Modeling the relationship between climate oscillations and drought by a multivariate GARCH model

R. Modarres[1] and T. B. M. J. Ouarda[1,2]

Received 12 March 2013; revised 12 November 2013; accepted 21 November 2013; published 27 January 2014.

[1] Typical multivariate time series models may exhibit comovement in mean but not in variance of hydrologic and climatic variables. This paper introduces multivariate generalized autoregressive conditional heteroscedasticity (GARCH) models to capture the comovement of the variance or the conditional covariance between two hydroclimatic time series. The diagonal vectorized and Baba-Engle-Kroft-Kroner models are developed to evaluate the covariance between drought and two atmospheric circulations, Southern Oscillation Index (SOI) and North Atlantic Oscillation (NAO) time series during 1954–2000. The univariate generalized autoregressive conditional heteroscedasticity model indicates a strong persistency level in conditional variance for NAO and a moderate persistency level for SOI. The conditional variance of short-term drought index indicates low level of persistency, while the long-term index drought indicates high level of persistency in conditional variance. The estimated conditional covariance between drought and atmospheric indices is shown to be weak and negative. It is also observed that the covariance between drought and atmospheric indices is largely dependent on short-run variance of atmospheric indices rather than their long-run variance. The nonlinearity and stationarity tests show that the conditional covariances are nonlinear but stationary. However, the degree of nonlinearity is higher for the covariance between long-term drought and atmospheric indices. It is also observed that the nonlinearity of NAO is higher than that for SOI, in contrast to the stationarity which is stronger for SOI time series.

Citation: Modarres, R., and T. B. M. J. Ouarda (2014), Modeling the relationship between climate oscillations and drought by a multivariate GARCH model, *Water Resour. Res.*, *50*, 601–618, doi:10.1002/2013WR013810.

1. Introduction

[2] Among natural hazards and disasters, drought is perhaps the most complex but least understood phenomenon with different characteristics in space and time which prohibit us to define its beginning and end. Drought spatial progress is slow and usually takes a long-time period to pass by a region. In addition, drought direct and indirect impacts on economic, social, and environmental systems are destructive [i.e., *Raziei et al*., 2009, among others]. These characteristics may have been the reasons for the development and application of a number of methods and approaches for drought definition, monitoring, modeling, and forecasting over the past decades [*Mishra and Singh*, 2011]. A large number of studies dealing with drought characterization can be seen in the literature. Four main types of droughts, namely, meteorological, hydrological, agricultural, and socioeconomic droughts have been discussed in the literature, and a number of indices and methods have been developed to identify drought conditions and characteristics for each type of drought.

[3] Among different approaches used for drought characterization, linear autoregressive moving average (ARMA) model is very popular in hydrology. The capability of modeling the seasonal characteristic of hydrologic variables, such as droughts, and an inherent advantage of having a model with a few parameters but a reasonable result have made the time series approaches popular for drought time series modeling [*Mishra and Desai*, 2005; *Durdu*, 2010].

[4] In spite of the popularity of multivariate analysis such as multivariate frequency distribution functions (i.e., copula functions) for hydrologic and drought probabilistic analysis [e.g., *Chebana and Ouarda*, 2007; *Shiau and Modarres*, 2009; *Shih-Chieh and Govindaraju*, 2010], multivariate time series modeling approaches have not been reasonably investigated for drought modeling and forecasting. The recent review by *Mishra and Singh* [2011] on drought modeling approaches indicates the lack of multivariate time series model application, such as a vector ARMA model, in the literature for drought analysis. However, a few applications of simple multivariate autoregressive time series model can be found for other hydrological variables such as streamflow [*Niedzielski*, 2007; *Sohail et al*., 2008; *Chaleeraktrakoon*, 2009]. Another gap is that univariate and multivariate "nonlinear" time series models,

[1]Hydroclimate Modeling Group, INRS-ETE, Quebec, Quebec, Canada.
[2]Institute Center for Water Advanced Technology and Environmental Research (iWATER), Masdar Institute of Science and Technology, Abu Dhabi, UAE.

