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SUNDRY AVERAGES METHOD (SAM)
FOR ESTIMATING PARAMETERS OF
THE LOG-PEARSON TYPE 3 DISTRIBUTION

by

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ABSTRACT

The log Pearson type 3 distribution has been recommended in the U.S. and Australia for representing annual flood flow series. Many methods have been proposed for fitting this distribution and among these the method of moments has been one of the most popular. This method is very broad and has already been applied to (1) the sample of observed (raw) flood values, (2) the sample of logarithmically transformed values and (3) a combination of raw and logarithmically transformed values. Some investigators have felt unconfortable about: (1) the use of transformed flood data in flood frequency estimation and (2) the use of high moments of the sample (moments of order 3 or more). In the present study we present a method which applies directly to the observed flood sample but which uses moments of low order, namely the harmonic mean (moment of order -1), geometric mean (moment of order "quasi zero") and arithmetic mean (moment of order 1). We call this method the "Sundry Averages Method" (S.A.M.) which is a special case of the "Generalized method of moments" (GMM) discussed by Ashkar and Bobée (1986). The method of Bobée (1975) which uses moments of order 1, 2 and 3 of the observed flood values (moments in real space) and one of the methods of "mixed moments" introduced by Rao (1980) which uses two moments (of order 1 and 2) in real space and one moment (of order 1) in log space are also special cases of the GMM. Each of these methods is expected to be particularly adequate for estimating river flows within a particular range of return periods (high flows, intermediate flows, low flows, etc..). A study is now being undertaken to show this result.

INTRODUCTION

Following the recommendation of the hydrology committee of the United States Water Resources Council (WRC, 1967; BENSON, 1968), the log-Pearson type 3 (LP) distribution has been largely used in many countries and especially in North America and Australia (I.E.A., 1977) for representing flood flows.

In recent years several methods of fitting this distribution have been proposed. In this paper we intend to review these methods from a critical point of view and to bring out the historical evolution and the improvements brought by the different proposed methods.

Also in the second part of the paper we shall propose a new method "Sundry averages method" based on the use of the three means (arithmetic, harmonic and geometric) for estimating the three parameters of the LP distribution.

This method as we shall show, follows logically the historical evolution of some of the recently proposed methods of fitting the LP distribution.

METHODS OF FITTING THE LP DISTRIBUTION: CRITICAL REVIEW

Before fitting a statistical distribution to a flood sample it is necessary to verify that the flood information is a reliable and representative time sample of random, independent and homogeneous events. It is then important to test the randomness, the independence and the homogeneity of the elements of the sample. These points are largely discussed elsewhere (Bobée and Ashkar 1986, Bobée 1976, WRC 1976).

Here we assume that these necessary conditions are respected for the sample of maximum annual flood flows and that a statistical distribution can be fitted to these data.

Our focus will be the LP distribution for which the first method of fitting was proposed when the recommendation of the systematic use of the distribution in the U.S. was made (WRC 1967, Benson 1968). This method consists in fitting three moments of the Pearson type 3 (P) distribution to the sample of logarithmically transformed data. This is based on the fact that: if X follows an LP distribution with parameters α, λ and m then Y = log X follows a P distribution with parameters α, λ and m. The moments considered in the WRC method are the arithmetic mean, the variance and the coefficient of skewness of the P distribution. This method gives the same weight to the logs of the flood values (y_i) but not to the flood values themselves (x_i).

For this reason Bobée (1975) proposed a method of moments which preserves the first three non-central moments of the LP variable, X (called

Bobée's method). This method conserves the moments of the sample in real space instead of the moments of the logarithmically transformed data and seems to be better suited for the estimation of events with a high return period.

