1. Introduction

Context
- RFA is used for predicting flood quantities at ungauged locations. The rational is that two sites having similar hydrological properties should behave similarly. In that case, valuable information can be transferred between sites.
- Hydrological dissimilarity between a gauged site and an ungauged location cannot be calculated. Instead, a physiographical space in which an associated metric between site characteristics must be used.
- In physiographical spaces, flood quantities can be predicted by interpolation methods, such as kriging.

Problematic
- Usually, flood quantities share a log-log relationship with site characteristics, which creates bias and suboptimal prediction with traditional kriging techniques.
- Traditional kriging does not account for non-constant variance.
- The problem associated with traditional kriging technique can be resolved by considering Spatial Copula, an extension of traditional geostatistical framework where spatial dependence is characterized by a copula.

2. Spatial copula

- A multivariate distribution G can be expressed as
  \[ G(x) = C \left( F_1(x_1), \ldots, F_n(x_n) \right) \]
  where \( F_i \) are margins for \( x = (x_1, \ldots, x_n) \) and \( C \) is a copula
- For spatial analysis, the copula has the same dimension as the number of site \( n \) and the strength of the dependence must be associated with a distance \( h \).
- Spatial copula must allows for strong dependence
  \[ C_n \to C^0 \text{ as } h \to 0 \]
  and perfect independence
  \[ C_n \to C^0 \text{ as } h \to \infty \]
- In spatial copula framework, the margins of the distribution \( G \) are treated separately from its dependence. Hence, there is 2 set of parameters: the marginal part \( \nu \) and the copula part \( \theta \).

3. Physiographical space

- There is no general agreement on what hydrological similarity between sites should be. Here the dissimilarity between two sites \( i \) and \( j \) is defined as the hydrological distance
  \[ h_{ij} = d(Z_i, Z_j) \]
  between the vectors of flood quantities \( Z_i = (Z_{i1}, \ldots, Z_{in}) \) with return periods 1, \ldots, \( r \)
- Let a physiographical space be a subspace of coordinates
  \[ S_i = AX \]
  where \( X \) are site characteristics and \( A \) is a matrix that projects \( X \) on the physiographical space.
- A metric is associated with a dissimilarity measure
  \[ \text{Small } d(S_i, S_j) \rightarrow \text{Small } d(Z_i, Z_j) \]

4. Model

Marginal part (\( \nu \))
- Regional distribution of the flood quantities is log-normal.
  \[ \log(Z) 
  
  = N \left[ \mu(S), \sigma^2(S) \right] \]
  A linear trend is added to account for the strong correlation between the first canonical coordinates \( S_1 \) and the flood quantities:
  \[ \mu(S_1) = \beta_0 + \beta_1 S_1 \]
  \[ \sigma(S_1) = \beta_2 + \beta_3 S_1 \]

Copula part (\( \theta \))
- The spatial dependence is characterized by a Gaussian copula with pairwise correlation
  \[ \rho(S_1, S_2 | \gamma) = (1 - \gamma) \exp \left[ -\frac{1}{2} d(S_1, S_2) \right] \]
  with \( \gamma > 0 \) (practical range) controls the correlation as \( d(S_1, S_2) \to \infty \) and \( \gamma \) is a local measurement error (nugget effect)
- The Goodness-of-fit test [1]
  - based on binned pairwise observations validates the choice of a Gaussian copula.
  - For each bins, \( \rho \)-values are larger than 20% are found.

5. Results

- The presentation of the prediction obtained by spatial copula is assessed by Leave-one-out cross-validation. In turn each gauged station is considered as ungauged and a predicted value is obtained as the median of the PPD.
- The analysis of the residuals shows the presence of large relative discrepancies (Fig. 5-Left). Note the presence of problematic stations previously identified for this database [3].
- The absolute residuals at logarithm scale (Fig. 5-Right) show a decreasing variance that is coherent with the trend of \( \nu(S) \).

6. Conclusion

- The spatial copula framework offers a full probabilistic model that can account for non-constant variance.
- The spatial copula framework appears more appropriate in presence of problematic stations in comparison with traditional kriging.
- The spatial copula framework has competitive performance with the best method. In particular, it improves over traditional kriging.
- The important relative bias observed with the traditional kriging approach is reduced greatly with the spatial copula approach.

References

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