Corresponding author: R. Modarres, Hydroclimate Modeling Group, INRS-ETE, 490 De La Couronne, Quebec, QC G1K9A9, Canada. (Reza.Modarres@ete.inrs.ca)

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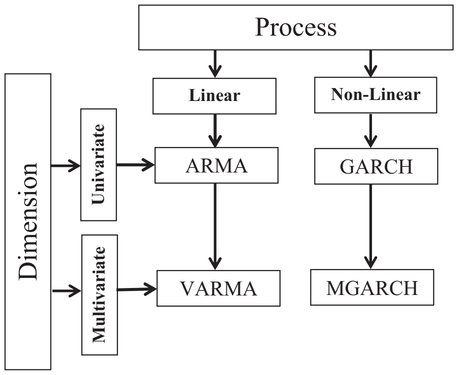

Figure 1. Categories of time series models.

which take into account the time-varying variance-covariance or the conditional variance-covariance structure of drought, have not been addressed in hydrologic and climatic literature yet.

[5] The nonlinear time series modeling approach usually refers to a popular econometrics generalized autoregressive conditional heteroscedasticity (GARCH) model. The GARCH model is widely used in finance for investigating the volatility and time-varying risk of the assets, stock markets, and returns. The theoretical aspects of the model were first introduced by *Engle* [1982] and developed by *Bollerslev* [1986]. The GARCH model has rarely been applied for hydrologic and climatic variables. *Wang et al*. [2005] and *Chen et al*. [2008] showed the advantage of univariate GARCH models over linear models. More recently, *Modarres and Ouarda* [2013] indicated that the GARCH model does not have superiority over seasonal autoregressive moving average model for rainfall time series modeling except for removing heteroscedasticity from the residuals of the linear model.

[6] Based on the literature review and different categories of time series methods (Figure 1), the multivariate generalized autoregressive conditional heteroscedasticity (GARCH) models, which are very popular in financial time series modeling, have not been applied in hydrology yet. The main application of multivariate GARCH models in econometrics is the study of the relationship between the conditional variances, the volatility, of different markets [*Bauwens et al*., 2006].

[7] The aim of this study is to introduce and develop univariate and multivariate GARCH models for drought conditional variance-covariance, or volatility-covolatility, relationship with atmospheric indices. Furthermore, this study also examines and compares the nonlinearity and nonstationarity of drought and its link to atmospheric circulations.

[8] This paper is organized as follows: the theoretical background of the univariate and multivariate GARCH models is given in the following section. The simulation procedure and testing methods applied in this study are presented in sections 3. and 4.. Section 5. is devoted to an example of the models used for drought analysis. The last sections are devoted to concluding remarks and recommendations for future work.

2. GARCH Models

[9] Time series models can be classified based on the space of variables (univariate or multivariate) and the hypothesis of the underlying process (linear or nonlinear). According to this perspective, we have four types of time series models (Figure 1). The focus of this paper is on univariate and multivariate nonlinear GARCH time series modeling approaches which have not been used for drought analysis yet. These models are described in the following sections.

2.1. Univariate Model

[10] The univariate nonlinear model was first introduced by *Engle* [1982] as a class of autoregressive conditional heteroscedasticity (ARCH) model to capture the volatility clustering of financial time series. In an ARCH model, the conditional variance (h_t^2) of the shocks that occurs at time t is a function of the squares of past shocks $(\varepsilon_{t-1}^2, \ \ldots, \varepsilon_{t-v}^2)$. Therefore, the ARCH model of order v or ARCH(v) model can be written as follows:

$$h_t^2 = \omega + \sum_{i=1}^{v} \alpha_i \varepsilon_{t-v}^2 \tag{1}$$

[11] *Bollerslev* [1986] suggested adding lagged conditional variance to the ARCH model to generalize the effect of past variances on the current variance, h_t^2 , in addition to the previous shocks. This model or the GARCH(v,m) model can then be specified as follows:

$$h_t^2 = \omega + \sum_{i=1}^{v} \alpha_i \varepsilon_{t-v}^2 + \sum_{j=1}^{m} \beta_j h_{t-m}^2 \tag{2}$$

where ω is a constant and α and β are parameters of the model to be estimated. In this model, the short-run persistency in conditional variance is defined by the ARCH parameter (α), while the long-run persistency in conditional variance is defined by (β) parameter. The high value of ($\alpha + \beta$) indicates a high intensity of persistence in the conditional variance of the time series.