The computation of var X_T using Bobée's method was done by Bobée and Boucher (1981) using the first 3 non central moments of X and by Hoshi (1979) for a modified version of the method which uses the mean, variance and skewness of X. Another class of methods so called "mixed moments" has been proposed by Rao (1980) and studied by Phien and Hsu (1985). These methods consider:

- The mean $\mu_1'(y)$ and the variance $\mu_2(y)$ of the variate Y = log X which follows a P distribution (moments in log space).

- the mean $\mu_1'(x)$ and the variance $\mu_2(x)$ of the variate X which follows an LP distribution (moments in real space).

In this manner one has four (4) moments and there are four ways of choosing three out of these four moments. The computation of var X_T for these methods has been given by Phien and Hsu (1985). The method which seems to perform best among these four methods is the one which involves:

· $\mu_1'(y)$, $\mu_1'(x)$ and $\mu_2(x)$; this method is called MM1 by Rao (1980).

However, it has been pointed out by Ashkar and Bobée(1986a) that:

- $\mu_1'(y)$ is in fact the logarithm of the geometric mean of the observed sample $x_1, \ldots, x_N$.

- based on Kendall and Stuart (1963) the geometric mean can be regarded as the moment of order "quasi zero", which for simplicity can be denoted by $\mu_{\underline{0}}'(x)$ (with a dash under the zero) (see equation 2 of the present study and Ashkar and Bobée, 1986b). All that the method MM1 calls for, therefore, are the moments $\mu_{\underline{0}}'(x)$, $\mu_1'(x)$, $\mu_2(x)$ all of which are moments of X (moments in real space) and hence this method cannot strictly be called MIXED-MOMENTS.

Using as criterion of comparison the asymptotic variance of the T-year event X_T (event corresponding to a return period T) Phien and Hsu (1985) conclude that MM1 performs better than Bobée's method. Ashkar and Bobée (1986a) show that this criterion can be misleading for small sample sizes as encountered in hydrology, and reach opposite conclusions by considering other criteria (observed standard error, and relative bias).

However in a general manner considering the asymptotic variance of X_T it appears useful to consider moments of order as low as possible at least for some values and not necessarily all possible values of return period T. With this in mind we have been led to propose a new method based on:

- The arithmetic mean (order 1)
- The geometric mean (order quasi zero)
- The harmonic mean (order-1)

This method calling for three means is termed the "SUNDRY AVERAGES METHOD "(S.A.M.).

THE SUNDRY AVERAGES METHOD (S.A.M.) APPLIED TO THE LOG-PEARSON TYPE 3 DISTRIBUTION

For a sample x_1, --- x_i, ..., x_N of size N consider the quantity

$$A(r) = \left| \frac{1}{N} \sum_{i=1}^{N} x_i^r \right|^{1/r} \quad (1)$$

if r = 1 we obtain the arithmetic mean

r = -1 we obtain the harmonic mean

It can be shown (Kendall and Stuart, 1963) that:

$$\cdot \lim_{r \to 0} \text{Log}_a [A(r)] = \frac{1}{N} \sum_{i=1}^{N} \text{Log}_a x_i = \text{Log}_a G \quad (2)$$

or

$$\cdot \lim_{r \to 0} A(r) = G$$

where G is the geometric mean of the sample, which therefore can be regarded as the moment of order "quasi zero"

· $A(1) \geqq A(0) \geqq A(-1)$ which represents the classical inequality between the three means (arithmetic, geometric and harmonic)

We consider now the sundry averages method (S.A.M.) applied to a sample (x_1, ---, x_i, ---, x_N) of size N drawn from a log-Pearson type 3 distribution. The variable X has an LP (Log-Pearson type 3) distribution if $Y = \log_a X$ has a P (Pearson type 3) distribution.

