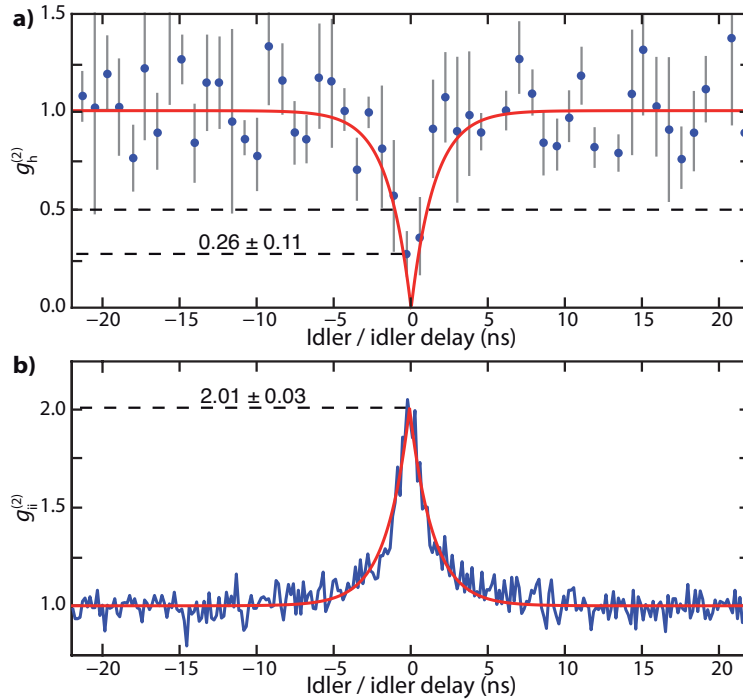


Fig. 3.10 Second-order coherence measurement of cross-polarized photons:
a) Measured heralded second-order coherence functions, showing a clear dip at zero delay below the limit for classical correlations (equal to 0.5). The curve clearly confirms the quantum nature and single-photon operation of the source. Ten bins were averaged for each point, displayed together with the statistical error (standard deviation of the 10 bin distribution). **b)** Measured second-order coherence function, showing a clear peak with a maximum at 2.01 ± 0.03 . This value verifies the single-mode operation and high purity of the photons. The red curves show the ideal case. Figure published in Ref [4].

We measured a coincidence rate of 4 Hz at 5 mW balanced pump power at the input of the chip (5 mW is the highest achievable pump power featured by a CAR > 10). Considering all losses of the detection system (8.5 dB for both signal and idler) as well as the quantum efficiency of the detectors (5 and 10%), this corresponds to a pair production rate of 40 kHz and a pair production probability

of 1.48×10^{-12} , accounting for the 1.6 dB coupling loss of the pump into the chip. To further characterize the performance of our device as a single-photon source, we measured the (heralded) second-order coherence of the photons. A clear dip of $g_h^{(2)}(0) = 0.26 \pm 0.11 < 0.50$ was recorded (see Fig. 3.10 a), showing that the source operates in a non-classical single-photon regime, while second-order coherence (see Fig. 3.10 b) shows a clear peak with a maximum of 2.01 ± 0.03 , resulting in $N_{\text{eff}} = 0.99 \pm 0.03$ effective modes, underlining the high purity of the source.

Finally, the production of cross-polarized photon pairs is not limited to only the adjacent resonances. Indeed, we measured cross-polarized photon pairs over 12 resonance pairs, limited by the available filters, each with pair production rates larger than 20 kHz at 5 mW balanced pump power (see Fig. 3.11).

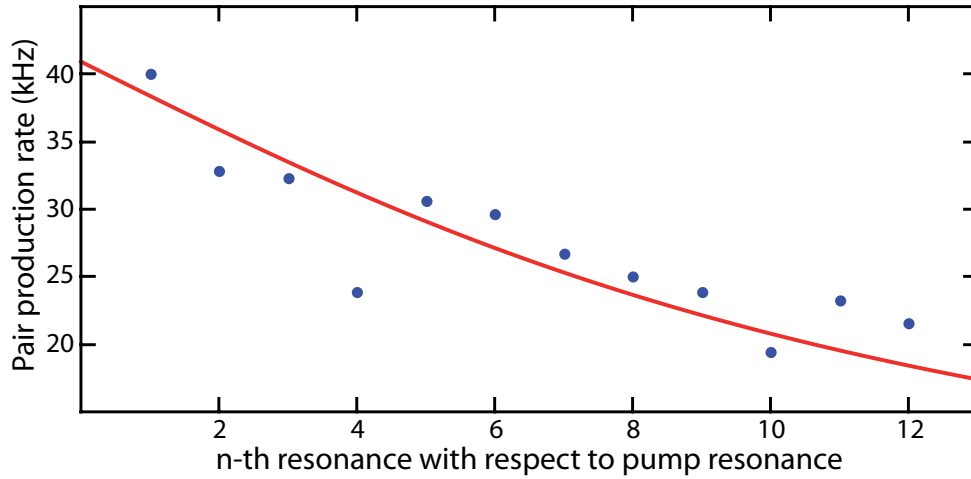


Fig. 3.11 Pair production rate at different resonances: Measured pair production rate (blue circles) associated to the Type-II SFWM process at different resonator lines symmetrically located with respect to the pumps at 5 mW balanced pump power, showing good agreement with the approximate predicted curve (red line). Figure published in Ref [4].

It can also be seen that the production rate decreases with increasing spectral separation from the pump. This is because of a mismatch in the phase-matching conditions, stemming from a slight difference in the FSRs of the TE and TM modes. In particular, to achieve SFWM, energy conservation must be fulfilled, i.e. the energy curves for the signal and idler fields have to overlap with the pump energy distribution. These bandwidths are defined by the microring resonator frequencies. The energy matching can be calculated through the overlap integral for the two Lorentzian curves, defined by the resonator bandwidths Γ :

$$1 - \frac{2}{\pi} \arctan \left(\frac{n \cdot |FSR_{TE} - FSR_{TM}|}{\Gamma_{TE} + \Gamma_{TM}} \right),$$

which becomes one for equal FSRs in TE and TM modes, and decreases when the mismatch between FSRs with respect to the FWHM of the resonances increases.

Using the measured values of the FSRs for the two modes, the pair production rate (PPR) for different resonances with respect to the offset can be approximated with:

$$PPR(n) = PPR_{max} \left(1 - \frac{2}{\pi} \arctan \left(\frac{n \cdot 120MHz}{410MHz + 820MHz} \right) \right)$$

Note that this approximation does not include higher-order dispersion terms and assumes a flat FWM gain spectrum. Even with these assumptions, the fit in Fig. 3.11 shows good agreement to the measured data, resulting in a $PPR_{max} = 40.92 \pm 1.33$ extracted from the fit.

3.2 Frequency comb of entangled photon pairs

Having demonstrated the generation of correlated photon pairs, the next important challenge is to generate entangled two-photon states, as these are the foundation for most applications of quantum information processing. As discussed in Sec. 2.8, particular interest is devoted to quantum states that can be generated and controlled within a single spatial mode, as this will allow to reduce the device complexity and provide favourable scalability. Additionally, for implementations of gate operations and most quantum communication protocols, discrete quantum states (qubits or quDits) are desired, as they allow direct access to the quantum modes. Discrete energy-time entanglement fulfils both criteria, and is therefore the focus of our investigations. In particular, we will discuss the demonstration of time-bin entanglement (Sec. 3.2.1), and frequency-bin entanglement (Sec. 3.2.2), both of which have not been demonstrated in integrated microring resonator sources before this work.

3.2.1 Generation of time-bin entangled photon pairs

As discussed in Sec. 2.8.1, time-bin entanglement can be generated by employing pulsed excitation with coherent double pulses (82). In particular, we generated these double pulses by passing the output of a pulsed pump laser through a stabilized unbalanced fiber interferometer with a 11.4 ns delay – significantly larger than the pulse duration of the laser – generating double pulses with a defined relative phase difference, see Sec. 2.9. These double pulses were then coupled into the integrated microring resonator (see Fig. 3.12), which acted as a filter, thus perfectly matching the spectral bandwidth of the pulse with that of a single ring resonance. The pump power was chosen in such a way that the probability of creating a photon pair from both pulses simultaneously was sufficiently low to be negligible. This configuration led to a quantum state $|\Psi_{time-bin}\rangle = \frac{1}{\sqrt{2}}(|S_s, S_i\rangle + |L_s, L_i\rangle)$, where the signal (s) and idler (i) photons are in a quantum superposition of the short (S) and long (L) time-bin. Most importantly, these entangled qubits are generated over all the microring resonances, thus leading to a quantum frequency comb of time-bin entangled photon pairs.

To perform projection measurements on the photons, they were each passed individually through different interferometers with an imbalance identical to that used for the pump laser (see Fig. 3.12). After the interferometer, photons can exit at three distinct time slots, where the first and

last slots correspond to projections on the short and long time-bin, respectively. The central measurement slot enables a projection measurement on $|\Psi_{\text{Proj.}}\rangle = \frac{1}{\sqrt{2}}(|S\rangle + e^{i\theta}|L\rangle)$, where the projection phase can be set by the interferometer phase. Selecting the projection measurement on the superposition of both S and L time-bins, quantum interference can be measured between both photons with respect to the set interferometer phases, which can be used to violate a Bell inequality (see Sec. 2.7).

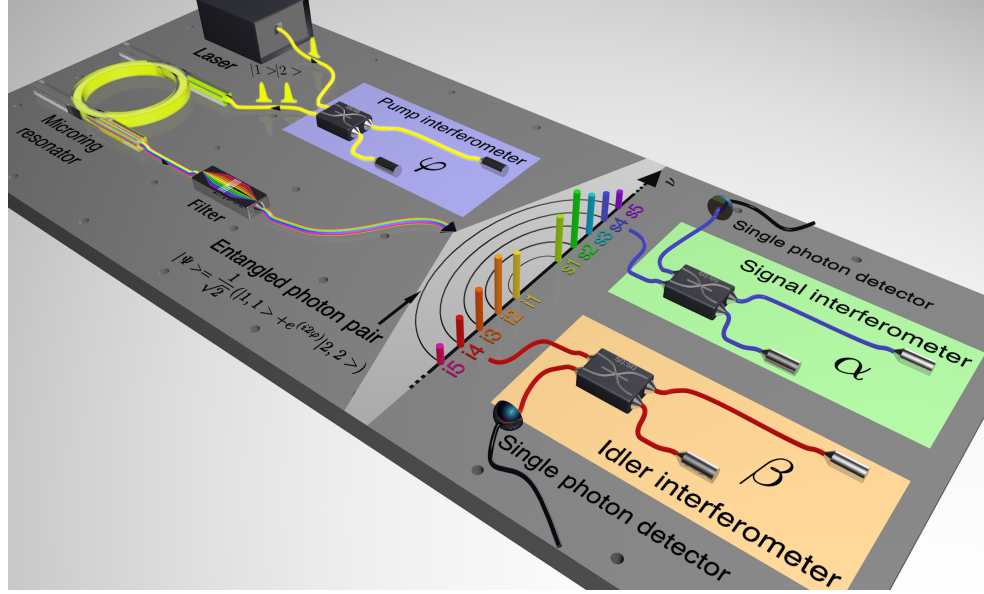


Fig. 3.12 Setup for generation and characterization of time-bin entangled states: A pulsed laser is passed through an unbalanced Michelson interferometer (consisting of a 50/50 beam splitter, a Faraday mirror, and a phase shifter), generating two pulses with a phase difference φ , in two respective time slots (time-bin $|S\rangle$ and $|L\rangle$). The pulses are fed into the microring resonator (see arrows for the propagation direction), exciting one microring resonance. The nonlinear SFWM process generates signal-idler photon pairs on several ring resonances symmetric to the excited resonance (optical frequency comb, indicated in multicolor), either within the first or the second time slot (the generation in both time-bins is made highly improbable by choosing a low enough excitation power). The superposition of the state generated in the first and the second time slot results in an entangled photon output $|\psi\rangle$, taking place simultaneously on several resonances and leading to a frequency comb of time-bin entangled photon pairs. For analysis purposes, each photon of the spectrally filtered photon pair (distributed on two resonances symmetric to the excitation frequency, here e.g., the resonance pair i4-s4) is individually passed (indicated by solid fibers) through an interferometer with the temporal imbalance equal to the time slot separation, and then detected with a single photon detector (the second and third interferometers can be individually controlled). Figure published in Ref. [6].

We selected 5 different frequency channel pairs symmetric to the excitation frequency and recorded quantum interference (see Fig. 3.13 for channel 1). We measured on channel pair one a raw visibility above $0.83 > 1/\sqrt{2} \approx 0.71$, thus confirming entanglement through the violation of a Bell-like inequality (109, 110). After subtracting the measured background (also shown in Fig. 3.13), the visibility was found to be above 98.5%. Indeed, all measured channels showed high enough visibility to violate a Bell-like inequality (see Table 3.2).

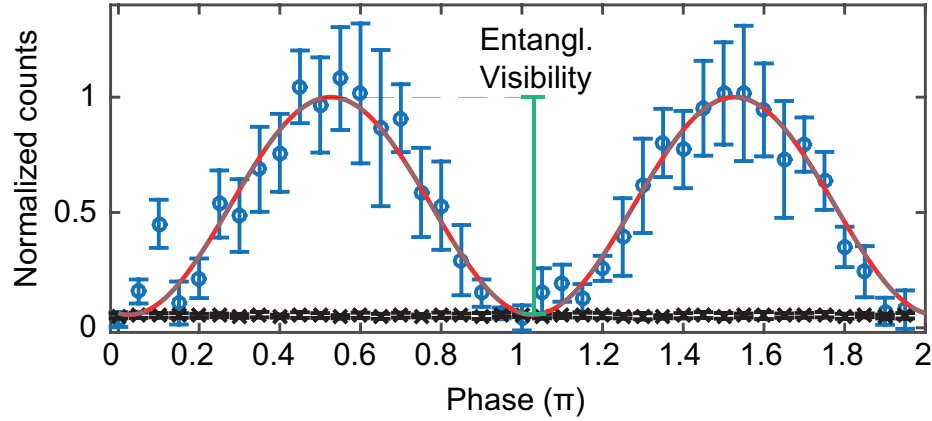


Fig. 3.13 Quantum interference and Bell inequality violation: Photon coincidence counts as a function of interferometer phase ($\alpha = \beta$). A doubling in the periodicity compared to classical interference is shown, and a visibility of 82.4% confirms the violation of a Bell inequality. The black crosses are the measured background for each point. Figure published in Ref [6].

Table 3.2 Measured visibilities on five channel pairs around the pump:

	Raw Visibility	Background Subtracted
Channel Pair 1	$83.0 \pm 2.1 \%$	$98.5 \pm 5.0 \%$
Channel Pair 2	$84.1 \pm 2.2 \%$	$93.2 \pm 4.9 \%$
Channel Pair 3	$86.5 \pm 2.2 \%$	$96.0 \pm 4.9 \%$
Channel Pair 4	$82.4 \pm 2.1 \%$	$96.0 \pm 4.9 \%$
Channel Pair 5	$83.1 \pm 2.1 \%$	$99.5 \pm 5.0 \%$

In addition to confirming time-bin entanglement, the quantum interference also reveals the phase-characteristic of the nonlinear generation process. Indeed, this phase dependency has been well described in theory (132) and has been exploited, for example, in optical squeezing (33). Performing quantum interference measurements, we found that this phase dependency is also manifested at the single photon level with a clear difference between second- and third-order nonlinear interactions. When SPDC in second-order nonlinear media is used to generate the

entangled photon pairs, quantum interference is expected to have a phase dependency proportional to $1 - V\cos(\alpha + \beta - \varphi)$ (82), where V is the fringe visibility, φ is the pump interferometer phase, and α and β are the phases for the signal and idler interferometers, respectively. In contrast, for photons generated through SFWM, quantum interference is expected to have a different dependency, of the form $1 - V\cos(\alpha + \beta - 2\varphi)$ for degenerate SFWM, or $1 - V\cos(\alpha + \beta - \varphi_1 - \varphi_2)$ for the more general case of non-degenerate SFWM, where $\varphi_{1,2}$ are the phases of the two pump fields.

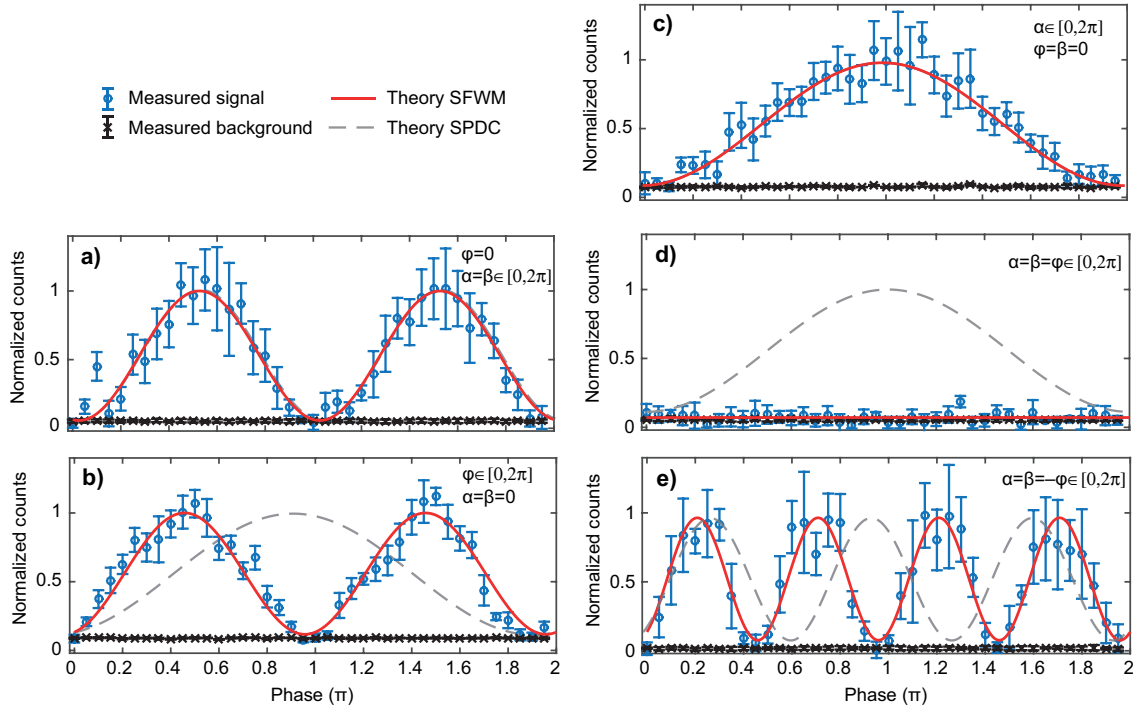


Fig. 3.14 Phase dependency of SFWM:

To demonstrate the difference between the phase characteristics of SPDC and FWM, five different quantum interference measurements are performed within the framework of time-bin entanglement. Three interferometer phases are adjusted: φ , α , β being the phases of the pump, signal and idler interferometers, respectively. **a)** $\varphi = 0$, $\alpha = \beta \in [0, 2\pi]$, **b)** $\varphi \in [0, 2\pi]$, $\alpha = \beta = 0$, **c)** $\alpha \in [0, 2\pi]$, $\varphi = \beta = 0$, **d)** $\alpha = \beta = \varphi \in [0, 2\pi]$, **e)** $\alpha = \beta = -\varphi \in [0, 2\pi]$. Taken together, the results confirm that the quantum interference is given by $1 - V\cos(\alpha + \beta - 2\varphi)$, where V is the visibility, showing a clear difference to what has been previously found for SPDC – where the quantum interference is dependent on $1 - V\cos(\alpha + \beta - \varphi)$. The error bars represent the standard deviation of 7 measurements. Figure published in Ref [6].

As shown in Fig. 3.14, five separate quantum interference measurements are used to confirm this important difference between SPDC and SFWM, where the phases of the interferometers are either tuned separately or simultaneously in a symmetric and anti-symmetric way. The expected

behaviour for SPDC and SFWM is plotted in Fig. 3.14 with dashed and solid lines, respectively. The measurements clearly confirm the difference between the two processes, which can be explained through the additional photon involved in SFWM. Indeed, in the generation of entangled photon pairs, the phase of the excitation photon(s) can be used to adjust the quantum state. If a single photon is involved (as in second-order processes), only the absolute phase of this photon can be used as a control parameter. In third-order nonlinear processes, however, two photons generate the quantum state, enabling an additional control parameter (i.e. the relative phase between the two photons). This additional degree of freedom can be exploited to allow all-optical reconfigurable quantum state generation, as discussed in Section 3.3.2.

Finally, to fully characterize the entangled states we performed quantum state tomography (107) to extract the state density matrix (see Sec. 2.6). We first measure the two-photon qubits generated on comb lines symmetric with respect to the pump wavelength (see Fig. 3.15) and find a fidelity of 96% (corrected for different losses in the arms of the interferometer, but not for losses or background), confirming that the generated quantum states are of high quality and very close to the ideal entangled state. We then repeated the tomography measurement after adding 40 km of fiber, to show that the entanglement is preserved after long fiber propagation (fidelity of 87% without compensation for losses), highlighting the immediate applicability of the generated two-photon qubits for quantum communications.