2.2. Multivariate Model

2.2.1. Overview

[12] Having a K-dimensional zero mean, serially uncorrelated process $\boldsymbol{\varepsilon}_\mathbf{t} = (\varepsilon_{1t}, \ldots, \varepsilon_{Kt})'$ is represented as

$$\boldsymbol{\varepsilon}_\mathbf{t} = \mathbf{H}_{t|t-1}^{1/2} z_t \tag{3}$$

where z_t is a K-dimensional independent and identically distributed (i.i.d.) white noise, $z_t \sim i.i.d(0, I_K)$, then we have $\mathbf{H}_t|t-1$ as the conditional covariance matrix of $\boldsymbol{\varepsilon}_t$, given ε_{t-1}, ε_{t-2}, $\ldots$ and $E[\boldsymbol{\varepsilon}_t|\Omega_{t-1}] = 0$ and $E\left[\boldsymbol{\varepsilon}_\mathbf{t}\boldsymbol{\varepsilon}'_\mathbf{t}|\Omega_{t-1}\right] = \mathbf{H}_t$. The above definition of conditional covariance matrix $\mathbf{H}_t$ needs to be parameterized now.

[13] Remembering the univariate GARCH(v,m) model, in a multivariate case one may want to allow $\mathbf{H}_t$ to depend on lagged shocks $\boldsymbol{\varepsilon}_{t-i}$, $i=1, \ . \ . \ . \ , \ v$ (i.e., the ARCH process of order v) and on lagged conditional covariance matrices $\mathbf{H}_{t-i}$, $i=1, \ . \ . \ . \ , \ m$ (i.e., the GARCH process of order m). Therefore, the general form of a multivariate GARCH model is written as follows:

$$vech(\mathbf{H}_t) = \mathbf{W} + \mathbf{A}_1 vech(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1}) + \mathbf{B}_1 vech(\mathbf{H}_{t-1}) \tag{4}$$

where $\mathbf{W}$ is a $^1/_2K(K+1) \times 1$ vector and $\mathbf{A}_1$ and $\mathbf{B}_1$ are $(^1/_2K(K+1) \times {}^1/_2K(K+1))$ parameter matrices. The VECH

() denotes the operator which stacks the lower portion of a matrix in a vector. As the conditional covariance matrix is symmetric, VECH($\mathbf{H}_t$) contains all unique elements of $\mathbf{H}_t$ and can therefore be written in a matrix form as follows:

$$vech\begin{bmatrix} h^2{}_{11,t|t-1} & h^2{}_{12,t|t-1} \\ h^2{}_{21,t|t-1} & h^2{}_{22,t|t-1} \end{bmatrix} = \begin{bmatrix} h^2{}_{11,t|t-1} \\ h^2{}_{12,t|t-1} \\ h^2{}_{22,t|t-1} \end{bmatrix} = \begin{bmatrix} w_{10} \\ w_{20} \\ w_{30} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon^2_{2,t-1} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} h^2{}_{11,t-1|t-2} \\ h_{12,t-1|t-2} \\ h^2{}_{22,t-1|t-2} \end{bmatrix} \quad (5)$$

[14] The number of parameters of the vectorized (VECH) model (w, a_{ii} *and* b_{jj}) is equal to $(v+m)\left(\frac{K(K+1)}{2}\right)^2+K(K+1)/2$.

[15] The main drawback of the VECH specification is that the number of parameters will become excessively large as K and the order of the model (v and m) increase. For example, the above bivariate GARCH(1,1) model has 21 parameters, while the trivariate model has 78 parameters. Estimation of this general model may therefore be quite problematic. Therefore, some specific diagonal parameterizations are introduced to reduce the number of parameters [*Frances and van Dijk*, 2000]. Among these specifications, in this study we develop and apply the diagonal VECH and diagonal Baba-Engle-Kroft-Kroner (BEKK) models. For simplicity, we discuss only the case $m = v = 1$.

2.2.2. Diagonal VECH Model

[16] As mentioned above, one of the main disadvantages of the full VECH model is the large number of parameters. To overcome this problem, *Bollerslev et al*. [1988] suggested a model by constraining the matrices $\mathbf{A}_1$ and $\mathbf{B}_1$ in (4) to be diagonal. In this case, the conditional covariance between $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$ depends only on lagged cross products of the two shocks involved and lagged values of the covariance itself:

$$\mathbf{H}_t=\mathbf{W}+\mathbf{A}_1\otimes(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1})+\mathbf{B}_1\otimes\mathbf{H}_{t-1} \quad (6)$$