We consider the general case of a logarithmic transformation with base a, in practice it is the common logarithm (base 10) and the natural logarithm (base e) which are most frequently used. The moment of order r about the origin $(\mu_r')_L$ of the LP distribution with parameters α, λ and m is given (Bobée, 1975) by:

$$(\mu_r')_L = \frac{e^{mr/k}}{(1 - r/\beta)^{\lambda}} \qquad (3)$$

where:

$$\beta = \alpha k \qquad \text{with } k = 1/\mathrm{Log}_e a = \mathrm{Log}_a e$$

Relationship (3) is valid for all real numbers r as long as the moment $(\mu_r')_L$ is defined. This moment is defined only if $(1- \frac{r}{\beta}) \geq 0$, that is:

- if $\beta \geq 0$: the moments are defined up to order $r \leq \beta$

- if $\beta \leq 0$: the moments are defined for orders $r \geq \beta$

In the proposed method (SAM) we consider moments of order $r = 1$ and $r = -1$. It follows that :

- if $\beta \geq 0$ we should have $\beta \geq 1$
- if $\beta \leq 0$ we should have $\beta \leq -1$

Putting these two conditions together means that for SAM to be applicable we should have $|\beta| \geq 1$.

Piegorsch and Casella (1985) give some general conditions for the existence of the first negative moment (r = -1) independantly of the distribution considered. These conditions lead to the same conclusions that have just been presented for the LP distribution.

It can be noted that in the case of the method MM1 (Rao, 1980) the condition of existence of the moments considered (r = 1,2) implies that $\beta \geq 2$ or $\beta \leq 0$.

When fitting an LP distribution with parameters α, λ and m to a sample x_1, ---, x_N, the three equations of the method of moments (SAM) are given by:

arithmetic mean: $$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i = (\mu_1')_L = \frac{e^{m/k}}{(1 - \frac{1}{\beta})^{\lambda}} \qquad (4)$$

harmonic mean : $$\frac{1}{H} = \frac{1}{N} \sum_{i=1}^{N} 1/x_i = (\mu_{-1}')_L = \frac{e^{-m/k}}{(1 + \frac{1}{\beta})^{\lambda}} \qquad (5)$$

geometric mean : $$\mathrm{Log}_a\, G = \frac{1}{N} \sum_{i=1}^{N} \mathrm{Log}_a x_i = \frac{1}{N} \sum_{i=1}^{N} y_i = m + \frac{\lambda}{\alpha} \qquad (6)$$

Equations (4) and (5) are obtained from relationship (3) by putting $r = 1$ and $r = -1$ respectively. Equation (6) is obtained, considering that $y_i = \log_a x_i$ follows a P distribution with parameters α, λ and m and that the mean of the P distribution $(m + \frac{\lambda}{\alpha})$ is estimated by $\frac{1}{N} \sum y_i = \frac{1}{N} \sum_{i=1}^{N} \mathrm{Log}_a x_i$. The parameter β appearing in (4) and (5) is given by:

$$\beta = \alpha k \quad \text{with } k = (\mathrm{Log}_e a)^{-1} = \mathrm{Log}_a e$$

After some computations it is possible to show that the system of equations (4) (5) and (6) is equivalent to equations (7) (8) and (9) such that :

$$f(\alpha) = \frac{\mathrm{Log}_a \left(1 - \frac{1}{\alpha k}\right) + \frac{1}{\alpha}}{\mathrm{Log}_a \left(1 - \frac{1}{\alpha^2 k^2}\right)} - \frac{\mathrm{Log}_a \bar{x} - \mathrm{Log}_a G}{\mathrm{Log}_a \bar{x} - \mathrm{Log}_a H} = 0 \qquad (7)$$

$$\lambda = \frac{\mathrm{Log}_a H - \mathrm{Log}_a \bar{x}}{\mathrm{Log}_a [1 - 1/\alpha^2 k^2]} \qquad (8)$$

$$m = \mathrm{Log}_a G - \frac{\lambda}{\alpha} \qquad (9)$$

Equation (7) can be solved for α by a Newton-Raphson method, considering $f'(\alpha)$ such that:

$$f'(\alpha) = \frac{\mathrm{Log}_a B}{\alpha^2} \left(\frac{1}{C} - 1\right) - \frac{2}{k\alpha^3 B} \left(\frac{1}{\alpha} + \mathrm{Log}_a C\right)$$

with:

$$B = \left(1 - \frac{1}{\alpha^2 k^2}\right) \text{ and } C = \left(1 - \frac{1}{\alpha k}\right)$$

After α is estimated, equations (8) and (9) can be used to calculate λ and m.