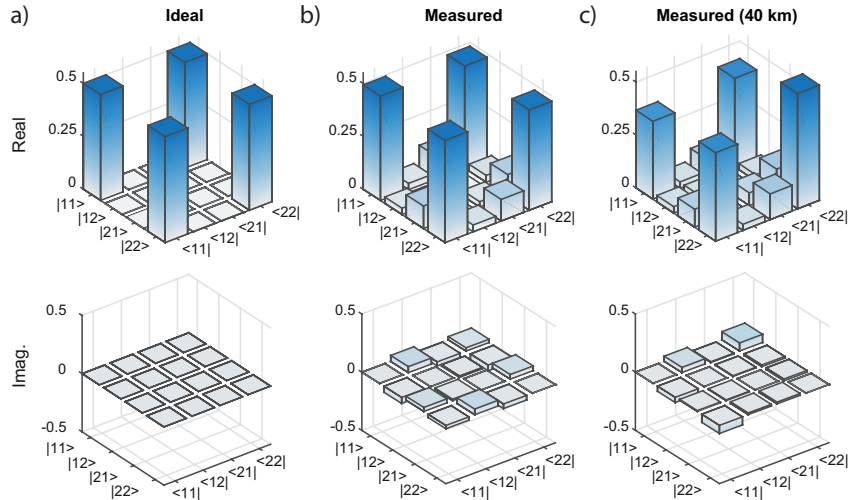


Fig. 3.15

Quantum state tomography of two-photon time-bin entangled states:

A quantum state can be fully described by its density matrix. **a)** The real and imaginary parts of the ideal density matrices of a two-photon entangled qubit state are shown. **b)** The measured density matrix of the two-photon state agrees very well with the ideal state, confirmed by a fidelity of 96%. **c)** The tomography was repeated after 40 km of fiber propagation, showing a slightly reduced fidelity of 87%. Figure published in Ref. [6].

3.2.2 Generation of frequency-bin entangled photon pairs

While time-bin entanglement was generated through a special choice of the excitation scheme, the multi-mode nature of optical frequency combs was not exploited. Remarkably, it can directly be used to generate optical frequency entanglement. In particular, if the SFWM phase matching bandwidth covers multiple ring resonances, photons are generated in a coherent superposition of these modes forming a frequency entangled state. If more than two frequency modes are involved, a high-dimensional superposition of multiple modes can be achieved. The frequency domain therefore offers a unique framework allowing to generate high-dimensional entangled quantum states on a photonic chip and to manipulate them in a single spatial mode.

To generate such a frequency entangled state, a spectrally-filtered mode-locked laser is used to excite a single resonance of the microring at ~ 1550 nm wavelength, in turn producing pairs of correlated signal and idler photons spectrally-symmetric to the excitation field covering multiple resonances, see Fig. 3.16.

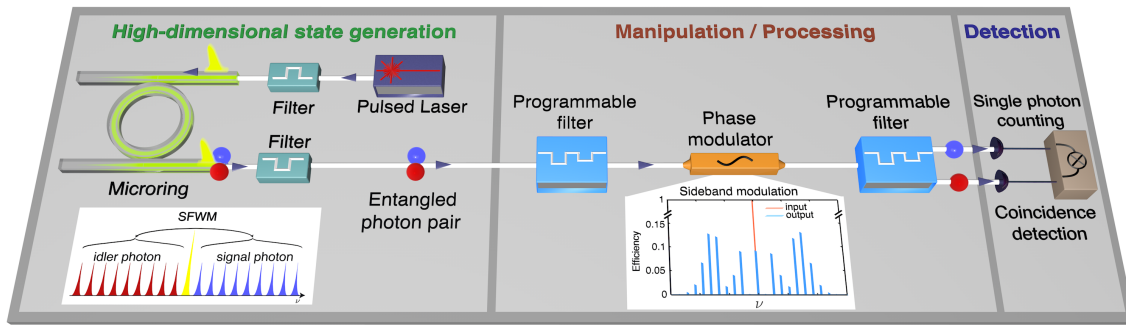


Fig. 3.16 Experimental setup for high-dimensional frequency-entangled quantum state generation and control:

A passively mode-locked laser was coupled into the integrated microring resonator after being spectrally filtered to precisely excite a single resonance. Spontaneous four-wave mixing (SFWM) generated photon pairs (signal and idler) symmetric to the excitation frequency and in a quantum superposition of the frequency modes defined by the resonances. Programmable filters and a modulator were used for manipulating the state, before the signal and idler photons were detected by two single photon counters. Figure published in Ref. [9].

To characterize the dimensionality of the generated state, two different experiments were performed. The large FSR of the ring cavity (~ 200 GHz), i.e. the spectral separation between adjacent resonance modes, enabled us to use a commercially available telecommunications programmable filter for individually selecting and manipulating the states in these modes (given the filter's operational bandwidth of 1527.4 to 1567.5 nm, we were able to access 10 signal and 10

idler resonances). Using a computer controlled programmable filter, the joint spectral intensity was measured, describing the two-photon state's frequency distribution (133, 134). Specifically, we routed different frequency modes of the signal and idler photons to two single photon detectors and counted photon coincidences for all sets of mode combinations. As shown in Fig. 3.17 a), photon coincidences were detected only for mode combinations symmetric to the excitation frequency, a characteristic of frequency-entangled states.

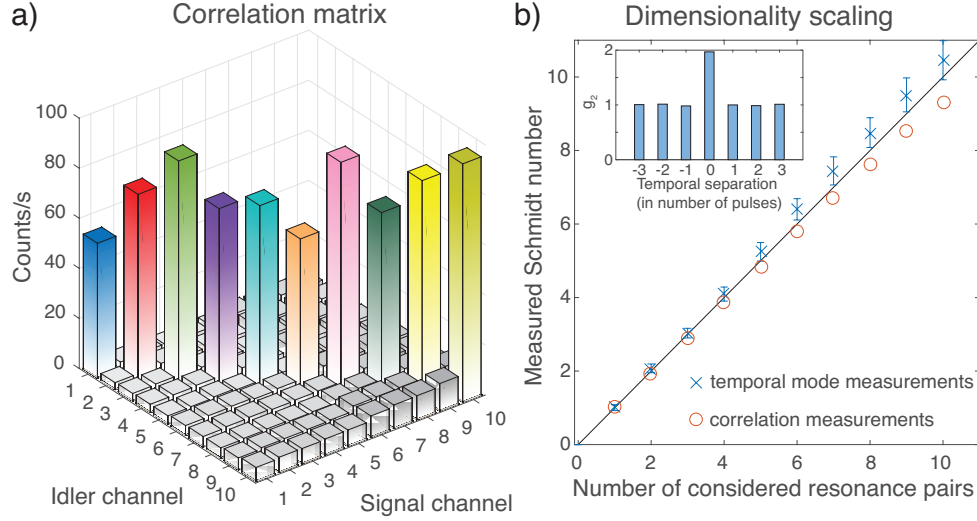


Fig. 3.17 Characterization of the quantum state dimensionality:
a) Measured joint spectral intensity of the high-dimensional quantum state, showing photon coincidences only at symmetric mode pairs (i.e. on the diagonal of the matrix) and revealing a frequency correlation. **b)** Two-photon state dimensionality (Schmidt number) as a function of considered resonance pairs symmetric to the excitation frequency with the upper bound (blue crosses), obtained from the second-order coherence, and the lower bound (red circles), calculated from the correlation matrix. Inset: Second-order coherence of a single photon emitted at one specific resonance with a maximum of 1.97, corresponding to 1.03 effective modes. The measurement was repeated for each signal and idler resonance, returning comparable values. Figure published in Ref [9].

In general, a frequency-entangled two-photon state can be described using its joint spectral amplitude (JSA) $F(\omega_s, \omega_i)$ (133). The dimensionality of such an entangled state can be estimated by performing a Schmidt mode decomposition, where the Schmidt number K represents the lowest amount of relevant orthogonal modes in the system (134) (defined as $K = (\sum \lambda_n^2)^{-1}$, where λ_n are the Schmidt eigenvalues with $\sum \lambda_n = 1$). However, it is experimentally challenging to extract the JSA as the measurement requires the reconstruction of the state's full phase information. Instead, a lower bound for the Schmidt number can be experimentally determined by measuring

the joint spectral intensity (JSI) $|F(\omega_s, \omega_i)|^2$ of the two photon state. The JSI can be for example measured by performing spectrally-resolved coincidence measurements (see Fig. 3.17 a). Even without the full phase information, the JSI can be used to approximate the JSA as $F(\omega_s, \omega_i) \approx \sqrt{|F(\omega_s, \omega_i)|^2}$. This assumption can in turn be used to determine the lower bound for the Schmidt number, by extracting the Schmidt eigenvalues λ_n using a singular value decomposition of $\sqrt{|F(\omega_s, \omega_i)|^2}$, and calculating $K = (\sum \lambda_n^2)^{-1}$. Through a Schmidt mode decomposition of the measured correlation matrix, we extracted the lower bound for the Schmidt number to be 9.4 for 10 selected modes (see Fig. 3.17 b).

Due to the narrow spectral linewidth of the photons (~ 800 MHz) and the related long coherence time (~ 0.6 ns), the effective time resolution of our full detection system (~ 100 ps) was sufficient to perform time-domain measurements and extract the maximal dimensionality of the state. In particular, the maximum of the second-order coherence function is directly related to the number of effective modes (96). By individually selecting 10 signal and 10 idler resonances, we confirmed that the photons from each single resonance are in a pure quantum state (one effective mode per resonance, measured within the detection uncertainty) (see Fig. 3.17 b). In turn, the two-photon state defined over a pair of single resonances (one signal and one idler) symmetric to the excitation frequency is separable and has a Schmidt number of one, i.e. it does not contain any frequency entanglement. Consequently, a two-photon entangled quantum state comprised of multiple such pure frequency mode pairs has a Schmidt number with an upper bound given by the sum of the measured individual Schmidt numbers. Using this technique, the maximal dimensionality was measured for states with up to 10 selected modes, leading to a maximal state Schmidt number of 10.45 ± 0.53 (see Fig 3.17 b). As the lower and upper bound coincide and scale linearly with the number of selected modes, we conclude that the number of relevant orthogonal modes is indeed the same as the number of selected frequency modes.

These measurements confirmed that one photon pair simultaneously spans multiple frequency modes, forming a high-dimensional entangled state of the form

$$|\Psi\rangle = \sum_{k=1}^D c_k |k\rangle_s |k\rangle_i, \text{ with } \sum |c_k|^2 = 1 \quad (\text{Eq. 3.1}).$$

Here $|k\rangle_s$ and $|k\rangle_i$ are pure, single-frequency quantum states of the signal (s) and idler (i) photons, and $k=1, 2, \dots, D$ is the mode number, as indicated in Fig. 3.18 a). Such a state (Eq. 1) is of particular importance for quantum information processing, as it is a two photon high dimensional cluster state in its compact form (135), which forms the foundation for measurement-based quantum

computation (136). Additionally, the presented state is formed by a superposition of pure states, which are relevant for two-photon quantum interference experiments. Indeed, only pure states show efficient two-photon Hong-Ou-Mandel interference, which is at the basis of linear optical quantum computation (81).

In general, the exploitation of quDit states for quantum information processing motivates the need for high-dimensional operations that enable access to multiple modes with a minimum of components. While the individual elements (phase shifters and beam splitters) employed in the framework of spatial quantum information processing usually operate on only one or two modes at a time (81), the frequency domain is ideally suited for ‘global’ operations, i.e. acting on all system modes simultaneously. Through the merging of the fields of quantum state manipulation and ultrafast optical signal processing, state-of-the-art, yet established radio-frequency (RF) and telecommunications technologies (137) can be used to simultaneously address multiple frequency modes and enact high-dimensional coherent control and optical gate operations, achieving a scalable and practical quantum platform (138). In particular, optical phase gates for manipulating high-dimensional quDits can be directly implemented using programmable phase filters. The coherent mixing of multiple modes (a high-dimensional ‘beam splitting’ in the frequency domain) can be achieved through deterministic frequency conversion in electro-optic modulators. More importantly, as these two components are electronically-tunable, versatile operations can be performed in real-time without modifying the experimental setup (or the device), by simply adjusting the filter control and electrical modulation signal. By combining these elements, basic high-dimensional photon operations can be implemented in a single and robust spatial mode.

To implement this concept, a setup to perform basic gate operations for coherent state control was developed using a configuration composed of two programmable filters and one electro-optic phase modulator, as schematized in Fig. 3.16 and explained in more detail in Fig. 3.18. The first programmable filter was used to impose an arbitrary spectral amplitude and phase mask on the high-dimensional state, see Fig. 3.18 b). This manipulated state was then sent to an electro-optic phase modulator, which was driven by an RF frequency synthesizer. The imposed phase modulation generated coherent sidebands from the input frequency modes, see inset in Fig. 3.16. When the sideband frequency shift was chosen to match the spectral mode separation of the quantum state, i.e. the FSR, these input frequency modes were coherently mixed, see Sec. 2.6 and Fig. 3.18 c). Then, a second programmable filter was used to select different, individual frequency components of the manipulated state through the application of a second amplitude mask (Fig. 3.18

d). Finally, each of the two photons was routed to a separate single photon detector for coincidence detection (Fig. 3.18 e).

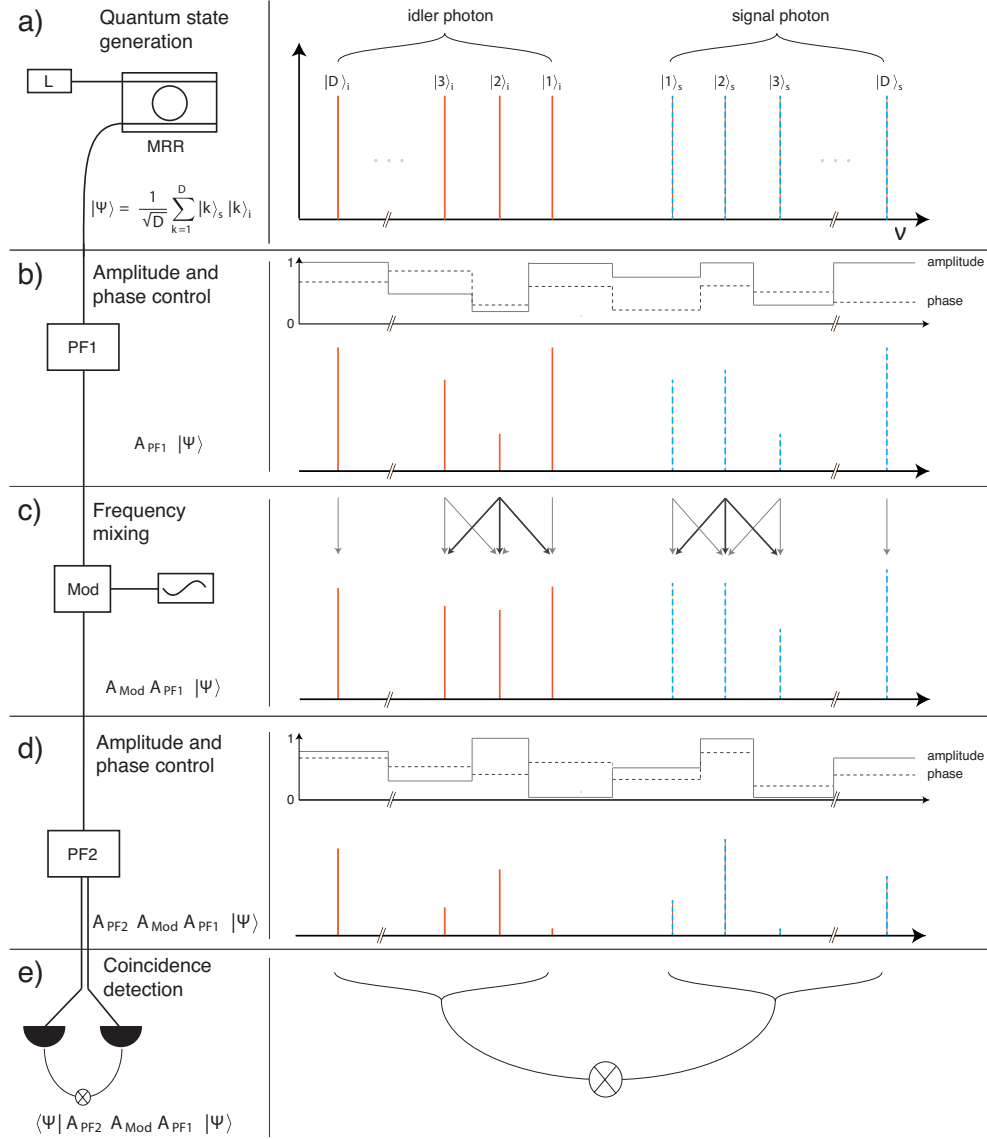


Fig. 3.18

Processing of frequency-entangled high-dimensional quantum states:

Steps to control, manipulate and characterize the high-dimensional quantum states are displayed, showing for each the equipment used and a schematic of the modification imposed on the quantum state in the spectral domain. **a)** The initial quantum states were generated using the operational principle illustrated in Fig. 3.16. **b)** Using a programmable filter (PF1) any arbitrary spectral phase and amplitude mask can be imposed on the non-classical states for manipulation. **c)** An electro-optic modulator (Mod) was used to coherently mix different frequency components of the high-dimensional states. **d)** A second programmable filter (PF2) can be used to impose an amplitude and phase mask and to route the signal and idler to two different paths. **e)** The photons were then detected using single photon counters and timing electronics. Figure published in Ref. [9].

This manipulation scheme allows to design well-defined quantum operations, which can be exploited for projection measurements. In particular, the RF driving signal was chosen in such a way to enable the mixing of four adjacent frequency modes, see Sec. 2.6. In combination with single photon detection, this allows to perform projection measurements of the form $|\Psi_{\text{Proj.}}\rangle = \alpha e^{i\varphi} |\bar{k}\rangle + \beta e^{i\theta} |\bar{k} + 1\rangle + \gamma e^{i\vartheta} |\bar{k} + 2\rangle + \delta e^{i\sigma} |\bar{k} + 3\rangle$, for a given frequency mode $\bar{k}$, where the projection amplitudes α, β, γ and δ , as well as the phases $\varphi, \theta, \vartheta$ and σ can be arbitrarily chosen for each photon.

Since the state amplitudes c_k in Eq. 3.1 were measured to be very similar, see Fig. 3.17 a), we assumed the generation of maximally-entangled states with equal amplitudes, i.e. $|\Psi\rangle = \sum_{k=1}^D c_k |k\rangle_s |k\rangle_i \approx \frac{1}{\sqrt{D}} \sum_{k=1}^D |k\rangle_s |k\rangle_i$. By considering any two, three or four adjacent frequency modes we prepared entangled quantum states of the form $|\Psi\rangle_{D=2} = \frac{1}{\sqrt{2}} (|\bar{k}\rangle_s |\bar{k}\rangle_i + |\bar{k} + 1\rangle_s |\bar{k} + 1\rangle_i)$, $|\Psi\rangle_{D=3} = \frac{1}{\sqrt{3}} (|\bar{k}\rangle_s |\bar{k}\rangle_i + |\bar{k} + 1\rangle_s |\bar{k} + 1\rangle_i + |\bar{k} + 2\rangle_s |\bar{k} + 2\rangle_i)$, or $|\Psi\rangle_{D=4} = \frac{1}{2} (|\bar{k}\rangle_s |\bar{k}\rangle_i + |\bar{k} + 1\rangle_s |\bar{k} + 1\rangle_i + |\bar{k} + 2\rangle_s |\bar{k} + 2\rangle_i + |\bar{k} + 3\rangle_s |\bar{k} + 3\rangle_i)$ and compared our measurements to these ideal cases. Using the projection measurements for both photons $|\Psi_{\text{Proj.}, D=2}\rangle = \frac{1}{\sqrt{2}} (|\bar{k}\rangle + e^{i\theta} |\bar{k} + 1\rangle)$, $|\Psi_{\text{Proj.}, D=3}\rangle = \frac{1}{\sqrt{3}} (|\bar{k}\rangle + e^{i\theta} |\bar{k} + 1\rangle + e^{i2\theta} |\bar{k} + 2\rangle)$, and $|\Psi_{\text{Proj.}, D=4}\rangle = \frac{1}{2} (|\bar{k}\rangle + e^{i\theta} |\bar{k} + 1\rangle + e^{i2\theta} |\bar{k} + 2\rangle + e^{i3\theta} |\bar{k} + 3\rangle)$, quantum interference was measured in the coincidence counts as a function of the phase θ , see Sec. 2.3.3. The measured quantum interference for the $D = 2, 3$, and 4 is shown in Fig. 3.19 a-c) with raw visibilities of 83.6%, 86.6%, and 86% (without background subtraction). These visibilities exceed the values of 71%, 77%, and 82%, respectively, thus violating Bell inequalities (*II0*) for two, three, and four-dimensional states and demonstrating their entanglement, see Sec. 2.3.3. Furthermore, a larger set of projection measurements was used to perform state tomography. The reconstructed density matrices show very good agreement with the ideal states, resulting in fidelities of 88.5%, 80.9%, and 69.5% respectively (see Fig. 19 d-f).

These measurements represent the first demonstration of high dimensional quantum state generation on a photonic chip. Furthermore, a flexible platform was developed to perform coherent control of these states through the manipulation of their frequency components using commercially available programmable telecommunications filters and RF photonics components. This makes it possible the simple execution of D -dimensional manipulations and mode-mixing operations with low experimental effort. Furthermore, the concatenation (*I39*) of the basic manipulation units shown here, as well as the implementation of arbitrary RF modulation waveforms and phase masks,

will enable the realization of more complex gates, thus allowing access to high-dimensional quantum computing logic in optical circuits of manageable experimental complexity.