COMPUTATION OF VAR (X_T)

Ashkar and Bobée (1986b) presented a general method for computing Var(X_T) when the parameters of the LP distribution are estimated by what was called a "generalized " method of moments GMM (s, t, u). Bobée's method or GMM (1, 2, 3), the method MM1, or GMM (0, 1, 2), and the method SAM, or GMM (-1, 0, 1), are all special cases of GMM (s, t, u). In the case of SAM we present the following example in which we shall refer frequently to (Ashkar and Bobée, 1986b).

EXAMPLE

Suppose that a sample of annual flood maxima of size:

$$N = 50 \qquad (10)$$

is available, and assume that it comes from an LP distribution (X) such that $Y = \log_{10} X$ is distributed according to the density function given in

(Ashkar and Bobée, 1986b, eq. 1) (denoted as "eq. AB1"), i.e. Pearson type 3 with parameters α, λ and m.

This means that the value of k in eq. AB2 (or equation 2 of Ashkar and Bobée, 1986b) is given by:

$$k = \log_{10} e \quad = .4343 \tag{11}$$

Suppose that the estimates of α, λ and m as calculated using SAM are:

$$\tilde{\alpha} = -10 \text{ (i.e. } \tilde{\beta} = k\tilde{\alpha} = -4.343\text{)}; \quad \tilde{\lambda} = 2; \quad \tilde{m} = 4 \tag{12}$$

We shall show how to calculate the variance of $\tilde{X}_T$ with:

$$T = 100 \tag{13}$$

(variance of the 100-year flood). These calculations are done in 9 steps:

(1) set the values of s, t and u; (eq. AB7)

(2) calculate μ_r' (x) for r = s, t, u, 2s, 2t, 2u; (eq. AB3)

(3) calculate the matrix A; (eq. AB21)

(4) calculate the matrix V and its inverse (V^{-1}); (eq. AB19)

(5) calculate the vector V_M; (eq. AB22 and AB23)

(6) calculate the vector V_P; (eq. AB24)

(7) calculate $\tilde{Y}_T$, $\tilde{X}_T$, $\partial\tilde{Y}_T / \partial\tilde{\alpha}$, $\partial\tilde{Y}_T / \partial\tilde{\lambda}$ and $\partial\tilde{Y}_T / \partial\tilde{m}$; (eq. AB15, AB8, AB25, AB26 and AB27)

(8) calculate Var $\hat{Y}_T$; (eq. AB16)

(9) calculate Var $\hat{X}_T$; (eq. AB14)

note: throughout these calculations the unknown values of α, λ and m will as usual be replaced by their respective estimates $\tilde{\alpha}$, $\tilde{\lambda}$ and $\tilde{m}$ given in (12).

step 1

SAM uses moments of order -1, $\underline{0}$ and 1 (GMM (-1, $\underline{0}$, 1)), so the values of s, t and u (eq. AB7) are set at:

$$s = \underline{0}; \quad t = -1; \quad u = 1 \tag{14}$$

step 2

The calculation of μ_s' and μ_{2s}' is done using equation (AB30b) because s = 2s = $\underline{0}$ while that of μ_t', μ_u', μ_{2t}' and μ_{2u}' is done using equation (AB3) (by successively setting r = t, u, 2t, 2u; with t = -1 and u = 1). These calculations yield:

$$\mu_s' = \mu_{2s}' = 3.8, \quad \mu_t' = 1.69 \times 10^{-4}, \quad \mu_u' = 6.61 \times 10^{3},$$