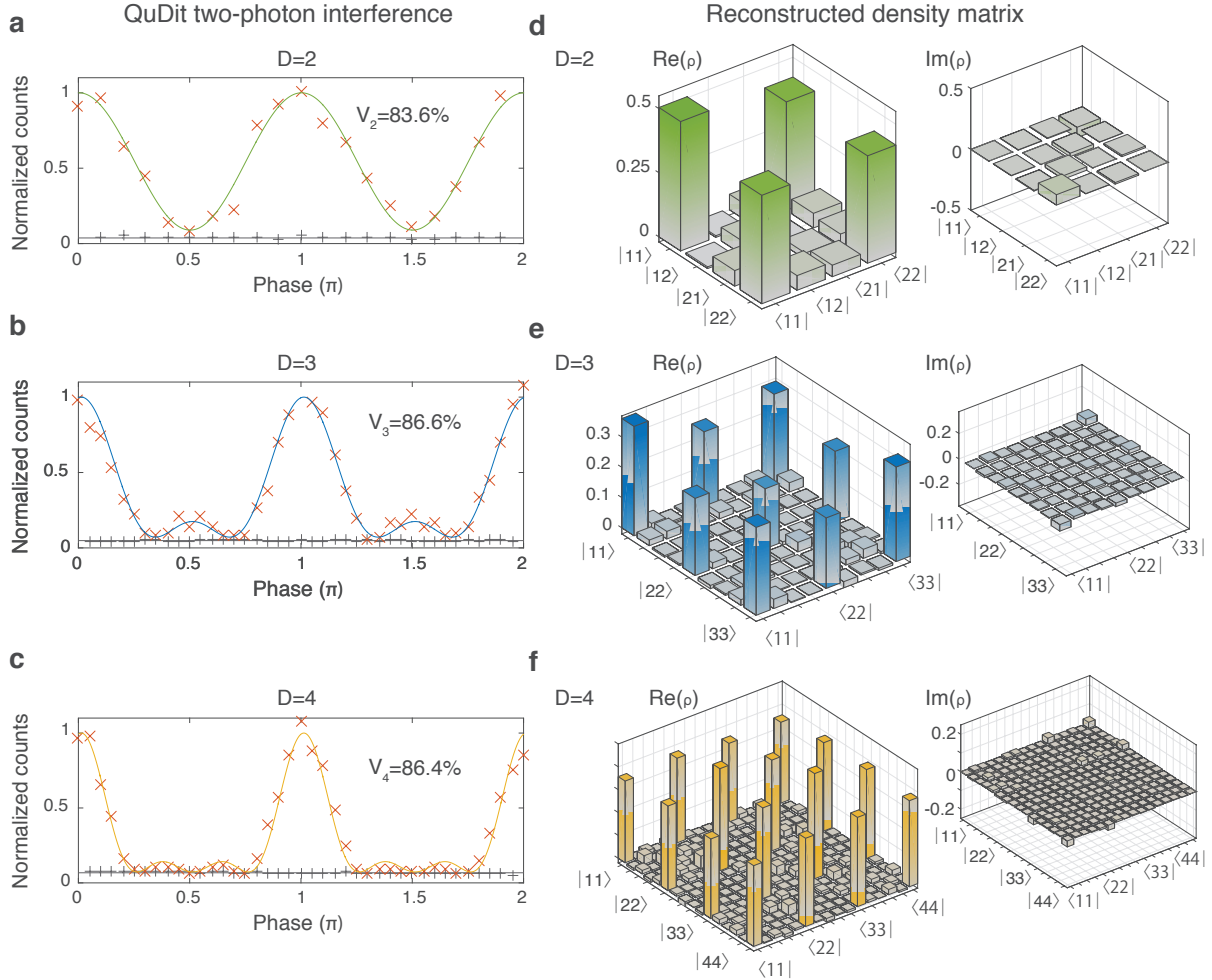


Fig. 3.19 Bell inequality violation and quantum state tomography of quDit states: To demonstrate the viability of the coherent control scheme described in Fig. 3, we performed a set of projection measurements. For the quantum interference characterization of quDit with $D=2$ **a**), $D=3$ **b**), and $D=4$ **c**), such states were projected on a superposition of D frequency modes with different phases. By changing these phases, a variation in the coincidence counts was measured (the flat black curve being the recorded background). Raw visibilities of 83.6%, 86.6%, and 86% for the quantum interference were obtained (without background subtraction), exceeding the visibilities of 71%, 77%, and 81.7% required to violate a Bell-like inequality for the $D=2$, $D=3$, $D=4$ states, respectively(110). Exploiting the ability to carry out arbitrary projection measurements on the signal and idler photon independently, we performed full quantum state tomography to experimentally reconstruct the density matrix of the entangled quDit states. We achieved fidelities of 88.5%, 80.9%, 69.5% for $D=2$ **d**) $D=3$ **e**), and $D=4$ **f**), respectively, demonstrating very good agreement between the measured and the expected maximally-entangled states. Figure published in Ref. [9].

3.3 Frequency comb of multi-partite entangled quantum states

The usable quantum resource of entangled states for quantum information processing is related to the size of the Hilbert space that the state occupies. In order to perform meaningful quantum information processing tasks (e.g. factorizing numbers (*140*), searching unsorted data (*141*)), very large Hilbert spaces (corresponding to at least 30 to 50 qubits) are required. In general, the Hilbert space of a state comprised of N D -dimensional entangled parties has a state dimensionality of D^N . The ultimate goal would therefore be to generate maximally entangled, high-dimensional multi-partite states, which span large Hilbert spaces, where the full dimensionality of the state can be effectively controlled and exploited.

In this section, the possibilities to generate different multi-partite quantum states using integrated frequency combs are discussed.

3.3.1 Generation of four-photon product states

One of the most basic multi-partite quantum states is a so-called product state of Bell states. As the name indicates, this state can be described as the product of two or more two-particle Bell states. In particular, four-photon product state can be described as the product of two individual two-photon states. The generation of such states can for example be achieved using either multiple sources, or multiple photon pairs generated from the same source. The generation of product states is of high interest as they form an important starting point to generate maximally entangled states such as GHZ (*142*) or cluster states (*143*), even though they do not carry genuine multi-partite entanglement themselves (*144*). A product state can for example be transformed into a different genuine multi-partite entangled state, e.g. by performing controlled phase gates (*145*), or passing them through a selective element such as a beam-splitter (*31*).

Before this work, no multi-photon entanglement had been generated using integrated sources. To demonstrate the first realization of multi-photon entangled quantum states generated on a photonic chip, we chose time-bin entangled product states generated within quantum optical frequency combs. By selecting two different signal-idler pairs symmetric to the excitation frequency, it is possible to generate individual two-photon qubit states given by $|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|S_{s1}, S_{i1}\rangle + e^{i2\varphi}|L_{s1}, L_{i1}\rangle)$ and $|\Psi_2\rangle = \frac{1}{\sqrt{2}}(|S_{s2}, S_{i2}\rangle + e^{i2\varphi}|L_{s2}, L_{i2}\rangle)$. By post-selecting four-photon events with one photon on each frequency channel, we can detect and characterize the

generated four-photon entangled product state, given by $|\Psi_{4\text{-photon}}\rangle = |\Psi_1\rangle \otimes |\Psi_2\rangle = \frac{1}{2} (|S_{s1}, S_{i1}, S_{s2}, S_{i2}\rangle + e^{i2\varphi}|S_{s1}, S_{i1}, L_{s2}, L_{i2}\rangle + e^{i2\varphi}|L_{s1}, L_{i1}, S_{s2}, S_{i2}\rangle + e^{i4\varphi}|L_{s1}, L_{i1}, L_{s2}, L_{i2}\rangle)$. It is worth noting that for the generation of a four-photon state, the coherence length of both generated photon pairs has to be the same and matched to the excitation field's coherence time. This requirement is intrinsically fulfilled through the resonant characteristics (i.e. equal resonance bandwidths) of the ring cavity in combination with the excitation scheme described above. Setting the pump power to 1.5 mW, a quadruple detection rate of 0.17 Hz was measured, leading to a calculated generation rate of 135 kHz – by taking into account the system and detection losses of 14.75 dB. To confirm the presence of four-photon entanglement, four-photon quantum interference measurements were performed (see Fig. 3.20). Indeed, four-photon interference is in general not present for two completely independent two-photon qubit states. The interference is expected to be proportional to $3 + \cos(4\alpha - 4\varphi) + 4\cos(2\alpha - 2\varphi)$, where α is the phase imposed on all four entangled photons and φ the pump interferometer phase. The measurement clearly follows the expected relation, having a visibility of 89% without compensation for background noise or losses. Finally, to fully characterize the entangled states we performed quantum state tomography, and obtained a fidelity of 64% without compensation for background noise or interferometer imperfections (see Fig. 3.21), which is comparable to the fidelity measured for non-integrated four-photon states (12). These results represent the first demonstration of any type of multi-photon entanglement on a photonic chip, and therefore open up the possibility to build on these results to achieve more complex states.

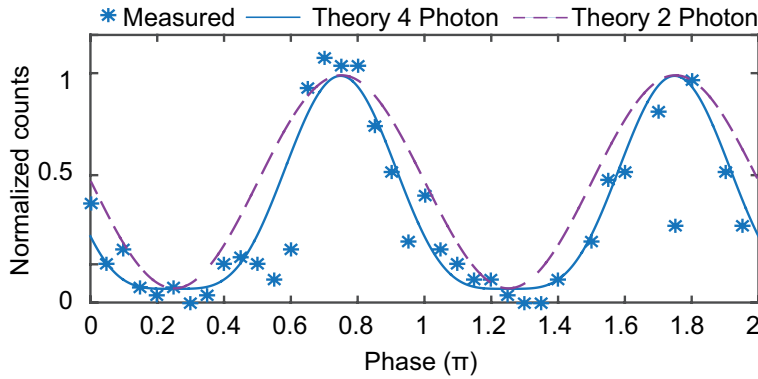


Fig. 3.20

Four photon time-bin quantum interference:

Four-photon entanglement measurement with all photon phases tuned simultaneously, showing clear four-photon quantum interference with a visibility of 89%. The solid line indicates the expected function; the dashed line shows the cosine interference in the two-photon case. Figure published in Ref [6].

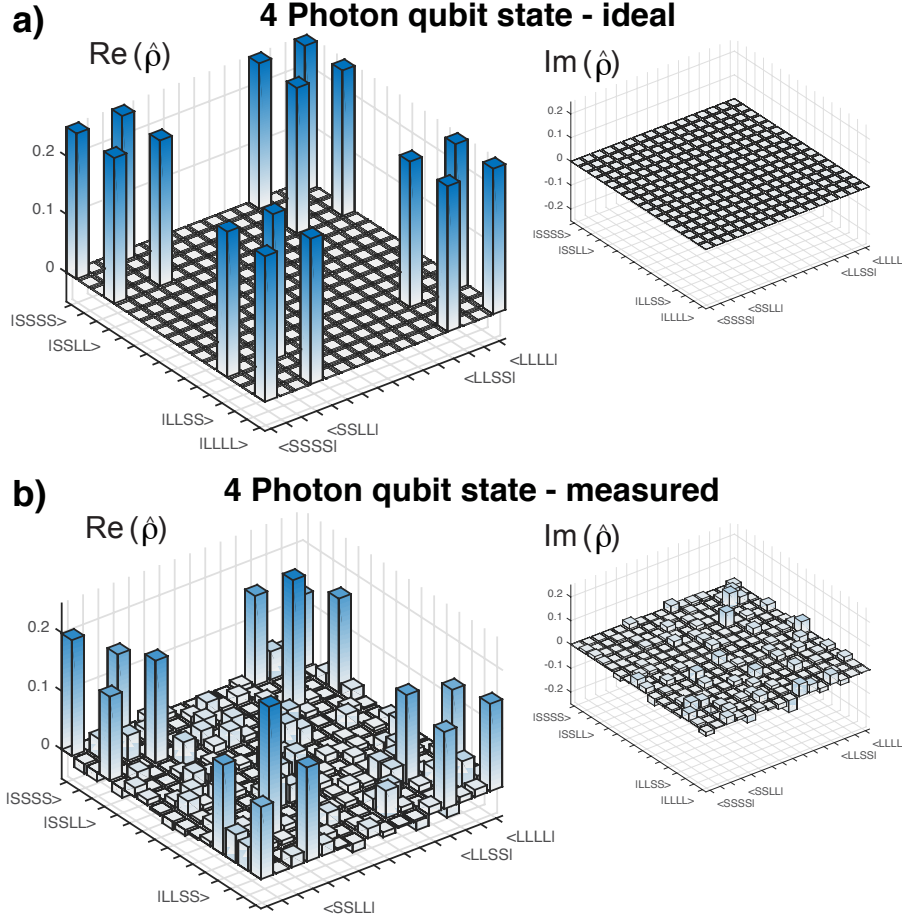


Fig. 3.21 Four photon time-bin tomography:
a) The real (Re) and imaginary (Im) parts of the ideal density matrices of a four-photon entangled product state, represented in the time-bin basis ($|SSSS\rangle$, $|SSSL\rangle$, ..., $|LLLL\rangle$). **b)** The measured density matrix reaches a fidelity of 64%, which is comparable in quality to other non-integrated four-photon states. Figure published in Ref [6].

3.3.2 Generation of multi-mode quantum states

After demonstrating separable multi-photon product states, the next challenge is to generate non-separable quantum states. As already mentioned above, this can be achieved by performing controlled phase-gates on product states.

A different and novel approach is enabled by the polarization mode dispersion of the integrated microring resonator. This approach makes use of multi-mode correlations, which are generated by the simultaneous exploitation of different nonlinear interactions, overlapping within the same frequency modes. As already described in Sec. 3.1.2, both Type-0 and Type-II SFWM are phase matched in the chosen microring resonator. We developed a scheme to use the presence of these two different nonlinear processes as a source to generate quantum states. In particular,

using the same resonator as in Section 3.1.2, we exploit the spatial/polarization multi-mode structure of the integrated microring resonator (pumped on two different modes) to superimpose two different SFWM processes in a single mode: Type-0 SFWM, for which two photons from one pump field generated two new photons with the same polarization; and Type-II SFWM, where two photons, one each from a different pump field, generated a cross-polarized photon pair, see Fig 3.22 a). The modes of the two SFWM processes spectrally overlapped, thus creating a multi-correlated set between four frequency modes, as shown in Fig. 3.22 b). In a single pump operation, multiplexed photon pairs are generated on a frequency comb symmetric to the pump. If both excitation fields are used at the same time, additional correlations are visible, confirming that each spectral mode is correlated with two other spectral modes, as summarized in Fig. 3.22 c).

To characterize the generated state and to determine the coupling network of the nonlinear interactions, photon coincidences between all combinations of 24 cavity modes were measured (6 signal and idler modes for both TE and TM polarizations), limited by the spectral bandwidth of the filter used to route different frequency modes to different detectors. Such measurements were performed for both single, as well as for the bi-chromatically pumped configuration. While only correlations in the TE mode are observed in the single frequency pumped quantum frequency comb, clear correlations are measured between TE modes (green), TM modes (blue) and TE/TM mode (red) in the bi-chromatically pumped frequency comb, separated into six individual two-photon four-mode multi-correlations.

In order to show entanglement between combinations of these four modes, we use a pulsed excitation to pump both the TE and TM mode with double pulses. If two-photon coincidences are selected, we confirm that each mode is entangled with two other frequency modes, forming a multi-mode entangled optical state (see Fig. 3.22 d). The visibilities of all four quantum interference measurements exceed the minimum requirement to violate a Bell-like inequality, confirming entanglement in all four mode combinations.

These first measurements so far only confirm that the quantum state is not an incoherent mixture of different mode combinations. Further investigations will have to be conducted, e.g. using frequency-bin entanglement, to confirm the exact nature of the generated quantum state. In particular, similar experiments, where two second-order nonlinear interactions overlapped in sets of modes resulted to the generation of cluster states (71, 74). If also cluster states can be generated using the overlap of two SFWM processes within the resonator is an open question and will be investigated in future work.

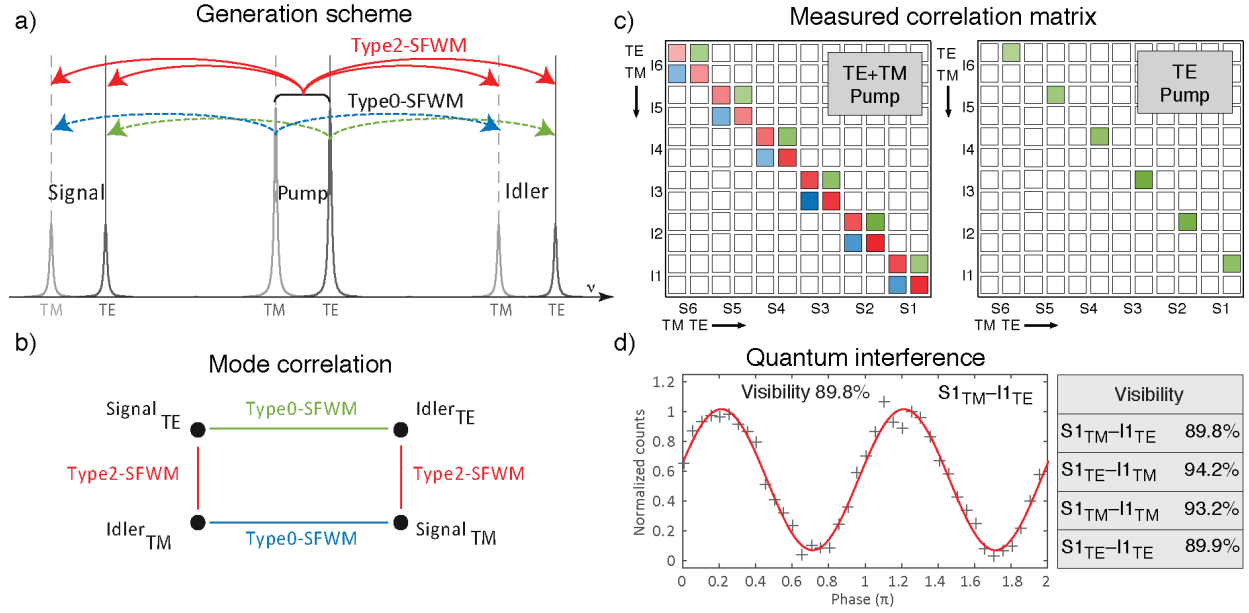


Fig. 3.22 Multi-correlated and entangled quantum states:
a) Schematic of the superimposed nonlinear processes in the bi-chromatically cross-polarized pumped OPO: Type-0 and type-II SFWM. **b)** Visualization of the four-mode correlated state. **c)** Measured correlation matrices of the bi-chromatically pumped OPO (TE+TM) using coincidence measurements. Correlations occur between TE modes (green), TM modes (blue) and TE/TM modes (red), clearly showing the formation of 6 individual 4-mode multi-correlated states. For comparison, we show the correlation matrix for the single-frequency pumped operation (TE), where only correlations between the TE modes occur. **d)** Quantum interference with a visibility above 89% was measured for all four two-photon states, confirming entanglement. Figure published in [C39].

In the future, this experiment can also be extended to the generation of four-photon entangled states. Indeed, as discussed below, it is predicted that such states will exhibit a high degree of control and particular properties. To generate multi-photon entanglement, the pump power will have to be raised to increase the probability of generating multiple photon pairs and post selected events where there is a photon on each of the four frequency modes. While such high pump powers and post selection enable the generation of product states if only one pump is used (Sec. 3.3.1), the overlap of the two nonlinear processes leads to additional components and a quantum state of the form:

$$\begin{aligned}
 |\Psi_{4\text{-phot,TE-TM}}\rangle = \frac{1}{\sqrt{6}} & \left(|S_{s,TE}, S_{i,TE}, S_{s,TM}, S_{i,TM}\rangle + e^{i2\varphi_{TM}} |S_{s,TE}, S_{i,TE}, L_{s,TM}, L_{i,TM}\rangle + \right. \\
 & e^{i2\varphi_{TE}} |L_{s,TE}, L_{i,TE}, S_{s,TM}, S_{i,TM}\rangle + e^{i(\varphi_{TE}+\varphi_{TM})} |S_{s,TE}, L_{i,TE}, L_{s,TM}, S_{i,TM}\rangle + \\
 & \left. e^{i(\varphi_{TE}+\varphi_{TM})} |L_{s,TE}, S_{i,TE}, S_{s,TM}, L_{i,TM}\rangle + e^{i2(\varphi_{TE}+\varphi_{TM})} |L_{s,TE}, L_{i,TE}, L_{s,TM}, L_{i,TM}\rangle \right),
 \end{aligned}$$

where the phases for the two excitation beams can be individually adjusted.

The simultaneous manipulation of two phases will give unprecedented control over the generated quantum state. It is worth noting that the generation of such a quantum state will not be possible using a second-order nonlinear process, as for that only a single excitation phase is available as a control parameter.

In the following, we theoretically investigated the properties of this quantum state in terms of its De Broglie wavelength. In general, it is possible to associate a De Broglie wavelength to an entangled state (146), where the quantum interference has a periodicity with respect to this wavelength. As an example, a two-photon entangled state shows a doubling in interference fringes compared to a single photon, resulting in a De Broglie wavelength of a two-photon entangled state that is half of that of single photons. The measured quantum interference of the four-photon product states in Sec. 3.3.2 has the same periodicity as the two-photon states, showing that the associated De Broglie wavelength of the four-photon product state is the same as that of the two-photon state. For this reason, product states of two-photon Bell states cannot be used for quantum metrology to enhance the sensitivity of interference experiments. Other four-photon states on the other hand have been shown to have a De Broglie wavelength four times smaller than that of an individual photon with the same wavelength (146).

By calculating the expected interference patterns generated by the above-described quantum state if all photons are sent through the same interferometer, we find that the De Broglie wavelength depends on the chosen pump interferometer phase settings. For this we consider the relative phase between the TE and TM pump interferometers, and calculate the interference patterns, see Fig. 3.23. It can be clearly seen that the periodicity changes from two to four times of that of a single photon, showing that the quantum state exhibits different De Broglie wavelengths for different pump phase settings. To further underline this transition, the quantum interference is shown for three different phase difference (0.5π , 0.75π , and π), see Fig. 3.23.

By overlapping two different third-order nonlinear processes, it will be possible to generate multi-partite quantum states on a photonic chip, which can be changed by adjusting the two input pump phases. The experimental realization is outside the time constraints of my thesis work, but preparations for the experiment are in progress. Once experimentally validated, the results will represent the first all-optically controllable quantum state demonstrated on a chip, where the De Broglie wavelength can be actively adjusted.

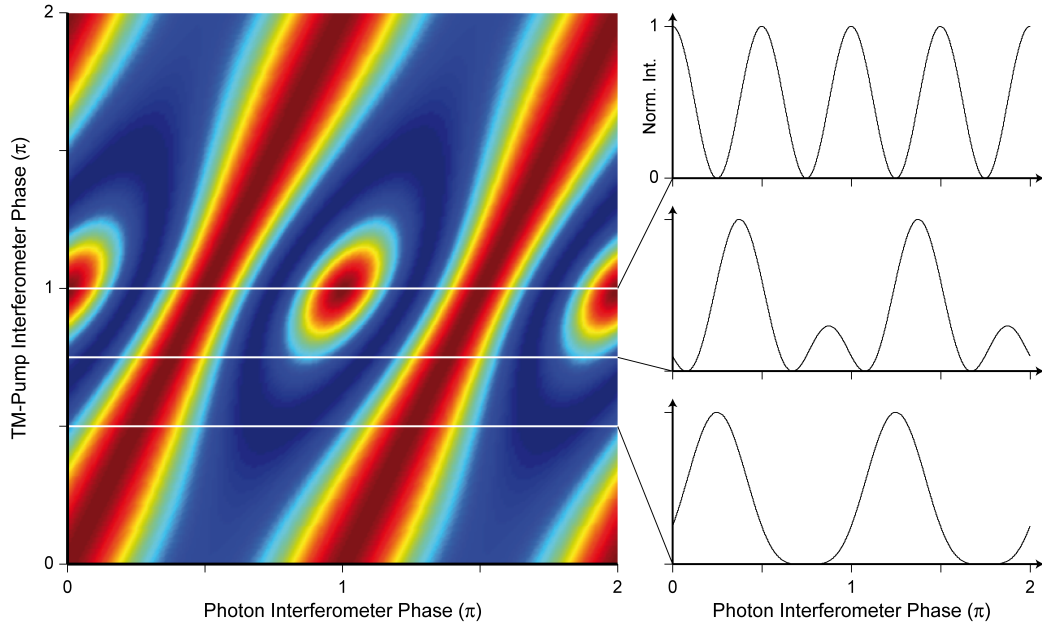


Fig. 3.23

De Broglie wavelength of four-photon multi-mode quantum state:

The expected interference pattern is calculated for the case where all photons pass the same interferometer (i.e. acquire the same phase) as a function of the pump interferometer phase (the TE-pump interferometer phase is set to zero, while the TM phase is adjusted from 0 to 2π). The particular TM pump phases (0.5π , 0.75π , and π) are shown on the right, with all amplitudes being normalized to one. It can be clearly seen that the state's De Broglie wavelength of the state becomes four (instead of two) times smaller than that of a single photon.

4. Conclusions and perspectives

The generation of optical quantum states with integrated microring-resonator-based frequency comb sources was investigated. In agreement with the project objectives, we successfully achieved the generation of heralded single photons, entangled photon pairs, as well as multi-partite entangled quantum states. All quantum states were realized within a single spatial mode, exploiting the temporal and frequency domain as a quantum resource. The single mode operation in particular enables highly compact sources and compatibility with fiber technology, enabling the use of telecommunications infrastructure and optical signal processing tools for quantum applications. Our results show that there is a large potential for quantum state generated using on-chip optical frequency comb sources. However, they also show that there is also significant work to be done to improve these devices in order to make them a viable candidate for practical quantum applications.

Objective 1, Single photons:

We have achieved the first generation of wavelength division multiplexed photon pairs generated from a single integrated source. The multiplexed nature could find applications in quantum communications platforms to achieve higher bit rates or to enable communications between multiple parties using the same source. In addition, we have demonstrated high photon purity and narrow spectral bandwidth, resulting in temporally long single photons. High photon purity is highly desired for quantum communications protocols, as well as linear optical quantum computation (LOQC), where pure photons interfere at beam splitters via HOM interference. Indeed, it is well known that the HOM interference scales inversely with the number of modes, putting a strong demand on pure single photons. Additionally, the narrow spectral bandwidth on the order of 100 MHz is comparable with those of atomic quantum memories, hinting at a potential compatibility. However, current memories operate at different wavelengths, which would require the development of telecom-band memories, or methods for efficiently changing the photon frequency from e.g. infrared to visible wavelength. What limits the immediate usability of the presented heralded single photon sources are the high losses, the resulting low CAR and heralding efficiencies. A large portion of these losses is caused by the resonator structure (4-port resonators), which implies an intrinsic 6 dB loss (as photons can exit different ports). This could be addressed by the use of two port resonators. Additionally, the coupling losses off the chip into optical fibers

(1.5 to 2dB) and losses of telecommunication filters (2-6 dB depending on the type of filter) significantly reduce the CAR, count rate and heralding efficiency. These limitations will have to be addressed before the device becomes a viable competitor as a source for heralded single photons. To circumvent the coupling issues, more filters and logical operations could be implemented directly on chip. This approach will become particularly attractive for applications in quantum information processing such as on-chip implementations of LOQC, but will require extensive development in terms of low loss integrated photonics waveguide structures, as well as on-chip single photon detectors.

Objective 2, Entangled photon pairs:

We have achieved the first generation of both time-bin, as well as frequency-bin entanglement using integrated microring resonators. The results also represent the first realization of multi-channel entanglement generation, as well as the first high-dimensional entanglement generated on a chip. Both achieved forms of entanglement can be used for quantum communications, where we show that the entanglement is preserved over significant fiber propagation. Time-bin entanglement is particularly suited for quantum communications, as the quantum state is resistant to fiber dispersion as well as polarization distortion due to fiber birefringence. Frequency-bin entanglement can also be used for quantum communications, where the high-dimensional nature can be used to increase the bit rate per photon. Unfortunately, the frequency-bin entangled states are sensitive to fiber dispersion, where different frequency components propagate with different velocities, resulting in different arrival times and eventually loss of indistinguishability. However, we have shown that the quantum coherence can be retrieved if the dispersion is compensated, e.g. by using dispersion compensating fibers or chirped fiber Bragg gratings. These additional elements will introduce additional loss, which, if not reduced, will eventually limit their application for communications applications. In addition to communications, both time-bin and frequency-bin entangled quantum states can be used for optical quantum computation. Models to perform LOQC have been experimentally demonstrated for time-bin entangled states (68), and theoretically discussed frequency-bin entanglement (139). After our first realization of multi-channel entanglement on a chip, other groups have published related work, achieving multi-channel continuous variable energy-time entanglement in silicon microring resonators (147, 148), as well as the full integration of the required interferometers for quantum state analysis (69). This shows the strong potential to combine, in the future, both sources and state manipulation devices. For the

frequency-bin entangled states, a main challenge will be to extend the coherent manipulation scheme to higher dimensional quantum states. This could be achieved by using microring resonators with a smaller FSR, such that the frequency mode spacing is in the 30 to 50 GHz range, which allows the use of RF signals matching the FSR, while still using standard wavelength filters to separate the modes. In addition, more advanced RF signals can be used instead of simple sine-waves to improve the frequency mixing, and to achieve custom mixing patterns.

Objective 3, Multi-partite entangled states:

We have achieved the first, and so far only, generation of multi-photon entanglement on a photonic chip. In addition, we realized the first multi-mode quantum states by means of integrated photonics. In particular, we demonstrated the first generation of four-photon entanglement on a photonic chip by making use of multiple frequency modes together with time-bin entanglement. These states are four-photon product states generated by the simultaneous generation of two Bell states on different frequency modes. In the future, these product states could be transformed into GHZ or cluster states by means of selective beam splitters or controlled phase gates. However, the realization of such beam splitters or gates using time-bin entanglement has not yet been achieved. The realization of product states is nevertheless an important step into the right direction and show that multi-photon entanglement can be generated using integrated sources.

The ability to generate nonlinear interactions between different mode families was used to generate quantum states with entanglement between sets of different modes. The presented concept could in the future be further exploited to realize a rich variety of quantum states. The scheme demonstrated here using polarization modes could be extended to higher order waveguide modes, to achieve interaction between 2,3,4 or more different mode families. In addition to extending this to multi-photon states, such nonlinear coupling between mode families is very promising for the generation of squeezed quantum states. In particular, it has been shown that architectures can generate continuous variable squeezed optical cluster states by making use of the overlap of nonlinear processes between two different mode families forming a set of four modes (71, 74), very similar to what we achieved in our on-chip devices. The ability to use polarization and higher-order waveguide modes within the resonators and their nonlinear interaction could therefore be used to generate highly complex multi-mode squeezed states. However, such states are very sensitive to losses, and the high internal and coupling losses will most likely drastically reduce the achievable squeezing. Indeed, two-mode squeezing from a two-port integrated resonator was

measured (149), where a maximum of 1.2 dB below the classical shot-noise limit was achieved. In order to perform error tolerant computation with squeezed states however, squeezing values exceeding 20.5 dB are required (21). With the resonators available during this project, it is unlikely that even squeezing below 1dB could be achieved due to the intrinsic 6 dB loss of the four-port resonator structure. The development of optimized resonators, low loss coupling, and/or integrated detectors would be required for such applications.

Finally, a very promising route to achieve complex high-dimensional multi-partite quantum states is to exploit so-called hyper-entangled quantum states. Such states are simultaneously entangled in multiple degrees of freedom and it becomes possible to achieve multi-partite quantum states by means of only two photons (76, 145). This has been in the past achieved using multiple degrees of freedom such as polarization, spatial modes, and energy-time. Using the integrated approach developed here, it will be possible to simultaneously achieve (and measure) time-bin and frequency-bin entanglement [P6]. The experimental realization will be the same as the generation of time-bin and frequency-bin entanglement. In particular, exciting the ring with double pulses will generate time-bin entanglement, and the broad phase-matching condition will at the same time lead to frequency-bin entanglement. It is therefore expected that hyper-entangled states can be directly generated without significantly increasing the generation process. To confirm this type of entanglement, simultaneous measurement temporal and frequency correlations will be required, which can be performed by the combination of interferometers and electro-optic phase modulation. Furthermore, this scheme could be readily extended to higher dimensions by considering multiple frequency modes together with the excitation with three, four or more coherent optical pulses. Remarkably, such hyper-entangled states would be more tolerant to losses, as only two photons are used to achieve large multi-partite quantum states. This in turn would drastically increase the detection rate and therefore also the usability of these quantum states.

In conclusion, we confirm the significant potential of microring resonator based integrated quantum sources for practical applications such as quantum communications, sensing and in the future also computation. In addition to the here-presented measurements of single photons, energy-time entangled photon pairs and multi-photon product states of Bell states, there is a clear path to achieve even more complex quantum states without the need to significantly increase the source complexity. This is achieved by making use of correlations in the frequency- and time-domain, which are present in a single spatial waveguide mode, and therefore minimize source footprint and complexity.

5. Bibliography

The literature is structured into the following categories: Publications that I have authored during the course of my Ph.D. studies are indicated in angled brackets ‘[1-11]’, patents are indicated with a *P*, conferences with a *C*, and book chapters with a *B* before the number. Journal publications from side projects, which are not discussed in Chapters 2 and 3, are indicated in italic with an upper case + as ‘[2]⁺’. All other references are cited with italic brackets ‘(1-147)’.

5.1 Journal Publications (* Equally contributing, joint first authors)

- [11]⁺ Yanbing Zhang, **Christian Reimer**, Jenny Wu, Piotr Roztock, Benjamin Wetzel, Brent E. Little, Sai T. Chu, David J. Moss, Benjamin J. Eggleton, Michael Kues, and Roberto Morandotti, “*Multi-channel phase-sensitive amplification in a low loss CMOS-compatible spiral waveguide,*” **Optics Letters**, accepted (2017)
- [10]⁺ Piotr Roztock, Michael Kues, **Christian Reimer**, Benjamin Wetzel, Stefania Sciara, Yanbing Zhang, Alfonso Cino, Brent E. Little, Sai T. Chu, David J. Moss, and Roberto Morandotti, “*Practical System for the Generation of Pulsed Quantum Frequency Combs,*” **Optics Express** 25, 18940-18949 (2017)
- [9] Michael Kues*, **Christian Reimer***, Piotr Roztock, Luis Romero Cortés, Stefania Sciara, Benjamin Wetzel, Yanbing Zhang, Alfonso Cino, Sai T. Chu, Brent E. Little, David J. Moss, Lucia Caspani, José Azaña, and Roberto Morandotti, “*On-chip generation of high-dimensional entangled quantum states and their coherent control,*” **Nature** 546, 622-626 (2017)
- [8]⁺ Michael Kues, **Christian Reimer**, Benjamin Wetzel, Piotr Roztock, Brent E. Little, Sai T. Chu, Tobias Hansson, Evgeny A. Viktorov, David J. Moss, and Roberto Morandotti, “*Passively mode-locked laser with an ultra-narrow spectral width,*” **Nature Photonics** 11, 159-162 (2017)
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- [6] **Christian Reimer***, Michael Kues*, Piotr Roztock, Benjamin Wetzel, Fabio Grazioso, Brent E. Little, Sai T. Chu, Tudor Johnston, Yaron Bromberg, Lucia Caspani, David J. Moss, and Roberto Morandotti, “*Generation of multiphoton entangled quantum states by means of integrated frequency combs,*” **Science** 351, 1176-1180 (2016)

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- [P6] **Christian Reimer (22.5%)**, Michael Kues (22.5%), Piotr Roztocki (22.5%), Stefania Sciara (22.5%), Yoann Jestin (5%), and Roberto Morandotti (5%), “*Method and system for the generation and control of high-dimensional multi-partite quantum states,*”
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US Provisional Patent
- [P4] **Christian Reimer (26%)**, Piotr Roztocki (26%), Michael Kues (26%), Robin Helsten (12%), Yoann Jestin (5%), Roberto Morandotti (5%), “*Method and system for phase-readout and active stabilization of optical interferometers,*”
PCT/CA2016/051060
- [P3] **Christian Reimer (29%)**, Michael Kues (29%), Benjamin Wetzel (18%), Piotr Roztocki (11%), Fabio Grazioso (3%), Yoann Jestin (5%), and Roberto Morandotti (5%), “*Method and system for the generation of optical multipartite quantum states,*”
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- [P2] **Christian Reimer (28%)**, Michael Kues (28%), Benjamin Wetzel (28%), Piotr Roztocki (6%), Yoann Jestin (5%), and Roberto Morandotti (5%), “*Method and system for the pulsed excitation of a nonlinear medium for photon pair generation*”
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5.3 Book Chapter

- [B1] Mayukh Lahiri, **Christian Reimer**, and Anton Zeilinger, “*Quantum interference and imaging,*” in “*Frontiers in Moderns Optics,*” Pages 159-170 (2016)
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5.4 Conference contributions

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- [C70] P. Roztock, M. Kues, C. Reimer, B. Wetzel, B.E. Little, S.T. Chu, D.J. Moss, and R. Morandotti, "Pulsed Quantum Frequency Combs from an Actively Mode-Locked Intra-Cavity Generation Scheme", CLEO Europe, Munich, Germany (2017). **(Invited)**
- [C69] C. Reimer, M. Kues, P. Roztock, L. Romero Cortes, S. Sciara, B. Wetzel, Y. Zhang, A. Cino, S.T. Chu, B.E. Little, D. Moss, L. Caspani, J. Azana, and R. Morandotti, "High-dimensional frequency-bin entanglement in on-chip optical frequency combs," Photonics North, Ottawa, Canada (2017). **(Invited)**
- [C68] P. Roztock, M. Kues, C. Reimer, B. Wetzel, F. Grazioso, B.E. Little, S.T. Chu, T. Johnston, Y. Bromberg, L. Caspani, D.J. Moss, and R. Morandotti, "On-chip Frequency Combs for Scalable Quantum State Generation", Photonics North, Ottawa, Canada (2017). **(Invited)**
- [C67] M. Kues, C. Reimer, B. Wetzel, P. Roztock, B.E. Little, S.T. Chu, T. Hansson, E.A. Viktorov, D.J. Moss, and R. Morandotti, "Transform-limited Nanosecond Passively Mode-locked Laser", Photonics North, Canada (2017). **(Invited)**
- [C66] C. Reimer, M. Kues, P. Roztock, B. Wetzel, B.E. Little, S.T. Chu, L. Caspani, D.J. Moss and R. Morandotti, "Generation of Complex Quantum States via Integrated Frequency Combs", Date 17, Lausanne, Switzerland (2017). **(Invited)**
- [C65] P. Roztock, M. Kues, C. Reimer, B. Wetzel, F. Grazioso, B.E. Little, S.T. Chu, T. Johnston, Y. Bromberg, L. Caspani, D.J. Moss, and R. Morandotti, "Quantum State Generation via Integrated Frequency Combs", SPIE Photonics West, San Francisco, USA (2017). **(Invited)**
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- [C63] P. Roztock, M. Kues, C. Reimer, B.E. Little, S.T. Chu, D.J. Moss, and R. Morandotti, "Four-Wave Mixing Photon Pair Generation Statistics for a Nonlinear Microcavity with Chaotic and Pulsed Excitation", CLEO Europe, Munich, Germany (2017).
- [C62] M. Kues, C. Reimer, B. Wetzel, P. Roztock, B.E. Little, S.T. Chu, T. Hansson, E.A. Viktorov, D.J. Moss, and R. Morandotti, "A Passively Mode-locked Nanosecond Laser with an Ultra-narrow Spectral Width", CLEO Europe, Munich, Germany (2017).

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- [C54] R. Morandotti, M. Kues, C. Reimer, P. Roztock, B. Wetzel, F. Grazioso, B.E. Little, S.T. Chu, T. Johnston, Y. Bromberg, L. Caspani, and D.J. Moss, "Complex Quantum State on a Chip", Congreso Nacional de Ingenieria Fisica, Medellin, Colombia (2016). **(Keynote)**
- [C53] C. Reimer, M. Kues, P. Roztock, B. Wetzel, B.E. Little, S.T. Chu, T. Johnston, Y. Bromberg, L. Caspani, D.J. Moss, and R. Morandotti, "Quantum State Generation with Integrated Frequency Combs", Photonics North, Quebec City, Canada (2016). **(Invited)**
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- [C50] C. Reimer, M. Kues, P. Roztock, B. Wetzel, Y. Bromberg, B.E. Little, S.T. Chu, D.J. Moss, L. Caspani, and R. Morandotti, "Integrated Quantum Frequency Comb Source of Entangled Qubits", META'16, 7th International Conference on Metamaterials, Photonic Crystals and Plasmonics, Torremolinos (Malaga), Spain (2016). **(Invited)**
- [C49] P. Roztock, M. Kues, C. Reimer, B. Wetzel, B.E. Little, S.T. Chu, T. Johnston, Y. Bromberg, L. Caspani, D.J. Moss, and R. Morandotti, "Quantum State Generation via Integrated Frequency Combs", International Conference on Optical, Optoelectronic and Photonic Materials and Applications, Montreal, Canada (2016). **(Invited)**
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- [C46] C. Reimer, and R. Morandotti, "Quantum frequency comb on a chip," Tecnolaser, Havana, Cuba (2016). **(Invited)**
- [C45] R. Morandotti, P. Roztock, C. Reimer, M. Kues, B. Wetzel, L. Caspani, F. Grazioso, Y. Bromberg, M. Clerici, M. Ferrera, M. Peccianti, A. Pasquazi, L. Razzari, B.E. Little, S.T. Chu, and D.J. Moss, "Integrated Optical Combs: Towards Quantum Optical Applications", SPIE Photonics West, San Francisco, USA (2016). **(Invited)**
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- Munro, S.T. Chu, B.E. Little, D.J. Moss, and R. Morandotti, “Multi-correlated Two-Photon States from a Bi-Modal Integrated Frequency Comb Source”, Integrated Photonics Research, Silicon and Nanophotonics, Vancouver, Canada (2016).
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- [C30] C. Reimer, M. Kues, L. Caspani, B. Wetzel, P. Roztock, M. Clerici, Y. Jestin, M. Ferrera, M. Peccianti, A. Pasquazi, B.E. Little, S.T. Chu, D.J. Moss, and R. Morandotti, “Novel Classical and Quantum Phenomena in Nonlinear Ring Resonators”, Energy Materials Nanotechnology (EMN) on Optoelectronics, Beijing, China (2015). **(Invited)**

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- [C14] L. Caspani, C. Reimer, M. Peccianti, A. Pasquazi, B.E. Little, S.T. Chu, D.J. Moss, and R. Morandotti, “CMOS Compatible Nonlinear Photonic Integrated Circuits”, Optoelectronics and Communications Conference (OECC), Melbourne, Australia (2014). **(Invited)**
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6 French summary

Résumé

L'exploitation de la mécanique quantique a le potentiel de résoudre d'importants défis et d'introduire, entre autres, de nouvelles technologies permettant une puissance de calcul inégalée, des communications parfaitement sécurisées, ainsi que l'émergence de systèmes de détection et d'imagerie possédant une précision et des résolutions non réalisables par des moyens classiques. Afin d'utiliser les propriétés uniques de la physique quantique, des plates-formes expérimentales fournissant l'accès et le contrôle des états quantiques doivent être développées. En particulier, les systèmes quantiques constitués de plusieurs parties sont requis, où chacune des parties est dans une superposition cohérente d'états, et où les dites parties sont superposées de façon cohérente l'une envers l'autre (par exemple, enchevêtrée). En outre, afin d'utiliser de tels systèmes quantiques enchevêtrés, il est nécessaire que chaque partie individuelle puisse être contrôlée et mesurée de manière cohérente. La facilité d'utilisation et la ressource quantique disponible du système enchevêtré sont liés à sa taille, ce qui est proportionnel au nombre de parties enchevêtrées, ainsi que la dimensionnalité de superposition dans laquelle chaque parties se trouvent. Pour cette raison, il existe une demande imminente pour la réalisation, l'étude et l'amélioration de la génération et du contrôle de larges systèmes quantiques, comprenant autant de parties enchevêtrées que possible et où chaque parties est idéalement dans une superposition à haute dimensionnalité. De nombreuses plates-formes quantiques sont actuellement soumises à des recherches approfondies, telles que celles relatives aux ions piégés, aux circuits supraconducteurs, aux centres de défauts dans des cristaux à l'état solide, aux oscillateurs mécaniques et aux états quantiques optiques (c'est-à-dire des photons). Bien que toutes les plates-formes offrent des avantages distincts (ainsi que certains challenges), les états optiques des photons sont particulièrement intéressants, car ils peuvent interagir avec d'autres systèmes quantiques et peuvent être transmis sur de longues distances tout en préservant leur cohérence quantique sans avoir besoin de protection excessives. Une grande variété de sources pour les états quantiques optiques ont été démontrées, mais la plupart de ces implémentations souffrent d'une grande complexité, ce qui limite leur évolutivité. La photonique intégrée est récemment devenue une plate-forme de premier plan pour la génération et le traitement d'états quantiques optiques sous une forme compact, rentable et stable. Cependant, à ce jour, les sources quantiques intégrées ont seulement pu générer des paires de photons, qui fonctionnaient sur des canaux et modes de polarisation uniques. Les quelques réalisations

d'enchevêtrement sur puce, étaient ainsi limités à deux parties enchevêtrées, où chaque partie était dans une superposition bidimensionnelle. Les limitations des sources actuelles de photons intégrées apparaissent en grande partie à partir des concepts exploités pour générer et contrôler ces états. En particulier, les sources enchevêtrées en chemin sont les plus couramment utilisées, mais l'augmentation de la complexité de l'état quantique nécessite une augmentation significative de la complexité du dispositif. Il existe donc un besoin urgent de développer de nouveaux concepts qui permettent d'obtenir la génération sur puce de grands et complexes états de photons sans augmenter la complexité de la source tout en permettant un contrôle et une détection cohérents de l'état quantique.