$$\mu_{2t}' = 3.44 \times 10^{-8}, \quad \mu_{2u}' = 4.69 \times 10^{7} \tag{15}$$

step 3

The first row of the matrix A is calculated using eqs. (AB34) through (AB36) because s = $\underline{0}$. This yields:

$$A_{11} = -.02; \quad A_{12} = -.1; \quad A_{13} = 1 \tag{16}$$

The second and third rows of the matrix A are calculated using eq. (AB21) because t and u are different from zero. This yields:

$$A_{21} = 1.010 \times 10^{-5}; \quad A_{22} = 4.416 \times 10^{-5}; \quad A_{23} = -3.886 \times 10^{-4}$$

$$A_{31} = -2.473 \times 10^{2}; \quad A_{32} = -1.369 \times 10^{3}; \quad A_{33} = 1.521 \times 10^{4} \tag{17}$$

step 4

The matrix V is easily obtained by substituting the values A_{ij} obtained in step 3 into eq. (AB19). The inversion of V is then easily done on a computer to yield V^{-1}.

step 5

Posing $M_1 = \mu_s'$, $M_2 = \mu_t'$ and $M_3 = \mu_u'$ and calculating Var (M_1), Cov

(M_1, M_2) and Cov (M_1, M_3) by eqs. AB31-AB33 because $s = \underline{0}$, gives:

$$\mathrm{Var}\,(M_1) = 4 \times 10^{-4};\ \mathrm{Cov}\,(M_1, M_2) = -2.019 \times 10^{-7};\ \mathrm{Cov}\,(M_1, M_3) = 4.946 \tag{18}$$

Calculating Var (M_2), Var (M_3) and Cov (M_2, M_3) by eqs. AB22 and AB23 because t and u are different from zero gives:

$$\mathrm{Var}(M_2)=1.175 \times 10^{-10};\ \mathrm{Var}(M_3)= 6.454\times10^{4};\ \mathrm{Cov}(M_2,M_3)=-2.302\times10^{-3} \tag{19}$$

Placing the variances and covariances obtained in (18) and (19) in a vector form gives the vector V_M (eq. AB18):

$$V_M = \begin{bmatrix} \mathrm{var}\ M_1 \\ \mathrm{var}\ M_2 \\ \mathrm{var}\ M_3 \\ \mathrm{cov}(M_1, M_2) \\ \mathrm{cov}(M_1, M_3) \\ \mathrm{cov}(M_2, M_3) \end{bmatrix} = \begin{bmatrix} 4 \times 10^{-4} \\ 1.175 \times 10^{-10} \\ 6.454 \times 10^{4} \\ -2.019 \times 10^{-7} \\ 4.946 \\ -2.302 \times 10^{-3} \end{bmatrix}$$

step 6

Multiplying the matrix V^{-1} that was obtained in step 4 on a computer, with the vector V_M obtained in step 5, gives the vector V_p:

$$V_p = \begin{bmatrix} \text{var } \alpha \\ \text{var } \lambda \\ \text{var } m \\ \text{cov } (\alpha, \lambda) \\ \text{cov } (\alpha, m) \\ \text{cov } (\lambda, m) \end{bmatrix} = \begin{bmatrix} 53.246 \\ 6.3371 \\ 0.01208 \\ -18.048 \\ -0.7599 \\ 0.27274 \end{bmatrix}$$

step 7

The frequency factor K (eq. AB15) is obtained from the Harter tables (1969) by entering the value $|C_S| = 2/\tilde{\lambda}^{\frac{1}{2}} = 1.414$ and probability level (1-1/T) = .99. The tables give $K \simeq -1.31$. Putting this in eq. (AB15) gives:

$$\hat{Y}_T = 3.985 \tag{20}$$

By eq. (AB8) we get:

$$\hat{X}_T = 10^{3\ 985} = 9.66 \times 10^3 \tag{21}$$

By eqs. (AB25) and (AB27) we get:

$$\partial\hat{Y}_T \;/\; \partial\tilde{\alpha} = -1.49 \times 10^{-3} \tag{22}$$

$$\partial\hat{Y}_T \;/\; \partial\tilde{m} = 1 \tag{23}$$

From the polynomial expression given by Bobée (1979) for K in terms of C_S we easily obtain $\partial K/\partial C_S$ to be equal to .6204. Putting this into eq. (AB26) gives:

$$\partial\hat{Y}_T \;/\; \partial\tilde{\lambda} = -2.266 \times 10^{-2} \tag{24}$$

step 8

The vector V_p obtained in step 6 along with eqs. (49), (50) and (51) are now put into eq. (AB16) to give Var $\hat{Y}_T$:

$$\text{Var } \hat{Y}_T = .41 \times 10^{-2}$$

step 9

The value of $\hat{X}_T$ obtained in step 7 and Var $\hat{Y}_T$ obtained in step 8 are finally substituted into eq. (AB14) to give Var $\hat{X}_T$:

$$\begin{aligned} \text{Var } \hat{X}_T &= (\hat{X}_T \,/\, k)^2 \text{ Var } \hat{Y}_T \\ &= (9.66 \times 10^3 / \ .4343)^2 * .41 \times 10^2 \\ &= 2.03 \times 10^6 \end{aligned}$$

EXTENSION OF THE METHOD TO OTHER DISTRIBUTIONS

This method (SAM) can be applied when it is possible to express the harmonic and geometric means as a function of the parameters of the distribution. Generally, the relation between harmonic mean 1/H (moment of order -1) and the parameters of a given distribution can be easily established when this mean exists. Some general conditions for the existence of the harmonic mean have been given by Piegorsh and Casella

(1985). The relation between the geometric mean and the parameters of the distribution can on the other hand be difficult to establish except in certain cases described below:

- distributions deduced by a logarithmic transformation (log normal, log Pearson, Log Gamma, Log Gumbel [Weibull] etc.). In this case let the distribution be denoted by X. If G is the geometric mean of the sample (x_i) in real space then we have log G = $\bar{y}$ where $\bar{y}$ is the arithmetic mean of the sample (y_i) (y_i = log x_i) in log space. Log G can therefore be taken as an estimate of the arithmetic mean (moment of order 1) of the distribution Y = log X which can generally be easily expressed in terms of the parameters.

  These kinds of distributions X deduced by logarithmic transformation have another characteristic property which can explain their practical flexibility. It can in fact easily be shown that if X follows an "LD" distribution, say, then X^p follows also an LD distribution. This proof is made in appendix in the case of the LP distribution.

- the (3-parameter) generalized gamma distribution which is deduced from the gamma distribution by a transformation of the form $X = Y^k$ where k is a parameter. For calculating the geometric mean of this distribution, one can use the results given in (Johnson and Kotz, 1970, pp. 197-198).

- the Halphen family of distributions (Morlat 1956) for which it can be proved that the arithmetic, geometric and harmonic means are sufficient statistics.

SUMMARY AND CONCLUSION

A new method (SAM) for estimating the parameters of the Log-Pearson type 3 distribution is presented here. This method considers the arithmetic, geometric and harmonic means which are respectively moments of order 1, quasi zero and -1. Reducing the order of the moments in this manner can lead to a lower value of Var X_T for certain values of T. This method is interesting in that it can be considered as an extension to the method MM1 ("mixed moments") proposed by RAO(1980) and used by Phien and Hira (1983) and Phien and HSU(1985).

The determination of Var X_T for this method has also been demonstrated by means of an example. A general comparison of several methods mentioned in the present study is now being undertaken and the results should be published in the near future.