Dans ce travail, nous visons à remédier à ces limitations en utilisant les fréquences multiples et les modes temporels à travers le mode de guide d'onde spatial unique d'un micro-résonateur cyclique sur puce. Cette approche contraste avec les techniques les plus couramment utilisées s'appuyant sur l'enchevêtrement en chemin et qui exploite de multiples modes spatiaux générant l'enchevêtrement au moyen de structures de guides d'ondes complexes. En travaillant avec un mode spatial unique, nous avons réalisé la première démonstration de photons unique annoncé et de paires de photons enchevêtrés, qui sont émis sur plusieurs modes de fréquence à partir d'une seule source. Nous avons ensuite démontré la première génération d'états quantiques enchevêtrés à plusieurs photons sur une puce photonique. En utilisant la nature spectrale multimode des peignes de fréquence optique, nous avons réalisé la génération de paires de photons dans une superposition cohérente de modes à fréquence multiple, représentant la première génération d'états enchevêtrés à haute dimension sur une puce photonique.

Notre travail représente une avancée significative dans la génération d'états quantiques complexes sur une puce photonique et dans un mode spatial unique. En outre, nous avons démontré la possibilité de contrôler et manipuler de façon cohérente l'état quantique pour tous les états générés au moyen de composants de télécommunications standard, qui pourraient à l'avenir être intégrés sur la même puce avec les sources. Nos résultats indiquent que l'exploitation de sources intégrées de peigne de fréquence optique pour la génération d'états quantiques optiques peut constituer une plate-forme évolutive et pratique pour les technologies quantiques.

Contexte

Au cours des dernières années, le domaine de l'optique quantique est devenu l'un des domaines les plus actifs pour la recherche fondamentale ainsi que pour les applications pratiques. Sur la base des capacités techniques disponibles et évolutives pour la génération, la manipulation et la détection de photons individuels, l'optique quantique permet aujourd'hui une grande variété d'expériences fondamentales. En effet, de nombreux tests fondamentaux de la mécanique quantique ont été mis en œuvre à l'aide de photons. En outre, la possibilité de transmettre des photons au moyen de liaisons optiques en espace libre ou en utilisant des réseaux fibrés a donné naissance au domaine des communications quantiques, ce qui permet l'échange sécurisé d'informations.

L'une des caractéristiques les plus prononcées de la lumière est qu'elle peut interagir avec d'autres systèmes (en général, la lumière peut être émise, absorbée ou dispersée, en imprimant les informations du système) suggérant donc un moyen d'observer et d'enquêter sur les propriétés du dit système. En effet, les applications d'imagerie, de détection, de microscopie et de spectroscopie peuvent toutes être améliorées en utilisant les propriétés quantiques de la lumière. En outre, les photons peuvent interagir, contrôler et enchevêtrer d'autres systèmes quantiques tels que les ions piégés, les centres de défauts dans les cristaux à l'état solide et les systèmes quantiques mécaniques, ce qui rend l'optique quantique un outil indispensable pour la science quantique.

En général, les sources quantiques optiques peuvent être décrites comme appartenant à l'une des trois catégories principales: sources d'états compressés, sources de photons uniques, sources de paires de photons.

Les *états compressés* sont des états quantiques optiques où l'amplitude et le bruit de phase de l'état sont à la limite définie par le principe d'incertitude de Heisenberg. Dans de tels états, l'amplitude ou le bruit de phase peut être "comprimé" plusieurs décibels en dessous de la limite classique au détriment de la variable complémentaire. De tels états compressés peuvent être exploités pour le calcul quantique à variable continue.

Les *sources de photons uniques* ne produisent idéalement qu'un seul photon à la fois et peuvent être réalisées par exemple en utilisant des points quantiques, des centres de défauts, des atomes uniques et des molécules uniques. Les défis actuels, sujets d'une recherche approfondie, comprennent la

génération déterministe des photons à la demande, ainsi que la génération de photons purs monomode.

Les *sources de paires de photons*, n'émettent idéalement que deux photons en même temps et peuvent être réalisées au moyen d'interactions optiques non linéaires. Les sources de paires de photons peuvent être utilisées comme des sources de photons annoncées dites «unique», où l'un des deux photons est détecté, prédisant de manière déterministe la présence de l'autre photon de la paire. En outre, et en raison de la conservation de l'énergie, du moment et du spin dans le processus de génération non linéaire, les sources de paires de photons peuvent générer directement une grande variété de différents états enchevêtrés et sont donc couramment utilisés pour des applications liées aux ressources quantiques.

Alors que les paires de photons enchevêtrées peuvent déjà trouver des applications directes, leur utilisation en tant que ressource quantique est quelque peu limitée. Afin d'augmenter la complexité de l'état quantique, deux approches principales peuvent être utilisées: l'augmentation du nombre de photons enchevêtrés et / ou l'augmentation de la dimensionalité de la superposition dans laquelle se trouvent les photons.

La génération d'états enchevêtrés à plusieurs photons peut être obtenue en interférant plusieurs paires de photons. En effet, la concaténation de plusieurs sources à deux photons a permis la préparation d'états composés de dix photons enchevêtrés. Cependant, l'utilisation exclusive de l'approche multi-photons comporte deux principaux inconvénients. Tout d'abord, la complexité de la source augmente de manière significative avec chaque paire de photons ajoutée, et deuxièmement, le taux de détection diminue avec une quantité croissante de photons enchevêtrés. Les photons sont idéaux pour réaliser des superpositions et des enchevêtrements à haute dimension, car les photons peuvent être enchevêtrés dans de nombreux degrés de liberté différents, tels que les modes spatiaux (par exemple le moment angulaire orbital et la position du moment) ou les modes de fréquence-temporelle. La plupart des sources avancées d'états quantiques optiques ont en commun qu'ils s'appuient principalement sur les montages en espace libre. Cependant, les pertes et les complexités de l'implantation (tel que la réalisation d'un chevauchement de mode parfait et une stabilité de phase dans les configurations de l'espace libre) posent des défis importants dans la réalisation de circuits quantiques complexes. Il est donc nécessaire de mettre en place des plates-

formes stables et évolutives pour générer et contrôler des états quantiques complexes multi-photons et de haute dimension.

Dans le but de diminuer la taille des sources quantiques, l'accent a été mis sur l'obtention de sources compactes à base de fibres optiques, ainsi que sur des sources intégrées basées sur des structures compactes de guides d'ondes. Des progrès dans la miniaturisation des cristaux non linéaires ont conduit à la réalisation de sources compactes à base de cristaux non linéaires traditionnels et couplées à la fibre. En plus du couplage des sources à base de cristaux avec les fibres optiques, la non-linéarité intrinsèque des fibres optiques peut être directement utilisée pour générer des paires de photons. Cependant, les taux de génération relativement faibles avec le bruit élevé des photons Raman nécessitent généralement un refroidissement cryogénique des sources quantiques fibrées. Ainsi, il est de nos jours possible d'affirmer que l'une des plus grandes avancées dans la réalisation de dispositifs quantiques compact et évolutifs a été réalisé en utilisant la technologie photonique sur puce.

La photonique intégrée utilise des puces photoniques compactes, principalement à base de semi-conducteurs, où les structures de guides d'ondes optiques peuvent être réalisées à l'aide de techniques de nanofabrication. La photonique intégrée a d'abord été utilisée pour les dispositifs optiques linéaires, où des guides d'ondes optiques monomode ont été réalisés dans sur des puces photoniques de silicium. En raison de la petite zone de mode des champs optiques, des intensités de champ optique élevées peuvent être atteintes, ce qui permet l'exploitation d'interactions optiques non linéaires.

L'évolution de la photonique intégrée (i.e. des applications linéaires aux applications non linéaires) démontre que celle-ci est en train de devenir une plate-forme importante également pour l'optique quantique. En effet, la manipulation, la génération et la détection des états quantiques ont pu être réalisées. Cependant, tous les types d'enchevêtrements possibles ne peuvent être générés ou exploités dans des structures intégrées. Par exemple, la dispersion de mode et la condition d'appariement de phase dans les fibres et les guides d'ondes intégrés interdisent généralement la génération ou l'exploitation de l'enchevêtrement de la position du moment, et le moment angulaire orbitaire n'est généralement pas guidé dans de telles structures. Les sources intégrées de paires de

photons enchevêtrés reposent de nos jours principalement sur la polarisation et l'enchevêtrement en chemin.

Alors que l'enchevêtrement en polarisation a été démontré dans des structures intégrées, les états enchevêtrés n'ont jusqu'à présent été générés qu'à la sortie de la puce. En raison de la dispersion de polarisation, l'enchevêtrement n'est maintenu que dans des guides d'ondes intégrés très spécialisés, c'est pourquoi la manipulation d'états enchevêtrés par polarisation générée sur puce a été effectuée à l'aide de dispositifs en espace libre. L'enchevêtrement en chemin est actuellement l'une des formes d'enchevêtrement les plus exploitées dans les structures intégrées, car il peut être généré en concaténant plusieurs sources de photons via des coupleurs de guides d'ondes optiques.

En outre, les états enchevêtrés peuvent également être caractérisés par des dispositifs sur puce au moyen de diviseurs de faisceau et de déphaseurs. Cependant, pour le concept d'enchevêtrement en chemin, la complexité / l'empreinte du dispositif évolue avec la complexité de l'état, ce qui pose des défis technologiques importants pour atteindre des complexités de source plus élevées telles que l'enchevêtrement de haute dimension.

Pour les applications futures, des états quantiques multi-photons et / ou de haute dimension seront nécessaires. Cependant, ni les états quantiques multi-photons ni les états enchevêtrés de haute dimension n'ont été générés dans des sources intégrées. En outre, il n'y a pas de vision très claire pour générer de grands états quantiques dans un avenir proche en utilisant l'enchevêtrement en chemin ou celui de la polarisation. De nouveaux concepts pour la génération de l'état quantique sur puce doivent donc être développés, permettant en même temps d'atteindre des états quantiques de haute dimension et une mise à l'échelle favorable en termes de complexité des sources, d'augmentation du nombre de photons et de dimensionnalité de l'état. L'optimum en termes de complexité et d'empreinte sont des sources qui peuvent générer des états quantiques complexes dans un seul mode de guide d'onde. De tels états quantiques devraient également être compatibles avec l'infrastructure de télécommunications par fibre, offrant ainsi un avantage supplémentaire.

Les états enchevêtrés en énergie-temps sont bien adaptés pour être générés dans un seul mode spatial et sont donc des candidats idéaux pour réaliser des états quantiques intégrés avec une source de faible complexité. En effet, les premières réalisations de l'enchevêtrement en énergie-temps dans les sources sur puce ont été réalisées. Cependant, alors que les sources étaient plus compactes, les

réalisations de l'enchevêtrement en énergie-temps dans les dispositifs à puce réalisés avant ce travail ne fournissaient pas d'états quantiques plus complexes par rapport aux schémas utilisés pour l'enchevêtrement en polarisation ou en chemin. En particulier, les réalisations antérieures n'ont exploité que des ensembles de modes temporels et de fréquences uniformes permettant d'obtenir des états variables bidimensionnels ou continus dans lesquels les modes quantiques individuels ne peuvent être traités individuellement.

Dans le but d'augmenter le nombre de modes temporels et fréquentiel accessibles pour la préparation de l'état quantique, la génération d'états quantiques optiques a récemment commencé à exploiter les concepts de peigne de fréquence optique. Les peignes de fréquence sont des sources lumineuses multi-fréquences caractérisées par la présence de multiples modes de fréquence spectralement équidistants et bloqués en phase. Dans le domaine classique, les peignes de fréquence ont révolutionné la détection et la métrologie, en raison de leur grande précision et de leur stabilité en tant que sources lumineuses. Au cours des dernières années, les corrélations quantiques dans les peignes de fréquence optique ont également été étudiées. En effet, il a été démontré que des corrélations quantiques complexes entre les modes de peigne de fréquence peuvent être obtenus dans des peignes de fréquence à oscillation paramétrique optique (OPO) pompés de façon synchrone ou bi-chromatique. Cependant, toutes les recherches effectuées sur les peignes de fréquence quantique étaient limitées aux états compressés, et seules des quantités modérées de compression (<10 dB) ont été obtenues en raison de pertes optiques élevées. En outre, les modes spectraux sont séparés par seulement quelques MHz dans les peignes de fréquence quantique, rendant les modes spectrales uniques inaccessibles (avec la plupart des techniques de filtrage optique). La manipulation de l'état quantique ne paraît donc pas entièrement réalisable dans de tels systèmes.

Une autre approche pour la réalisation des peignes de fréquence repose sur l'utilisation de structures photoniques intégrées, pour lesquelles la génération classique de peigne de fréquence sur puce fait l'objet d'une recherche approfondie. De tels peignes de fréquence basés sur les microrésonateurs peuvent générer des émissions de lumière cohérentes à très large bande avec une séparation du mode spectral entre les dizaines et les centaines de GHz, ce qui permet un accès facile aux modes de fréquence individuels. Cependant, à ce jour, toutes les études concernant les peignes sur puce

se sont concentrées sur leurs propriétés classiques et les sources intégrées de peigne de fréquence n'ont pas été utilisées pour la génération d'états quantique.

Dans le contexte du travail présenté dans cette thèse, nous avons proposé d'utiliser des peignes de fréquence optique intégrés comme sources quantiques, car ils ont le potentiel de résoudre de nombreuses limitations liées aux sources quantiques intégrées actuelles. Tout d'abord, la fabrication et l'optimisation des composants sont déjà à un stade avancé en raison des précédentes réalisations utilisant les peignes de fréquence classiques. En outre, les sources de peigne de fréquence sont basées sur un seul résonateur cyclique, et sont donc des structures beaucoup moins compliquées que par exemple les sources enchevêtrées en chemin. L'utilisation de peignes sur puce va également bien au-delà de l'avantage de la miniaturisation des peignes de fréquence quantique, qui ont montré la génération d'états quantiques compressés très complexes. En effet, l'espacement du mode de fréquence plus large dans les sources de peigne intégrées permet pour la première fois un accès facile aux différents modes de fréquence individuels au moyen de filtres optiques. Cela permet également d'étudier d'autres états quantiques autres que les états compressés et pourrait conduire à la mise en œuvre d'opérations quantiques dans le domaine fréquentiel. Les peignes de fréquence intégrés sont donc des candidats idéaux pour la génération d'états quantiques évolutifs et complexes dans des sources compactes, où la complexité de l'état quantique ne se mesure pas avec la complexité de la source et où tous les états quantiques sont émis dans un seul mode spatial.

Ce travail représente la première étude systématique des propriétés quantiques des photons générés dans les peignes de fréquence optique intégrés. Au lieu d'étudier la génération d'états compressés, nous nous concentrons sur la génération de paires de photons, et en particulier des états enchevêtrés en énergie-temps.

Il existe trois différents types d'enchevêtrement énergie-temps en termes de mode temporel et de fréquence, à savoir l'enchevêtrement en continu, en intervalle de temps et en intervalle de fréquence. Si une paire de photons à large bande spectrale est générée, les photons peuvent être générés dans une superposition cohérente d'un continuum de modes temporel et de fréquence, formant des états enchevêtrés variables continues. Cependant, étant donné qu'il est très difficile (voire impossible) d'aborder les modes quantiques individuels des états variables continus, leur mise en œuvre dans les applications est très difficile. Pour cette raison, il est souvent préférable de

travailler avec des états quantiques discrets, en particulier pour la mise en place de portes quantiques. La forme discrète de l'enchevêtrement énergie-temps (où les modes discrets temporels et / ou de fréquence sont présents) peut être classé en tant qu'enchevêtrement en intervalle de temps et intervalle de fréquence, en fonction de la variable (mode temporel ou fréquentiel) pouvant être contrôlée. En particulier, si les photons sont dans une superposition de deux (ou plus) modes temporels distincts, qui ne se chevauchent pas et ne peuvent pas être manipulés indépendamment, l'état est appelé enchevêtrement en intervalle de temps. Si les deux (ou plus) modes de fréquence ne se chevauchent pas et sont suffisamment séparés de façon spectrale pour pouvoir être sélectionnés et manipulés au moyen de filtres optiques, l'état quantique est appelé enchevêtrement en intervalle de fréquence.

Résultats: génération de pair de photons avec des peignes de fréquence optique intégrés

Les photons corrélés forment les états quantiques les plus élémentaires qui peuvent être générés par des processus optiques non linéaires spontanés. Ces états sont également à la base d'états quantiques enchevêtrés plus complexes. Néanmoins, même sans présence d'enchevêtrement, la génération de deux photons corrélés peut déjà être exploitée pour des applications quantiques. Principalement, la corrélation des photons dans le temps d'arrivée permet de prévoir la présence d'un photon unique en détectant l'autre. Une telle source de photons unique annoncée peut par exemple être implémentée dans des protocoles de communication quantique, qui reposent sur l'impression d'informations sur des photons unique.

Pour démontrer la première source intégrée multiplexée de photons uniques annoncés, nous avons choisi d'utiliser un résonateur en forme d'anneau intégré avec une gamme spectrale libre (FSR) de 200 GHz, un facteur de qualité Q de 1,3 million et une dispersion très faible et anormale.

Pour montrer la nature multiplexée en fréquence de la source de photons, cinq couples de longueur d'onde signal / complémentaire différents et symétriquement situés autour de la longueur d'onde d'excitation ont été sélectionnés par des filtres de télécommunication. Les canaux de fréquence individuels ont ensuite été envoyés à des détecteurs de photons uniques pour la détection de coïncidence. Des pics de coïncidence clairs ont été mesurés sur les cinq paires de canaux symétriques, alors qu'aucune coïncidence n'a été mesurée entre des combinaisons de canaux non symétriques. Afin de démontrer la présence de photons individuels annoncés, nous mesurons la

cohérence annoncée du second ordre des photons. Pour une source de photons unique annoncée parfaite, un seul photon complémentaire devrait être présent lors de la détection d'un photon de signal. Cette caractéristique se manifeste par un pic dans la fonction de cohérence du second ordre, où un pic inférieur à 0,5 confirme la nature quantique du photon. Un pic dans la cohérence annoncée du second ordre a été mesurée, ce qui confirme que la source a fonctionné dans un régime de photon unique. En utilisant l'excitation pulsée et en augmentant la puissance de la pompe pour augmenter la probabilité de générer plusieurs paires de photons, nous avons mesuré la cohérence du deuxième ordre du photon pour extraire le nombre effectif de modes. Nous avons observé un pic aussi élevé que 1.537, correspondant à $N = 1.86$ modes efficaces, pour le micro résonateur avec un facteur Q de 1.3M. La valeur non idéale de moins de deux a été attribuée à une résonance partagée provoquée par une réflexion arrière dans l'anneau. En répétant les mesures avec un résonateur différent possédant un facteur Q inférieur i.e. 240 000 mais avec une résonance non cassée, une pureté parfaite du photon dans l'incertitude de mesure a été obtenue, ce qui montre que, dans ce cas, notre micro résonateur avec un Q inférieur est mieux adapté pour les expériences quantiques. Toutes les expériences consécutives ont donc été effectuées avec des micro résonateurs à faible Q, offrant une meilleure pureté du photon.

Dans une étape suivante, nous avons étudié la bande passante spectrale du peigne de fréquence quantique. Le résonateur a été excité à l'aide d'un laser à verrouillage de mode filtré de manière spectrale, et le taux de comptage de photon unique en fonction de la longueur d'onde a été mesurée à l'aide d'un filtre à haute résolution réglable en longueur d'onde dans la bande C, ainsi qu'un analyseur de spectre à base de réseaux. À partir de ces mesures, nous avons calculé le taux de production de la paire de photons par impulsion. Une large bande de photons a été émise, couvrant l'ensemble des bandes S, C et L de l'Union International des Télécommunications (ITU) (longueurs d'ondes allant de 1470 à 1620 nm). Le processus de mélanges à quatre ondes spontané (SFWM) génère un spectre symétrique en fréquence, tandis que l'asymétrie spectrale dans le nombre de photons mesuré a été expliquée par la diffusion Raman, ce qui pourrait être réduit en refroidissant la puce. En raison de la grande condition d'accord de phase, obtenue grâce à la dispersion du guide d'onde proche de zéro, le peigne émis présente un spectre très plat et à large bande avec des taux de production de paires quasiment uniforme, allant de 0,02 à 0,04 paires par impulsion, sur la totalité peigne mesurée. En plus de la génération de photons uniques annoncés sur une seule polarisation, nous avons également étudié la génération de paires de photons à

polarisation croisée. La polarisation est un degré de liberté couramment utilisé pour le traitement quantique de l'information. Cependant, l'utilisation de la polarisation dans les structures intégrées est très difficile en raison de la dispersion du mode de polarisation habituellement désavantageuse dans les guides d'ondes, car ceux-ci ne sont généralement conçus que pour une polarisation unique. Contrairement à d'autres dispositifs sur puce à base de silicium ou de nitrure de silicium, les guides d'onde d'oxynitrure de silicium utilisés ici ont des propriétés de dispersion similaires pour les deux modes de polarisation, ce qui est une conséquence des dimensions du guide d'onde presque carrées ($1,5 \times 1,45 \mu\text{m}$). En raison de ces propriétés du guide d'ondes, nous avons pu exploiter les deux modes de polarisation pour les interactions optiques non linéaires.

Pour réaliser la génération de paires de photons à polarisation croisée, nous avons développé un nouveau schéma pour supprimer le mélange à quatre ondes stimulé entre les deux champs d'excitation en choisissant un micro résonateur présentant un décalage de fréquence entre les résonances TE et TM, tout en ayant une gamme spectrale libre presque identiques entre les deux modes de polarisation, permettant ainsi au mélange à quatre ondes spontané à polarisation croisée de se produire à des résonances ciblées. Pour effectuer des mesures de coïncidence de photons, les photons générés ont été séparés par un diviseur de faisceau polarisant et détectés avec des détecteurs de photons unique. Un pic de coïncidence évident avec un CAR de 12 sans soustraction de fond a été mesuré. La nature du processus de génération a été confirmée au moyen de mesures de mise à l'échelle de puissance, confirmant la première génération directe de paires de photons à polarisation croisée sur une puce photonique. Pour caractériser davantage la performance de notre dispositif en tant que source de photon unique, nous avons mesuré la cohérence (annoncée) de second ordre des photons. Un net pic dans la cohérence annoncée du second ordre a été enregistrée, montrant que la source opère dans un régime à photon unique non classique, tandis que la cohérence du second ordre montre un pic clair avec un maximum de $2,01 \pm 0,03$, résultant en $N_{\text{eff}} = 0,99 \pm 0,03$ modes efficaces, soulignant la pureté élevée de la source.

Enfin, la production de paires de photons à polarisation croisée n'est pas seulement limitée aux résonances adjacentes. En effet, nous avons mesuré des couples de photons à polarisation croisée sur 12 couples de résonance, limités seulement par la disponibilité de filtres.

Résultats: génération d'enchevêtrement avec des peignes de fréquence intégrés

Après avoir démontré la génération de paires de photons corrélées, le prochain défi important est de générer des états à deux photons enchevêtrés, car ce sont les bases de la plupart des applications du traitement de l'information quantique. Les sources intégrées ont réussi à produire différents types d'états quantiques enchevêtrés tels que l'enchevêtrement en chemin, l'enchevêtrement en polarisation, ainsi que l'enchevêtrement continu en énergie et en temps. La plupart des sources s'appuient principalement sur des guides d'ondes simples, des guides d'ondes à cristaux photoniques et des cavités, ainsi que des résonateurs cycliques. Il existe un intérêt particulier pour la génération d'états quantiques enchevêtrés discrets variables, car ils fournissent une manipulation facile et un accès aux modes quantiques. Pour cette raison, ce travail se concentre sur la réalisation des deux versions discrètes de l'enchevêtrement énergie-temps, appelé l'enchevêtrement en intervalle de temps et en intervalle de fréquence.

L'enchevêtrement en intervalle de temps est très utilisé pour les applications dans les communications quantiques, mais il peut également être exploité pour les protocoles de calcul quantique. L'enchevêtrement en intervalle de temps peut être généré en excitant une source non linéaire avec des impulsions doubles cohérentes. De telles impulsions peuvent par exemple être générées à l'aide d'un interféromètre non-équilibré, où la différence de longueur de trajet est beaucoup plus grande que la longueur de cohérence de l'impulsion d'entrée (donc aucune interférence classique de photon unique ne peut être mesurée avec un tel interféromètre). Pour une impulsion d'entrée unique, un tel interféromètre générera deux impulsions à la sortie, qui sont retardées dans le temps (définies par la différence de longueur de trajet de l'interféromètre) et qui ont une relation de phase fixe (définie par la phase de l'interféromètre). Si une source non linéaire est excitée par une telle impulsion double, et la puissance choisie de telle sorte que la probabilité que chacune des deux impulsions génère une paire de photons peut être négligée, alors une paire de photons générée à partir d'une telle double impulsion est générée dans une superposition cohérente de deux modes temporels. En particulier, nous avons généré ces doubles impulsions en passant la sortie d'un laser de pompe pulsé à travers un interféromètre stabilisé à fibre non-équilibré avec un retard de 11,4 ns - significativement plus grand que la durée d'impulsion du laser générant les doubles impulsions avec une différence de phase relative définie. Ces deux impulsions ont ensuite été couplées dans le micro-résonateur intégré. La puissance de la pompe a été choisie de telle sorte que la probabilité de créer une paire de photons simultanément à partir des deux

impulsions était suffisamment faible pour être négligeable. Cette configuration a conduit à la génération d'états de qubit enchevêtrés. Plus important encore, ces qubits enchevêtrés sont générés sur toutes les résonances du micro résonateur, conduisant ainsi à un peigne de fréquence quantique de paires de photons enchevêtrés.

Pour caractériser les photons enchevêtrés, chacun d'entre eux a été passé individuellement par différents interféromètres avec un déséquilibre identique à celui utilisé pour le laser de la pompe. En changeant les phases de l'interféromètre, on peut effectuer différentes mesures de projection pour mesurer l'interférence quantique entre les deux photons par rapport aux phases définies de l'interféromètre. Ceci a été utilisé pour violer l'inégalité de Bell sur cinq paires de canaux différentes et symétriques par rapport à la longueur d'onde d'excitation. En plus de mesurer les violations de l'inégalité de Bell, nous avons effectué une tomographie d'état quantique pour mesurer la matrice de densité d'état. Nous mesurons d'abord les qubits de deux photons générés sur les lignes du peigne de fréquence symétriques par rapport à la longueur d'onde de la pompe et trouvons une fidélité de 96%, confirmant que les états quantiques générés sont de haute qualité et très proches de l'état enchevêtré idéal. Nous avons ensuite répété la mesure tomographique après avoir ajouté 40 km de fibres, pour montrer que l'enchevêtrement est conservé après une longue propagation dans la fibre (fidélité de 87% sans compensation pour les pertes), ce qui met en évidence l'applicabilité immédiate des qubits à deux photons générés pour les communications quantiques.

Par rapport à l'enchevêtrement en intervalle de temps, l'enchevêtrement en intervalle de fréquence est très rarement utilisé pour les applications. Cela découle de deux problèmes principaux liés à sa génération et à sa détection. En particulier, les mesures de projection requises dans le domaine fréquentiel sont difficiles car une modification de la fréquence du photon est requise. Afin d'exploiter efficacement ce type d'enchevêtrement, nous avons développé une nouvelle génération, manipulation et approche de détection. Compte tenu de la génération d'état, les micro-résonateurs en forme anneau intégrée sont idéaux pour générer des états enchevêtrés en fréquence. En effet, si la condition d'appariement de phase est telle que les paires de photons peuvent être générées sur des résonances multiples symétriques à la pompe, les paires de photons sont intrinsèquement générées dans des états enchevêtrés en fréquence. L'avantage de la génération est triple: premièrement, la gamme spectrale libre des micro-résonateurs en forme d'anneau peut être choisie

dans la gamme GHz, permettant de sélectionner des modes de fréquence individuels avec des filtres de télécommunications standard. Deuxièmement, les photons sont directement générés dans les modes de fréquence des résonateurs, de sorte qu'aucun photon n'est perdu en raison du filtrage externe. Et troisièmement, les photons peuvent être générés en états purs, si une configuration de pompe pulsée est choisie. Cela signifie que de vrais états enchevêtrés peuvent être générés, où chaque mode de fréquence est un état pur.

En particulier, pour générer un tel état enchevêtré en fréquence, un laser à verrouillage de modes spectralement filtré est utilisé pour exciter une seule résonance du micro-résonateur à la longueur d'onde de ~ 1550 nm, produisant à son tour des paires de photons corrélés signaux et complémentaires spectralement symétriques au champ d'excitation couvrant des résonances multiples. Pour caractériser la dimensionnalité de l'état généré, deux expériences différentes ont été réalisées. Tout d'abord, à l'aide d'un filtre programmable commandé par ordinateur, l'intensité spectrale commune a été mesurée, décrivant la répartition des fréquences de l'état des deux photons. L'intensité spectrale commune peut être utilisée pour déterminer la limite inférieure du nombre de Schmidt, qui fournit une mesure de la dimensionnalité du système. Nous avons extrait la limite inférieure du nombre de Schmidt à 9,4 pour 10 modes sélectionnés. Deuxièmement, nous avons effectué des mesures dans domaine temporel et extrait la dimensionnalité maximale de l'état. En particulier, le maximum de la fonction de cohérence de second ordre est directement lié au nombre de modes efficaces. À l'aide de cette technique, la dimensionnalité maximale a été mesurée pour les états et ceci jusqu'à 10 modes sélectionnés, conduisant à un nombre de Schmidt maximal de $10,45 \pm 0,53$. Comme la limite inférieure et supérieure coïncident et s'élèvent linéairement avec le nombre de modes sélectionnés, nous concluons que le nombre de modes orthogonaux significatifs est en effet directement lié au nombre de modes de fréquence sélectionnés.

Même si la génération de paires de photons enchevêtrés en fréquence peut presque être réalisée naturellement en utilisant une source intégrée non linéaire, la question de mesurer efficacement ces états reste un défi. Pour résoudre ce problème, nous avons développé un nouveau système de contrôle cohérent, basé sur des modulateurs de lumière spatiale couplés par fibres et une modulation de phase électro-optique. En général, l'exploitation des états quDit pour le traitement quantique de l'information motive le besoin d'opérations de grande dimension qui permettent l'accès à plusieurs modes avec un minimum de composants. Grâce à la fusion des disciplines liées

à la manipulation de l'état quantique et au traitement du signal optique ultra-rapide, les technologies de la radiofréquence et des télécommunications les plus modernes peuvent être utilisées pour adresser simultanément les modes de fréquence multiples et autoriser le contrôle cohérent de grande dimension ainsi que les opérations sur les portes optiques, pour la réalisation d'une plateforme quantique évolutive et pratique. En particulier, les portes de phase optique pour manipuler des quDits à haute dimension peuvent être directement implémentées à l'aide de filtres de phase programmables. Le mélange cohérent de plusieurs modes (un « fractionnement de faisceau » à grande dimension dans le domaine de la fréquence) peut être atteint grâce à une conversion de fréquence déterministe dans les modulateurs électro-optiques. Plus important encore, étant donné que ces deux composants sont accessibles par voie électronique, des opérations polyvalentes peuvent être effectuées en temps réel sans modifier la configuration expérimentale (ou le dispositif), en réglant simplement le contrôle du filtre et le signal de modulation électrique. En combinant ces éléments, les opérations basiques sur les photons de haute dimension peuvent être implémentées dans un mode spatial unique et robuste.

Pour implémenter ce concept, une configuration pour effectuer des opérations de base de la porte pour un contrôle d'état cohérent a été développée à l'aide d'une configuration composée de deux filtres programmables et d'un modulateur de phase électro-optique. Ce schéma de manipulation nous a permis de concevoir des opérations quantiques bien définies, qui peuvent être exploitées pour des mesures de projection. Nous avons utilisé ces mesures de projection pour effectuer des violations de l'inégalité de Bell et la tomographie d'état quantique. L'interférence quantique mesurée pour $D = 2, 3$ et 4 a donné des visibilités brutes de 83,6%, 86,6% et 86% (sans soustraction de fond). Ces visibilités dépassent respectivement 71%, 77% et 82%, violant ainsi les inégalités de Bell pour les états à deux, trois et quatre dimensions et démontrant leur enchevêtrement. Les matrices de densité reconstituées topographiquement montrent un très bon accord avec les états idéaux, donnant lieu à des fidélités de 88,5%, 80,9% et 69,5% respectivement.

Résultats: enchevêtrement multipartite dans les peignes à fréquence optique sur puce

La ressource quantique utilisable des états enchevêtrés pour le traitement de l'information quantique est liée à la taille de l'espace de Hilbert que l'état occupe. Afin d'effectuer des tâches significatives de traitement de l'information quantique (par exemple, factoriser les nombres, rechercher dans les ensembles de données), des espaces Hilbert très importants (correspondant à

au moins 30 à 50 qubits) sont nécessaires. En général, l'espace de Hilbert d'un état composé de N parties enchevêtrées de dimension D a un état de dimensionnalité DN . Le but ultime serait donc de générer des états multipartites à haute dimension, enchevêtrés au maximum, qui couvrent de grands espaces de Hilbert, et où la totalité de la dimension de l'état peut être efficacement contrôlée et exploitée.

Pour démontrer la première réalisation des états quantiques enchevêtrés multi-photons générés sur une puce photonique, nous avons choisi des états de produit enchevêtrés en fréquence générés dans les peignes de fréquence optique quantique. Les états de produit sont l'un des états quantiques multipartites les plus basiques. Comme son nom l'indique, cet état peut être décrit comme le produit de deux ou plusieurs états quantiques avec une quantité plus faible de parties impliquées. En particulier, l'état du produit à quatre photons peut être décrit comme le produit de deux états individuels à deux photons. La génération d'états de produit est d'un grand intérêt car ils constituent un point de départ important pour générer des états enchevêtrés maximum, de l'ordre du GHZ ou des états de cluster, même s'ils ne comportent pas eux-mêmes d'authentiques enchevêtrements multiples. En sélectionnant deux paires différent signal/complémentaire symétriques par rapport à la fréquence d'excitation, il est possible de générer des états qubit individuels à deux photons. En post-sélectionnant des événements à quatre photons avec un photon sur chaque canal de fréquence, nous pouvons détecter et caractériser l'état de produit enchevêtré à quatre photons générés. Il convient de noter que pour la génération d'un état à quatre photons, la longueur de cohérence des deux paires de photons générées doit être identique et adaptée au temps de cohérence du champ d'excitation. Cette exigence est intrinsèquement réalisée à travers les caractéristiques de résonance (c'est-à-dire des largeurs de bande de résonance égales) de la cavité cyclique en combinaison avec le schéma d'excitation décrit ci-dessus. En augmentant la puissance d'excitation, nous avons activé la génération d'états à quatre photons avec un taux de détection quadruple de 0,17 Hz, ce qui a conduit à un taux de génération calculé de 135 kHz - en tenant compte du système et des pertes de détection de 14,75 dB. Pour confirmer la présence d'un enchevêtrement à quatre photons, des mesures d'interférence quantique à quatre photons ont été effectuées. En effet, l'interférence à quatre photons n'est généralement pas présente pour deux états qubit à deux photons complètement indépendants. La mesure suit clairement le modèle d'interférence attendu pour les états produits, ayant une visibilité de 89% sans compensation pour le bruit de fond ou les pertes. Pour caractériser pleinement les états enchevêtrés, nous avons effectué une tomographie d'état quantique et obtenu

une fidélité de 64% sans compensation pour le bruit de fond ou les imperfections de l'interféromètre. Ces résultats représentent la première démonstration de tout type d'enchevêtrement multi-photons sur une puce photonique, et ouvre donc la possibilité de construire sur ces résultats pour obtenir des états plus complexes.

Une approche différente et nouvelle pour générer des états enchevêtrés multi-mode est étudiée, en fonction de la dispersion du mode de polarisation du micro-résonateur cyclique intégré. Cette approche utilise des corrélations multi-mode, qui sont générées par l'exploitation simultanée de différentes interactions non linéaires, se chevauchant dans les mêmes modes de fréquence. En particulier, les processus SFWM co-polarisé et à polarisation croisées sont en phase avec le micro-résonateur choisi. Les modes des deux processus SFWM se chevauchent spectralement, créant ainsi un ensemble multi-corrélé entre quatre modes de fréquence. Pour caractériser l'état généré et pour déterminer le réseau de couplage des interactions non linéaires, nous avons mesuré les coïncidences de photons entre toutes les combinaisons des 24 modes de cavité (6 modes de signal / complémentaire pour les polarisations TE et TM), limité par la bande passante spectrale du filtre utilisé pour acheminer les différents modes de fréquence vers les différents détecteurs. De telles mesures ont été effectuées pour la configuration de pompe à la fois mono-chromatique et bi-chromatique. Alors que seules les corrélations en mode TE sont observées dans le peigne de fréquence quantique à pompage à fréquence unique, les corrélations claires sont mesurées entre le mode TE, TM et TE / TM dans le peigne de fréquence pompé bi-chromatiquement, séparé en six multi-corrélations d'individus de deux photons à quatre modes. Afin de montrer l'enchevêtrement entre les combinaisons de ces quatre modes, nous utilisons une excitation pulsée pour pomper le mode TE et TM avec des impulsions doubles. Si les coïncidences à deux photons sont sélectionnées, nous confirmons que chaque mode est enchevêtré avec deux autres modes de fréquence, formant un état optique enchevêtré multimode. Les visibilités des quatre mesures d'interférence quantique dépassent l'exigence minimale pour violer une inégalité en deux modes de Bell, confirmant l'enchevêtrement dans les quatre combinaisons en deux modes.

Conclusions et perspectives

Nous avons étudié la génération d'états quantiques optiques avec des sources de peigne de fréquence basées sur des micro-résonateurs intégrés. En particulier, nous avons réussi à générer des photons uniques annoncés, des paires de photons enchevêtrés, ainsi que des états quantiques

multipartites enchevêtrés. Tous les états quantiques ont été réalisés dans un seul mode spatial, exploitant le domaine temporel et fréquentiel comme ressource quantique. L'opération en mode unique, permet en particulier, des sources très compactes et une compatibilité avec la technologie des fibres, permettant l'utilisation d'une infrastructure de télécommunications et des outils de traitement de signaux optiques pour les applications quantiques. Nos résultats montrent qu'il existe un grand potentiel pour l'état quantique généré en utilisant des sources de peigne de fréquence optique sur puce, mais qu'il existe également un travail important à accomplir pour améliorer ces dispositifs afin de les rendre viable pour des applications quantiques pratiques.

Nous avons réalisé la première génération de paires de photons multiplexées par répartition en longueur d'onde générées à partir d'une seule source intégrée. La nature multiplexée pourrait trouver des applications dans des plates-formes de communication quantique pour atteindre des débits binaires plus élevés ou permettre des communications entre plusieurs parties utilisant la même source. En outre, nous avons démontré une grande pureté de photons et une bande passante spectrale étroite, ce qui entraîne des photons uniques temporellement longs. La haute pureté du photon est fortement souhaitée pour les protocoles de communications quantiques, ainsi que le calcul quantique optique linéaire, où les photons purs interfèrent dans les diviseurs de faisceau via une interférence de type HOM. En outre, la bande passante spectrale étroite de l'ordre de 100 MHz est comparable à celle des mémoires quantiques atomiques, ce qui suggère une compatibilité potentielle. Cependant, les mémoires actuelles fonctionnent à différentes longueurs d'ondes, ce qui nécessiterait le développement de mémoires de bande de télécommunication, ou des procédés pour changer efficacement la fréquence de photons de l'infrarouge à la longueur d'onde visible. Ce qui limite la convivialité immédiate des sources de photons unique annoncées présentées, ce sont les pertes élevées, entraînant une diminution de l'efficacité du CAR et de l'efficacité. Une grande partie de ces pertes est causée par la structure du résonateur (résonateurs à 4 ports), ce qui implique une perte intrinsèque de 6 dB (car les photons peuvent quitter différents ports). Cela pourrait être résolue par l'utilisation de résonateurs à deux ports. En outre, les pertes de couplage hors de la puce en fibres optiques (1,5 à 2dB) et les pertes des filtres de télécommunication (2-6 dB selon le type de filtre) réduisent considérablement le CAR, le taux de comptage et l'efficacité. Ces limitations devront être traitées avant que le périphérique ne devienne un concurrent viable comme source pour les photons individuels annoncés.