The reaction to the present study by some readers might be that "here comes another method of parameter estimation to add to the long list of methods which have been developed over the years but which did not bring any significant improvement to the problem of flood frequency estimation".

This reaction is not justified because although many of the existing methods are usually found capable of "describing" or "fitting" the data with quite the same degree of accuracy, experience shows that beyond the range covered by the data these methods often produce significantly different results. Much confusion has been felt in the hydrologic literature in recent years as a result of the many distributions and methods proposed for flood frequency estimation but this should not discourage new ideas and especially less classical ones which can bring some freshness into the way the problem of flood estimation is handled. Methods that are becoming more and more sophisticated and refined such as regional flood frequency estimation, detection and treatment of outliers, correction of measurement errors, use of historical flood information, in addition to the basic single site studies which do not involve any of the above, all form one coherent unit. It is dangerous to ignore or loose hope in any one of these areas when dealing with a natural phenomenon as catastrophic in its effects on human life and property as floods.

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APPENDIX I

Proof that if X follows a Log pearson type 3 distribution (LP) then X^p follows an LP distribution.

If X follows an LP (α, λ, m) distribution (Log Pearson type 3 distribution with parameters α, λ and m), then Y = Log X follows a P (α, λ, m) distribution.

If we consider $Z = X^p$ we have:

$$\text{Log } Z = p \text{ Log } X = pY = U$$

The density function of the variate Y is :

$$g(Y) = \frac{|\alpha|}{\Gamma(\lambda)} \; e^{-\alpha(Y-m)} \; [\; \alpha(Y-m)]^{\lambda-1}$$

The density function of the variate U is such that:

$$f(U) = g[Y(U)] \frac{dY}{dU}$$

$$f(U) = \frac{|\alpha|}{p} \cdot \frac{1}{\Gamma(\lambda)} \cdot e^{-\alpha(\frac{U}{p} -m)} [\alpha (\frac{U}{p} - m)]^{\lambda-1}$$

$$f(U) = \frac{|\alpha|}{p} \cdot \frac{1}{\Gamma(\lambda)} e^{-\frac{\alpha}{p} (U-mp)} [\frac{\alpha}{p} (U-mp)]^{\lambda-1}$$

$$f(U) = \frac{|\alpha'|}{\Gamma(\lambda)} e^{-\alpha' (U-m')} [\alpha' (U-m')]^{\lambda-1}$$

In this form it can be seen that $U = \text{Log } X^p$ follows a P distribution with parameters $\alpha' = \frac{\alpha}{p}$,λ and $m' = mp$.

So X^p follows an LP distribution with parameters α', λ, m' $[LP(\frac{\alpha}{p}, \lambda, mp)]$

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APPENDIX III - NOTATION

a	:	logarithmic base; if X has LP distribution then $Y = Log_a X$ has P distribution
β	:	$= \alpha k$ (eq.3)
GMM (s,t,u)	:	Generalized method of moments which uses moments of order s,t,u in the estimation
G	:	geometric mean of the sample
H	:	$= \log_a e$ (eq. 3).
k	:	$= \log_a e$ (eq.3)
MM1	:	method of "mixed moments" (RAO, 1980): MM1=GMM (0, 1, 2)
m, α, λ	:	parameters of the P and LP distributions
$(\mu'_r)_L$	:	r^{th} population moment about the origin for the LP distribution
$\mu_2(x)$, $\mu_2(y)$	:	variance of X and Y respectively

$\mu'_{\underline{0}}(x)$: moment of order quasi zero for the LP distribution (X)

N : sample size

SAM : "Sundy Averages Method" = GMM ($\underline{0}$, -1, 1)

T : return period

X : LP random variable

X_T : event with return period T under the random variable X

$\bar{x}$: sample mean under an LP distribution X

Y : Pearson type 3 (P) random variable

$\bar{y}$: sample mean under a P distribution Y

$\underline{0}$: notation for "quasi zero": "r = $\underline{0}$" means "r→0".

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