Nous avons réalisé la première génération d'enchevêtrement en intervalle de temps et en intervalle de fréquence en utilisant des micro-résonateurs cyclique intégrés. Les résultats représentent également la première réalisation de génération d'enchevêtrements multicanaux, ainsi que le premier enchevêtrement à haute dimension généré sur une puce. Les deux formes d'enchevêtrement peuvent être utilisées pour les communications quantiques, où nous montrons que l'enchevêtrement est conservé sur une importante propagation des fibres. L'enchevêtrement en intervalle de temps est particulièrement adapté aux communications quantiques, car l'état quantique résiste à la dispersion des fibres ainsi qu'à la distorsion de polarisation due à la biréfringence des fibres. L'enchevêtrement en intervalle fréquence peut également être utilisé pour les communications quantiques, en particulier la nature à haute dimension peut être utilisée pour augmenter le débit par photon. Malheureusement, les états enchevêtrés en fréquence sont sensibles à la dispersion des fibres, où différentes composantes de fréquence se propagent avec des vitesses différentes, entraînant des temps d'arrivée différents et éventuellement une perte d'indistinguabilité. Cependant, nous avons montré que la cohérence quantique peut être récupérée si la dispersion est compensée, par ex. en utilisant des fibres compensatrices de dispersion ou des réseaux de Bragg fibrés chirpés. En plus des communications, les états quantiques enchevêtrés en temps et en fréquence peuvent être utilisés pour le calcul quantique optique. Les modèles pour effectuer un calcul quantique linéaire avec états enchevêtrés en temps ou en fréquence ont été respectivement démontrés expérimentalement, ou théoriquement discutés.

Nous avons réalisé la première génération d'enchevêtrement à quatre photons sur une puce photonique en utilisant des modes de fréquence multiples avec l'enchevêtrement en temps. Les états démontrés sont des états de produits à quatre photons, obtenus par la génération simultanée de deux états Bell sur différents modes de fréquence. Les états de produit ne comportent pas d'enchevêtrement authentique à quatre photons, mais ils constituent une étape importante vers la réalisation de différents types d'états multi-photons maximalelement enchevêtrés. Par exemple, en utilisant des diviseurs de faisceaux sélectifs, les états de produit peuvent être transformés en états GHZ, mais ces séparateurs de faisceau n'ont pas encore été réalisés pour l'enchevêtrement en temps. La réalisation des états de produit est néanmoins une étape importante et montre que l'enchevêtrement multi-photons peut être généré à l'aide de sources intégrées.

Nous avons effectué la première réalisation d'états quantiques multi-mode dans les sources de peigne de fréquence intégrées. En particulier, la possibilité d'activer des interactions non linéaires entre deux familles de mode différentes a été utilisée pour générer des états quantiques avec un enchevêtrement entre des ensembles de modes différents. Le concept présenté pourrait à l'avenir être exploité pour générer des états quantiques riches. Le schéma démontré ici et en utilisant les modes de polarisation pourrait être étendu aux modes de guide d'onde d'ordre supérieur, afin d'obtenir une interaction entre une multitude de familles de modes différents. En plus d'étendre cela aux états multi-photons, ce couplage non linéaire entre les familles de mode pourrait être très prometteur pour la génération d'états quantiques compressés. En particulier, il a été démontré que ces architectures peuvent générer des états de clusters optiques continus à compression variable en utilisant le chevauchement des processus non linéaires entre deux familles de mode différentes formant un ensemble de quatre modes, très similaire à l'architecture ici effectuée.

Enfin, une voie très prometteuse pour atteindre des états quantiques multipartite complexes à haute dimension est d'exploiter les soi-disant états quantiques hyper-enchevêtrés. De tels états sont simultanément enchevêtrés sur plusieurs degrés de liberté et il devient possible d'atteindre des états quantiques multipartites au moyen de seulement deux photons. Dans le passé, cela a été atteint en utilisant de multiples degrés de liberté tels que la polarisation, les modes spatiaux et le temps d'énergie. À l'aide de l'approche intégrée développée ici, il sera possible d'obtenir simultanément (et de mesurer) l'enchevêtrement en intervalle de temps et de fréquence. La réalisation expérimentale sera la même que pour la génération d'enchevêtrement en intervalle de temps et de fréquence. En particulier, l'excitation de l'anneau avec des impulsions doubles générera un enchevêtrement en temps, et la large condition d'accord de phase entraînera en même temps l'enchevêtrement en fréquence. Nous nous attendons à ce que les états hyper-enchevêtrés puissent être générés directement sans augmenter considérablement le processus de génération. Pour confirmer ce type d'enchevêtrement, des corrélations temporelles et de fréquence simultanée seront nécessaires, ce qui peut être effectué par la combinaison d'interféromètres et de modulation de phase électro-optique. De plus, ce schéma pourrait être facilement étendu à des dimensions plus élevées en considérant les multiples modes de fréquence lors de l'excitation avec trois, quatre ou plus impulsions optiques cohérentes. Remarquablement, ces états hyper-enchevêtrés seraient plus tolérants aux pertes, car seuls deux photons sont utilisés pour atteindre de grands états quantiques

multipartites. Cela augmenterait considérablement le taux de détection et donc la facilité d'utilisation de ces états quantiques.

En conclusion, nous confirmons le potentiel important des sources quantiques intégrées à base de résonateur en forme d'anneau pour des applications pratiques telles que les communications quantiques, la détection représentant ainsi que le futur du calcul. En plus des mesures présentées ici sur les photons uniques, les paires de photons enchevêtrés en énergie et les états multiphonons des états Bell, il existe un chemin clair pour obtenir des états quantiques encore plus complexes sans avoir à augmenter de manière significative la complexité de la source. Ceci est obtenu en utilisant des corrélations dans le domaine de la fréquence et du temps, qui sont présentes dans un seul mode de guide d'onde spatial, minimisant ainsi l'empreinte et la complexité de la source.

Appendix A

A.1 Generators for the Special Unitary Groups SU(2), SU(3) and SU(4)

The T matrices (see Sec. 2.6) can be found with the Kronecker multiplication of the generators.

Generators of the Special Unitary Group SU(2)

$$\lambda_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \lambda_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Generators of the Special Unitary Group SU(3)

$$\begin{aligned} \lambda_0 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \lambda_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \\ \lambda_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ i & 0 & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \end{aligned}$$

Generators of the Special Unitary Group SU(4)

$$\begin{aligned} \lambda_0 &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \lambda_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ \lambda_4 &= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ \lambda_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \lambda_9 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \lambda_{10} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \lambda_{11} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \\ \lambda_{12} &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}, \lambda_{13} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \lambda_{14} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \lambda_{15} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix} \end{aligned}$$

A.2 Projections used for tomography measurements

Two-photon, two-dimensional tomography

The two photons are projected onto every combination of the 4 listed projections, leading to 16 measurements.

$$|1\rangle, |2\rangle, \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle), \frac{1}{\sqrt{2}}(|1\rangle + i|2\rangle)$$

Two-photon, three-dimensional tomography

The two photons are projected onto every combination of the 9 listed projections, leading to 81 measurements.

$$\begin{aligned} \alpha &= e^{\frac{2\pi}{3}i}, \beta = e^{-\frac{2\pi}{3}i} \\ \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle), \frac{1}{\sqrt{2}}(\alpha|1\rangle + \beta|2\rangle), \frac{1}{\sqrt{2}}(\beta|1\rangle + \alpha|2\rangle) \\ \frac{1}{\sqrt{2}}(|1\rangle + |3\rangle), \frac{1}{\sqrt{2}}(\alpha|1\rangle + \beta|3\rangle), \frac{1}{\sqrt{2}}(\beta|1\rangle + \alpha|3\rangle) \\ \frac{1}{\sqrt{2}}(|2\rangle + |3\rangle), \frac{1}{\sqrt{2}}(\alpha|2\rangle + \beta|3\rangle), \frac{1}{\sqrt{2}}(\beta|2\rangle + \alpha|3\rangle) \end{aligned}$$

Two-photon, four-dimensional tomography

The two photons are projected onto every combination of the 16 listed projections, leading to 256 measurements.

$$\begin{aligned} \alpha &= e^{\frac{2\pi}{3}i}, \beta = e^{-\frac{2\pi}{3}i} \\ \frac{1}{\sqrt{3}}(|1\rangle + |2\rangle + |3\rangle), \frac{1}{\sqrt{3}}(\alpha|1\rangle + |2\rangle + |3\rangle), \frac{1}{\sqrt{3}}(|1\rangle + \alpha|2\rangle + \beta|3\rangle), \frac{1}{\sqrt{3}}(|1\rangle + \beta|2\rangle + \alpha|3\rangle) \\ \frac{1}{\sqrt{3}}(|1\rangle + |2\rangle + |4\rangle), \frac{1}{\sqrt{3}}(|1\rangle + |2\rangle + \alpha|4\rangle), \frac{1}{\sqrt{3}}(|1\rangle + \alpha|2\rangle + \beta|4\rangle), \frac{1}{\sqrt{3}}(|1\rangle + \beta|2\rangle + \alpha|4\rangle) \\ \frac{1}{\sqrt{3}}(|1\rangle + |3\rangle + |4\rangle), \frac{1}{\sqrt{3}}(|1\rangle + \alpha|3\rangle + |4\rangle), \frac{1}{\sqrt{3}}(|1\rangle + \alpha|3\rangle + \beta|4\rangle), \frac{1}{\sqrt{3}}(|1\rangle + \beta|3\rangle + \alpha|4\rangle) \\ \frac{1}{\sqrt{3}}(|2\rangle + |3\rangle + |4\rangle), \frac{1}{\sqrt{3}}(\alpha|2\rangle + |3\rangle + |4\rangle), \frac{1}{\sqrt{3}}(|2\rangle + \alpha|3\rangle + \beta|4\rangle), \frac{1}{\sqrt{3}}(|2\rangle + \beta|3\rangle + \alpha|4\rangle) \end{aligned}$$

Four-photon, two-dimensional tomography

The four photons are projected onto every combination of the listed 4 projections, leading to 256 measurements.

$$|1\rangle, |2\rangle, \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle), \frac{1}{\sqrt{2}}(|1\rangle + i|2\rangle)$$

Appendix B

B.1 English version of the French summary

Abstract

The exploitation of quantum mechanics has the potential to solve important challenges and introduce novel technologies enabling, among others, powerful computation, perfectly secure communications, as well as sensing and imaging with precision and resolutions not achievable by classical means. In order to make use of the unique properties of quantum physics, experimental platforms that provide access to, and control of quantum states, need to be developed. In particular, quantum systems composed of multiple parties are required, where each party is in a coherent superposition of states, and where multiple such parties are in a coherent superposition with each other (e.g. being entangled). Additionally, in order to make use of such entangled quantum systems, it is required that each individual party can be coherently controlled and measured. The usability and available quantum resource of the entangled system is related to its size, which is proportional to the number of entangled parties, as well as the dimensionality of the superposition each party is in. For this reason, there is an imminent demand to achieve, investigate, and improve the generation and control of large quantum systems, comprised of as many entangled parties as possible, where each party is in an ideally high-dimensional superposition. Many quantum platforms are currently subject to extensive research, such as trapped ions, superconducting circuits, defect centers in solid-state crystals, mechanical oscillators, and optical quantum states (i.e. photons). While all platforms provide distinct advantages (as well as challenges), optical photon states are of particular interest, because they can interact with other quantum systems, and can be transmitted over long distances while preserving their quantum coherence without the need for extensive shielding. A large variety of sources for optical quantum states has been demonstrated, however most such implementations suffer from high complexity, ultimately limiting their scalability. Integrated photonics has recently become a leading platform for the compact, cost-efficient, and stable generation and processing of optical quantum states. However, to date, integrated quantum sources have only been able to generate photon pairs, which operated on single channels, single polarization modes, and in the few realizations of on-chip entanglement, they were limited to two entangled parties, where each party was in a two-dimensional superposition. These limitations of current integrated photon sources arise in large parts from the exploited concepts to generate and control these states. In

particular, path-entangled sources are most commonly used, but increasing the quantum state complexity requires a significant, and very challenging increase in device complexity. There is therefore an urgent need to develop new concepts that can achieve the on-chip generation of large and complex photon states without increasing source complexity, while still enabling coherent quantum state control and detection.

In this work, we aim to address these limitations by making use of multiple frequency and temporal modes within a single spatial waveguide mode of an on-chip microring resonator. This approach is in contrast to more commonly used techniques relying on path entanglement, which exploits multiple spatial modes and generates entanglement by means of complex waveguide structures. Working within a single spatial mode, we achieve the first demonstration of pure heralded single photons and entangled photon pairs, which are emitted on multiple frequency modes from a single source. We then demonstrate the first generation of multi-photon entangled quantum states on a photonic chip. By making use of the spectral multi-mode nature of optical frequency combs, we achieve the generation of photon pairs in a coherent superposition of multiple frequency modes, representing the first generation of high-dimensional entangled states on a photonic chip.

Our work represents a significant step forward in the generation of complex quantum states on a photonic chip and in a single spatial mode. In addition, we demonstrated coherent quantum state control and manipulation for all generated states by means of standard telecommunications components, which could in the future be also integrated on the same chip together with the sources. Our results indicate that the exploitation of integrated optical frequency comb sources for the generation of optical quantum states can provide a scalable and practical platform for quantum technologies.

Background

In recent years, the field of quantum optics has become one of the most active avenues for fundamental research as well as practical applications. Based on the available and evolving technical capabilities for the generation, manipulation and detection of single photons, quantum optics nowadays enables a large variety of fundamental experiments. Indeed, many foundational tests of quantum mechanics have been implemented using photons. In addition, the possibility to

transmit photons by means of free-space optical links or using fiber networks has given birth to the field of quantum communications, which enables the secure exchange of information.

One of the most pronounced properties of light is that it can interact with other systems (in general, light can be emitted, absorbed or scattered, imprinting information of the system on the light), and therefore suggests a means to observe and investigate their properties. Indeed, imaging, sensing, microscopy and spectroscopy applications can all be improved by making use of the quantum properties of light. Additionally, photons can interact with, control, and entangle other quantum systems such as trapped ions, defect centers in solid-state crystals, and mechanical quantum systems, making quantum optics an indispensable tool for quantum science.

In general, optical quantum sources can be described as belonging to one of three main categories: sources of squeezed states, sources of single photons, sources of photon pairs.

Squeezed states are optical quantum states where the amplitude and phase noise of the state are at the limit given by the Heisenberg uncertainty limit. In such states, the amplitude or phase noise can be ‘squeezed’ several decibels below the classical limit at the expense of the complementary variable. Among several applications, such squeezed states can be exploited for continuous-variable quantum computation.

Single photon sources ideally only emit a single photon at a time, and can be realized e.g. using quantum dots, defect centers, single atoms, and single molecules. Current challenges, subject to extensive research, include the deterministic generation of on demand photons, as well as the generation of pure single-mode photons.

Photon pair sources ideally only emit two photons at the same time, and can be realized by means of nonlinear optical interactions. Sources of photon pairs can be used as so-called heralded single photon sources, where one of the two photons is detected, in turn deterministically predicting the presence of the other photon of the pair. Additionally, and due to the conservation of energy, momentum, and spin in the nonlinear generation process, photon pair sources can directly generate a large variety of different entangled states, and are therefore commonly used for applications relying on entanglement as a quantum resource.

While entangled photon pairs can already find direct applications, their usability as a quantum resource is somewhat limited. In order to increase the quantum state complexity, two main

approaches can be used: increasing the number of entangled photons, and/or increasing the dimensionality of the superposition the photons are in.

The generation of multi-photon entangled states can be achieved by interfering multiple photon pairs. Indeed, the concatenation of multiple two-photon sources has enabled the preparation of states consisting of as many as ten entangled photons. However, the use of solely the multi-photon approach has two main draw-backs. First, the source complexity increases significantly with every added photon pair, and second, the detection rate decreases with an increasing amount of entangled photons. Photons are ideally suited to achieve high-dimensional superpositions and entanglement, as photons can be entangled in many different degrees of freedom, such as spatial modes (e.g. orbital angular momentum and momentum-position) or frequency-temporal modes.

Most advanced sources of optical quantum states have in common that they mainly rely on free-space realizations. However, losses and implementation complexities (such as achieving perfect mode-overlap and phase stability in free-space setups) place significant challenges towards achieving complex quantum circuits. There is therefore a need to achieve stable and scalable platforms to generate and control complex multi-photon and high-dimensional quantum states.

With the goal to decrease the size of quantum sources, in recent years there has been an increased focus on achieving compact optical fiber-based sources, as well as integrated sources based on compact waveguide structures. Advances in the miniaturization of nonlinear crystals has led to the realization of compact and fiber-coupled sources based on traditional nonlinear crystals. In addition to coupling crystal-based sources to optical fibers, the intrinsic nonlinearity of optical fibers can be directly used to generate photon pairs. However, the relatively low generation rates together with high Raman photon noise, typically necessitates cryogenic cooling of fiber-based quantum sources. One of the largest advances in achieving compact and scalable quantum devices has been achieved by making use of on-chip photonics technology.

Integrated photonics makes use of compact, mainly semiconductor-based photonic chips, where optical waveguide structures can be realized using nanofabrication techniques. Integrated photonics was first used for linear optical devices, where single-mode optical waveguides were achieved in silicon photonic chips. Due to the small mode area of the optical fields, high optical field intensities can be reached, enabling the exploitation of nonlinear optical interactions.

Advancing further from linear and nonlinear applications, integrated photonics is becoming an important platform also for quantum optics. Indeed, the manipulation, generation, and detection of quantum states have been achieved. However, not all possible types of entanglement can be generated or exploited in integrated structures. For example, the mode dispersion and phase-matching condition in fibers and integrated waveguides usually prohibits the generation or exploitation of momentum-position entanglement, and orbital angular momentum is usually also not guided in such structures. Integrated sources of entangled photon pairs have mainly relied on polarization and path entanglement.

While polarization entanglement has been demonstrated in integrated structures, the entangled states have until now only been generated at the output of the chip. Due to polarization dispersion, entanglement is only maintained in very specialized integrated waveguides, which is why state manipulation of on-chip generated polarization entangled states has been performed using free-space devices. Path entanglement is currently one of the most exploited forms of entanglement in integrated structures, as it can be generated by concatenating multiple photon sources through optical waveguide couplers. Furthermore, path-entangled states can also be characterized with on-chip devices by means of beam splitters and phase shifters. However, for the path-entanglement concept, the device complexity/footprint scales with the state complexity, placing significant technological challenges to achieve higher source complexities such as high-dimensional entanglement.

For powerful future applications, multi-photon and/or high-dimensional quantum states will be required. However, neither multi-photon nor high-dimensional entangled states have been generated in integrated sources. Furthermore, there is no clear path to generate large quantum states in the near future using path or polarization entanglement. New concepts for on-chip quantum state generation have to therefore be developed, which can at the same time achieve high-dimensional quantum states and favourable scaling in terms of source complexity in terms of increasing the number of photons and state dimensionality. The optimum in terms of source complexity and footprint are sources that can generate complex quantum states within a single waveguide mode. Such quantum states will then also be compatible with the fiber telecommunications infrastructure, which provides an additional advantage.

Energy-time entangled states are well suited to be generated within a single spatial mode, and are therefore an ideal candidate to achieve integrated quantum states with low source complexity. Indeed, the first realizations of energy-time entanglement in on-chip sources have been achieved. However, while the sources were more compact, realizations of energy-time entanglement in on-chip devices prior to this work did not provide more complex quantum states compared to polarization or path-entangled schemes. In particular, previous realizations only exploited single sets of temporal and frequency modes achieving either two-dimensional, or continuous variable states where individual quantum modes cannot be addressed individually.

Towards the goal of increasing the number of temporal and frequency modes accessible for quantum state preparation, the generation of optical quantum states has recently begun to exploit optical frequency comb concepts. Frequency combs are multi-frequency light sources characterized by the presence of multiple, spectrally-equidistant, phase-locked frequency modes. In the classical domain, frequency combs have revolutionized sensing and metrology, due to the high precision and stability of these light sources. In recent years, also the quantum correlations within optical frequency combs have been investigated. Indeed, it has been shown that complex quantum correlations between frequency comb modes can be achieved in synchronously pumped, as well as bi-chromatically excited optical parametric oscillator (OPO) frequency combs. However, all performed investigations of quantum frequency combs were limited to squeezed states, and only moderate amounts of squeezing (< 10 dB) was achieved due to high optical losses. In addition, the spectral modes are separated by only a few MHz in the demonstrated quantum frequency combs, making single spectral modes inaccessible (with most optical filtering techniques). Quantum state manipulation is therefore not fully achievable in such systems.

Another approach towards realizing frequency combs relies on the use of integrated photonics structures, for which classical on-chip frequency comb generation is a subject of extensive research. Such microresonator-based frequency combs are able to generate very broadband coherent light emission with spectral mode separation in the tens to hundreds of GHz, enabling easy access to individual frequency modes. However, all investigations to date of these on-chip combs have focused on their classical properties and integrated frequency comb sources have not been used for quantum state generation.

In the context of the work presented in this thesis, we proposed to use integrated optical frequency combs as quantum sources, as they have the potential to address many limitations of current integrated quantum sources. First, the device fabrication and optimization is already at an advanced stage due to the rich history of classical comb realizations. Additionally, frequency comb sources are based on a single microring resonator, and are therefore much less complicated structures than for example path entangled sources. Second, the use of on-chip combs goes well above the advantage of simply miniaturizing quantum frequency combs, which have shown the generation of very complex squeezed quantum states. Indeed, the larger frequency mode spacing in integrated comb sources provides for the first time easy access to individual frequency modes by means of optical filters. This enables also the investigation of other quantum states other than squeezed states, and could lead to the implementation of quantum operations in the frequency domain. Integrated frequency combs are therefore ideal candidates for the generation of scalable and complex quantum states in compact sources, where the quantum state complexity does not scale with source complexity, and all quantum states are emitted within a single spatial mode.

This work represents the first systematic investigation of the quantum properties of photons generated in integrated optical frequency combs. Instead of investigating the generation of squeezed states however, we focus on the generation of photon pairs, and particularly of energy-time entangled states.

The three different types of energy-time entanglement in terms of their temporal and frequency modes, namely continuous, time-bin and frequency-bin entanglement. If a spectrally-broadband photon pair is generated, the photons can be generated in a coherent superposition of a continuum of temporal and frequency modes, forming continuous-variable entangled states. However, since it is very difficult (or even impossible) to address the individual quantum modes of continuous variable states, their implementation into applications is very challenging. For this reason it is often preferred to work with discrete quantum states, especially for the implementation of quantum gates. The discrete form of energy-time entanglement (where discrete temporal and/or frequency modes are present) can be categorized into time-bin and frequency-bin entanglement, depending on which variable (temporal or frequency mode) can be controlled. In particular, if the photons are in a superposition of two (or more) distinct temporal modes, which do not overlap and can be

manipulated independently, the state is referred to as time-bin entangled. If the two (or more) frequency modes do not overlap and are spectrally separated far enough so that they can be selected and manipulated by means of optical filters, the quantum state is referred to as frequency-bin entangled.

Results: Photon pair generation with integrated optical frequency combs

Correlated photons form the most basic quantum states that can be generated through spontaneous nonlinear optical processes. These states form the foundation for more complex entangled quantum states. Nevertheless, even without the presence of entanglement, the generation of two correlated photons can already be exploited for quantum applications. Mainly the photons' correlation in arrival time allows to herald the presence of a single photon by detecting the other. Such a heralded single photon source can for example be implemented into quantum communication protocols, which rely on imprinting information on single photons.

To demonstrate the first multiplexed integrated source of heralded single photons, we chose to use an integrated microring resonator with 200 GHz free spectral range (FSR), 1.3 million Q-factor and very low and anomalous dispersion. To show the frequency-multiplexed nature of the photon source, five different signal/idler wavelength pairs symmetrically located around the excitation wavelength were selected via telecommunication filters. The individual frequency channels were then sent to single photon detectors for coincidence detection. Clear coincidence peaks were measured on all five symmetric channel pairs, while no coincidences were measured between non-symmetric channel combinations. In order to demonstrate heralded single photons, we measure the heralded second-order coherence of the photons. For a perfect heralded single photon source only one idler photon is expected to be present upon detection of a signal photon. This characteristic manifests itself in a dip in the second-order coherence function, where a dip below 0.5 confirms the quantum nature of the photon. A clear dip in the heralded second-order coherence was measured, confirming that the source operated in the single-photon regime. Using pulsed excitation and increasing the pump power to increase the probability to generate multiple photon pairs, we measured the second-order coherence of the single photon to extract the effective number of modes. We observed a bunching peak as high as 1.537, corresponding to $N = 1.86$ effective modes, for the 1.3M Q-factor ring. The non-ideal value of below was attributed to a split resonance caused by back-reflection within the ring. Repeating the measurements with a different resonator with a lower

Q-factor of around 240,000, perfect photon purity within the measurement uncertainty was measured, showing that in this case our lower-Q microring resonator is better suited for the quantum experiments. All consequent experiments were therefore performed with the low-Q ring, providing better photon purity.

In a next step, we investigated the spectral bandwidth of the quantum frequency comb. The resonator was pumped using a spectrally-filtered mode-locked lasers, and the single photon count rate as a function of wavelength was measured using a high-resolution tunable C-band wavelength filter, as well as a grating-based spectrum analyzer. From these measurements, the photon pair production rate per pulse was calculated. It can be clearly seen that a very broad frequency comb of photons is emitted, covering the full S, C, and L International Telecommunication Union (ITU) bands (wavelengths ranging from 1470 to 1620 nm). The SFWM process generates a spectrum symmetric in frequency, while the spectral asymmetry in the measured photon counts can be explained by Raman scattering, which could be further reduced by cooling the chip. Due to the broad phase-matching condition, achieved through the close-to-zero waveguide dispersion, the emitted comb exhibits a very flat and broadband spectrum with close to uniform pair production rates, ranging from 0.02 to 0.04 pairs per pulse, over the full measured comb.

In addition to the generation of heralded single photons on a single polarization, we also investigated the generation of cross-polarized photon pairs. Polarization is a commonly used degree of freedom for quantum information processing. However, making use of polarization in integrated structures is very challenging due to the commonly disadvantageous polarization mode dispersion of waveguides, which are usually only designed for a single polarization. In contrast to other on-chip devices based on silicon or silicon nitride, the here used silicon oxynitride waveguides have similar dispersion properties for the two polarization modes, which is enabled by the almost square waveguide dimensions ($1.5 \times 1.45 \mu\text{m}$). Because of these waveguide properties, we were able to exploit both polarization modes for nonlinear optical interactions. To achieve cross-polarized photon pair generation, we developed a novel scheme to suppress stimulated FWM between the two excitation fields by choosing a resonator that has a frequency offset between the TE and TM resonances, while having almost identical FSRs of both polarization modes, thus allowing cross-polarized spontaneous FWM to take place at targeted resonances. To perform photon coincidence measurements, the generated photons were separated by a polarizing beam splitter and detected

with single-photon detectors. A clear coincidence peak with a CAR of up to 12 without any background subtraction was measured. The nature of the generation process was confirmed by means of power scaling measurements, confirming the first direct generation of cross-polarized photon pairs on a photonic chip. To further characterize the performance of our device as a single-photon source, we measured the (heralded) second-order coherence of the photons. A clear dip in the heralded second-order coherence was recorded, showing that the source operates in a non-classical single-photon regime, while the second-order coherence shows a clear peak with a maximum of 2.01 ± 0.03 , resulting in $N_{\text{eff}} = 0.99 \pm 0.03$ effective modes, underlining the high purity of the source. Finally, the production of cross-polarized photon pairs is not limited to only the adjacent resonances. Indeed, we measured cross-polarized photon pairs over 12 resonance couples, limited by the available filters.

Results: Entanglement generation with integrated frequency combs

Having demonstrated the generation of correlated photon pairs, the next important challenge is to generate entangled two-photon states, as these are the foundation for most applications of quantum information processing. Integrated sources have managed to produce different types of entangled quantum states such as path entanglement, polarization entanglement, as well as continuous variable energy-time entanglement. Most sources have mainly relied on simple waveguides, photonic crystal waveguides and cavities, as well as microring resonators. There is particular interest in the generation of discrete variable entangled quantum states, as they provide easy manipulation and access to the quantum modes. For this reason, this work focuses on the realization of both discrete versions of energy-time entanglement, called time-bin and frequency-bin entanglement are investigated.

Time-bin entanglement is very commonly used for applications in quantum communications, but it can also be exploited for quantum computation protocols. Time-bin entanglement can be generated by exciting a nonlinear source with coherent double pulses. Such pulses can for example be generated using an unbalanced interferometer, where the path length difference is much larger than the coherence length of the input pulse (therefore no classical, single photon interference can be measured with such an interferometer). For a single input pulse, such an interferometer will generate two pulses at the output, which are delayed in time (defined by the path length difference of the interferometer) and which have a fixed phase relationship (defined by the phase of the

interferometer). If a nonlinear source is excited with such a double pulse, and the power is chosen in such a way that the probability can be neglected that each of the two pulses generates a photon pair, then a photon pair generated from such a double pulse is generated in a coherent superposition of two temporal modes. In particular, we generated these double pulses by passing the output of a pulsed pump laser through a stabilized unbalanced fiber interferometer with a 11.4 ns delay – significantly larger than the pulse duration of the laser – generating double pulses with a defined relative phase difference. These double pulses were then coupled into the integrated microring resonator. The pump power was chosen in such a way that the probability of creating a photon pair from both pulses simultaneously was sufficiently low to be negligible. This configuration led to the generation of time-bin entangled qubit states. Most importantly, these entangled qubits are generated over all the microring resonances, thus leading to a quantum frequency comb of time-bin entangled photon pairs.

To characterize the entangled photons, they were each passed individually through different interferometers with an imbalance identical to that used for the pump laser. Changing the interferometer phases, different projection measurement can be performed to measure quantum interference between both photons with respect to the set interferometer phases. This was used to violate a Bell inequality on 5 different channel pairs symmetric to the excitation wavelength. In addition to measuring Bell inequality violations, we performed quantum state tomography to measure the state density matrix. We first measure the two-photon qubits generated on comb lines symmetric with respect to the pump wavelength and find a fidelity of 96%, confirming that the generated quantum states are of high quality and very close to the ideal entangled state. We then repeated the tomography measurement after adding 40 km of fiber, to show that the entanglement is preserved after long fiber propagation (fidelity of 87% without compensation for losses), highlighting the immediate applicability of the generated two-photon qubits for quantum communications.

Compared to time-bin entanglement, frequency-bin entanglement is very rarely used for applications. This arises from two main issues related to their generation and detection. In particular the required projection measurements in the frequency domain are challenging because a change in photon frequency is required. In order to efficiently exploit this type of entanglement, we developed a novel generation, manipulation and detection approach. Considering the state

generation, integrated microring resonators are ideally suited to generate frequency-entangled states. Indeed, if the phase-matching condition is such that photon pairs can be generated over multiple resonances symmetric to the pump, photon pairs are intrinsically generated into a frequency-bin entangled states. The advantage in the generation is threefold: First, the free-spectral range of the microring resonators can be chosen in the GHz range, making it possible to select individual frequency modes with standard telecommunications filters. Second, the photons are directly generated into the frequency modes of the resonators, so no photons are lost due to external filtering. And third, the photons can be generated into pure states, if a pulsed pump configuration is chosen. This means that true frequency-bin entangled states can be generated, where each frequency mode is a pure state.

In particular, to generate such a frequency entangled state, a spectrally-filtered mode-locked laser is used to excite a single resonance of the microring at ~ 1550 nm wavelength, in turn producing pairs of correlated signal and idler photons spectrally-symmetric to the excitation field covering multiple resonances. To characterize the dimensionality of the generated state, two different experiments were performed. First, using a computer controlled programmable filter, the joint spectral intensity was measured, describing the two-photon state's frequency distribution. The joint spectral intensity can be used to determine the lower bound for the Schmidt number, which provides a measure for the dimensionality of the system. We extracted the lower bound for the Schmidt number to be 9.4 for 10 selected modes. Second, we performed time-domain measurements and extract the maximal dimensionality of the state. In particular, the maximum of the second-order coherence function is directly related to the number of effective modes. Using this technique, the maximal dimensionality was measured for states with up to 10 selected modes, leading to a maximal state Schmidt number of 10.45 ± 0.53 . As the lower and upper bound coincide and scale linearly with the number of selected modes, we conclude that the number of significant orthogonal modes is indeed directly related to the number of selected frequency modes.

Even though the generation of frequency-bin entangled photon pairs can be almost naturally achieved using nonlinear integrated source, the issue to efficiently measuring these states remains a challenge. To address this issue, we developed a new coherent control scheme, based on fiber-coupled spatial light modulators and electro-optical phase modulation. In general, the exploitation of quDit states for quantum information processing motivates the need for high-dimensional

operations that enable access to multiple modes with a minimum of components. Through the merging of the fields of quantum state manipulation and ultrafast optical signal processing, state-of-the-art, yet established radio-frequency (RF) and telecommunications technologies can be used to simultaneously address multiple frequency modes and enact high-dimensional coherent control and optical gate operations, achieving a scalable and practical quantum platform. In particular, optical phase gates for manipulating high-dimensional quDits can be directly implemented using programmable phase filters. The coherent mixing of multiple modes (a high-dimensional ‘beam splitting’ in the frequency domain) can be achieved through deterministic frequency conversion in electro-optic modulators. More importantly, as these two components are electronically-tunable, versatile operations can be performed in real-time without modifying the experimental setup (or the device), by simply adjusting the filter control and electrical modulation signal. By combining these elements, basic high-dimensional photon operations can be implemented in a single and robust spatial mode.

To implement this concept, a setup to perform basic gate operations for coherent state control was developed using a configuration composed of two programmable filters and one electro-optic phase modulator. This manipulation scheme allowed us to design well-defined quantum operations, which can be exploited for projection measurements. We used these projection measurements to perform Bell inequality violations and quantum state tomography. The measured quantum interference for the $D = 2, 3$, and 4 resulted in raw visibilities of 83.6%, 86.6%, and 86% (without background subtraction). These visibilities exceed the values of 71%, 77%, and 82%, respectively, thus violating Bell inequalities for two, three, and four-dimensional states and demonstrating their entanglement. The topographically reconstructed density matrices show very good agreement with the ideal states, resulting in fidelities of 88.5%, 80.9%, and 69.5% respectively.

Results: Multi-partite entanglement in on-chip optical frequency combs

The usable quantum resource of entangled states for quantum information processing is related to the size of the Hilbert space that the state occupies. In order to perform meaningful quantum information processing tasks (e.g. factorizing numbers, searching in datasets), very large Hilbert spaces (corresponding to at least 30 to 50 qubits) are required. In general, the Hilbert space of a state comprised of N D -dimensional entangled parties has a state dimensionality of D^N . The

ultimate goal would therefore be to generate maximally entangled, high-dimensional multi-partite states, which span large Hilbert spaces, where the full dimensionality of the state can be effectively controlled and exploited.

To demonstrate the first realization of multi-photon entangled quantum states generated on a photonic chip, we chose time-bin entangled product states generated within quantum optical frequency combs. Product states are one of the most basic multi-partite quantum states. As the name indicates, this state can be described as the product of two or more quantum states with a lower amount of parties involved. In particular, four-photon product state can be described as the product of two individual two-photon states. The generation of product states is of high interest as they form an important starting point to generate maximally entangled states such as GHZ or cluster states, even though they do not carry genuine multi-partite entanglement themselves. By selecting two different signal-idler pairs symmetric to the excitation frequency, it is possible to generate individual two-photon qubit states. By post-selecting four-photon events with one photon on each frequency channel, we can detect and characterize the generated four-photon entangled product state. It is worth noting that for the generation of a four-photon state, the coherence length of both generated photon pairs has to be the same and matched to the excitation field's coherence time. This requirement is intrinsically fulfilled through the resonant characteristics (i.e. equal resonance bandwidths) of the ring cavity in combination with the excitation scheme described above. Setting the pump power to 1.5 mW, a quadruple detection rate of 0.17 Hz was measured, leading to a calculated generation rate of 135 kHz – by taking into account the system and detection losses of 14.75 dB. To confirm the presence of four-photon entanglement, four-photon quantum interference measurements were performed. Indeed, four-photon interference is in general not present for two completely independent two-photon qubit states. The measurement clearly follows the expected interference pattern for product states, having a visibility of 89% without compensation for background noise or losses. To fully characterize the entangled states we performed quantum state tomography, and obtained a fidelity of 64% without compensation for background noise or interferometer imperfections. These results represent the first demonstration of any type of multi-photon entanglement on a photonic chip, and therefore open up the possibility to build on these results to achieve more complex states.

A different and novel approach to generate multi-mode entangled states is investigated, based on the polarization mode dispersion of the integrated microring resonator. This approach makes use of multi-mode correlations, which are generated by the simultaneous exploitation of different nonlinear interactions, overlapping within the same frequency modes. In particular, both co- and cross-polarized SFWM are phase matched in the chosen microring resonator. The modes of the two SFWM processes spectrally overlapped, thus creating a multi-correlated set between four frequency modes. To characterize the generated state and to determine the coupling network of the nonlinear interactions, photon coincidences between all combinations of 24 cavity modes were measured (6 signal and idler modes for both TE and TM polarizations), limited by the spectral bandwidth of the filter used to route different frequency modes to different detectors. Such measurements were performed for both single, as well as for the bi-chromatically pumped configuration. While only correlations in the TE mode are observed in the single frequency pumped quantum frequency comb, clear correlations are measured between TE, TM, and TE/TM mode in the bi-chromatically pumped frequency comb, separated into six individual two-photon four-mode multi-correlations. In order to show entanglement between combinations of these four modes, we use a pulsed excitation to pump both the TE and TM mode with double pulses. If two-photon coincidences are selected, we confirm that each mode is entangled with two other frequency modes, forming a multi-mode entangled optical state. The visibilities of all four quantum interference measurements exceed the minimum requirement to violate a two-mode Bell-like inequality, confirming entanglement in all four two-mode combinations.

Conclusions and perspectives

The generation of optical quantum states with integrated microring resonators based frequency combs sources was investigated. In particular, we successfully achieved the generation of heralded single photons, entangled photon pairs, as well as multi-partite entangled quantum states. All quantum states were realized within a single spatial mode, exploiting the temporal and frequency domain as a quantum resource. The single mode operation in particular enables highly compact sources and compatibility with fiber technology, enabling the use of telecommunications infrastructure and optical signal processing tools for quantum applications. Our results show that there is a large potential for quantum state generated using on-chip optical frequency comb sources, but that there is also significant work to be done to improve these devices in order to make them a viable candidate for practical quantum applications.

We have achieved the first generation of wavelength division multiplexed photon pairs generated from a single integrated source. The multiplexed nature could find applications in quantum communications platforms to achieve higher bit rates or to enable communications between multiple parties using the same source. In addition, we have demonstrated high photon purity and narrow spectral bandwidth, resulting in temporally long single photons. The high photon purity is highly desired for quantum communications protocols, as well as linear optical quantum computation (LOQC), where pure photons interfere at beam splitters via HOM interference. Additionally, the narrow spectral bandwidth in the order of 100 MHz is comparable with those of atomic quantum memories, which hints a potential compatibility. However, current memories operate at different wavelengths, which would require the development of telecom-band memories, or methods for efficiently changing the photon frequency from e.g. infrared to visible wavelength. What limits the immediate usability of the presented heralded single photon sources are the high losses, resulting low CAR and heralding efficiencies. A large portion of these losses is caused by the resonator structure (4-port resonators), which implies an intrinsic 6 dB loss (as photons can exit different ports). This could be addressed by the use of two port resonators. Additionally, the coupling losses off the chip into optical fibers (1.5 to 2dB) and losses of telecommunication filters (2-6 dB depending on the type of filter) significantly reduce the CAR, count rate and heralding efficiency. These limitations will have to be addressed before the device becomes a viable competitor as a source for heralded single photons.

We have achieved the first generation of both time-bin, as well as frequency-bin entanglement using integrated microring resonators. The results also represent the first realization of multi-channel entanglement generation, as well as the first high-dimensional entanglement generated on a chip. Both achieved forms of entanglement can be used for quantum communications, where we show that the entanglement is preserved over significant fiber propagation. Time-bin entanglement is particularly suited for quantum communications, as the quantum state is resistant to fiber dispersion as well as polarization distortion due to fiber birefringence. Frequency-bin entanglement can also be used for quantum communications, where especially the high-dimensional nature can be used to increase the bit rate per photon. Unfortunately, the frequency-bin entangled states are sensitive to fiber dispersion, where different frequency components propagate with different velocities, resulting in different arrival times and eventually loss of indistinguishability. However,

we have shown that the quantum coherence can be retrieved if the dispersion is compensated, e.g. by using dispersion compensating fibers or chirped fiber Bragg gratings. In addition to communications, both time-bin and frequency-bin entangled quantum states can be used for optical quantum computation. Models to perform LOQC with time-bin, or frequency-bin entangled states have been experimentally demonstrated, or theoretically discussed, respectively.

We achieved the first generation of four-photon entanglement on a photonic chip by making use of multiple frequency modes together with time-bin entanglement. The demonstrated states are four-photon product states, achieved by the simultaneous generation of two Bell states on different frequency modes. Product states do not carry genuine four-photon entanglement, but they are an important step towards realizing different types of maximally entangled multi-photon states. For example, by making use of selective beam splitters, product states can be turned into GHZ states, however such beam splitters have not yet been achieved for time-bin entanglement. The realization of product states are never the less an important step into the right direction and show that multi-photon entanglement can be generated using integrated sources.

We achieved the first realization of multi-mode quantum states in integrated frequency comb sources. In particular, the ability to enable nonlinear interactions between two different mode families was used to generate quantum states with entanglement between sets of different modes. The presented concept could in the future be further exploited to generate a rich variety quantum states. The scheme demonstrated here using polarization modes could be extended to higher order waveguide modes, to achieve interaction between 2,3,4 or more different mode families. In addition to extending this to multi-photon states, such nonlinear coupling between mode families could be very promising for the generation of squeezed quantum states. In particular, it has been shown that architectures can generate continuous variable squeezed optical cluster states by making use of the overlap of nonlinear processes between two different mode families forming a set of four modes, very similar to the here achieved architectures.

Finally, a very promising route to achieve complex high-dimensional multi-partite quantum states is to exploit so-called hyper-entangled quantum states. Such states are simultaneously entangled in multiple degrees of freedom and it becomes possible to achieve multi-partite quantum states by means of only two photons. This has been in the past achieved using multiple degrees of freedom

such as polarization, spatial modes, and energy-time. Using the integrated approach developed here, it will be possible to simultaneously achieve (and measure) time-bin and frequency-bin entanglement. The experimental realization will be the same as the generation of time-bin and frequency-bin entanglement. In particular, exciting the ring with double pulses will generate time-bin entanglement, and the broad phase-matching condition will at the same time lead to frequency-bin entanglement. It is therefore expected that hyper-entangled states can be directly generated without significantly increasing the generation process. To confirm this type of entanglement, simultaneous measurement temporal and frequency correlations will be required, which can be performed by the combination of interferometers and electro-optic phase modulation. Furthermore, this scheme could be readily extended to higher dimensions by considering multiple frequency modes together with the excitation with three, four or more coherent optical pulses. Remarkably, such hyper-entangled states would be more tolerant to losses, as only two photons are used to achieve large multi-partite quantum states. This in turn would drastically increase the detection rate and therefore also the usability of these quantum states.

In conclusion, we confirm the significant potential of microring resonator based integrated quantum sources for practical applications such as quantum communications, sensing and in the future also computation. In addition to the here-presented measurements of single photons, energy-time entangled photon pairs and multi-photon product states of Bell states, there is a clear path to achieve even more complex quantum states without the need to significantly increase the source complexity. This is achieved by making use of correlations in the frequency- and time-domain, which are present in a single spatial waveguide mode, and therefore minimize source footprint and complexity.