where $\otimes$ denotes the Hadamard or element-by-element product. This model is called a diagonal VECH model and has $3k(k+1)/2$ parameters. For example, the bivariate diagonal VECH(1,1) model can be given as follows:

$$vech\begin{bmatrix} h^2{}_{11,t|t-1} & \\ h^2{}_{12,t|t-1} & h^2{}_{22,t|t-1} \end{bmatrix} = \begin{bmatrix} W_{11} & \\ W_{21} & W_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1,t-1} & \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon^2_{2,t-1} \end{bmatrix} + \begin{bmatrix} b_{11} & \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} h^2{}_{11,t-1|t-2} & \\ h^2{}_{21,t-1|t-2} & h^2{}_{22,t-1|t-2} \end{bmatrix} \quad (7)$$

[17] This diagonal VECH model has 9 parameters to be estimated which is much less than the full VECH model with 21 parameters. To get the conditional variance and covariance equations from the above specification we can write

$$h^2{}_{11,t}=W_{11}+a_{11}\varepsilon^2_{1,t-1}+b_{11}h^2{}_{1,t-1} \quad (8)$$

$$h^2{}_{21t}=W_{21}+\varepsilon_{1,t-1}\varepsilon_{2,t-1}+b_{21}h^2{}_{21,t-1} \quad (9)$$

$$h^2{}_{22,t}=W_{22}+a_{22}\varepsilon^2_{1,t-1}+b_{22}h^2{}_{1,t-1} \quad (10)$$

2.2.3. Diagonal BEKK Model

[18] BEKK is the acronym for the work by Baba, Engle, Kraft, and Kroner which was the early version of *Engle and Kroner*'s [1995] paper. The diagonal BEKK model is an alternative for the diagonal VECH presentation.

[19] In this case, the diagonal BEKK(1,1) model can be written in a diagonal form where the off-diagonal elements are all equal to zero (apart from the constant term):

$$\begin{aligned} \begin{bmatrix} h^2{}_{11,t|t-1} & h^2{}_{12,t|t-1} \\ & h^2{}_{22,t|t-1} \end{bmatrix} &= \begin{bmatrix} w_{11} & w_{12} \\ & w_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & \\ & a_{22} \end{bmatrix}' \begin{bmatrix} \varepsilon^2_{1,t-1} & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ & \varepsilon^2_{2,t-1} \end{bmatrix} \begin{bmatrix} a_{11} & \\ & a_{22} \end{bmatrix} \\ &+ \begin{bmatrix} b_{11} & \\ & b_{22} \end{bmatrix}' \begin{bmatrix} h^2{}_{11,t-1|t-2} & h^2{}_{12,t-1|t-2} \\ & h^2{}_{22,t-1|t-2} \end{bmatrix} \begin{bmatrix} b_{11} & \\ & b_{22} \end{bmatrix} \end{aligned} \quad (11)$$

[20] And therefore, the conditional variance and covariance equations can be written as the followings [*Baur*, 2006]:

$$h^2{}_{11,t}=W_{11}+a^2_{11}\varepsilon^2_{1,t-1}+\mathrm{b}^2_{11}h^2{}_{1,t-1} \quad (12)$$

$$h^2{}_{21t}=W_{12}+a_{11}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1}+\mathrm{b}_{11}b_{22}h^2{}_{21,t-1} \quad (13)$$

$$h^2{}_{22,t} = W_{22} + a_{22}^2 \varepsilon_{1,t-1}^2 + b_{22}^2 h^2{}_{1,t-1} \quad (14)$$

[21] In this study, we develop and apply the two diagonal multivariate GARCH specifications to establish the covolatility relationship between drought and atmospheric indices.

3. Simulation and Model Verification

[22] As multivariate GARCH specifications have been developed to estimate the conditional covariance between two time series, different specifications are supposed to result in similar covariance estimation. The performance of different multivariate GARCH models for estimating conditional covariance is analyzed and compared using simulation procedures and three performance criteria, normalized bias (NBIAS), normalized root-mean-square error (NRMSE), and Diebold and Mariano (DM) criteria.

[23] The NBIAS and NRMSE criteria allow one to sort among models based on the covariance estimating accuracy. These criteria can be written as follows:

$$\mathrm{NBIAS} = \frac{1}{k} \sum_{i=1}^{k} \frac{s_{CCOV} - e_{CCOV}}{e_{CCOV}} \quad (15)$$

$$\mathrm{NRMSE} = \sqrt{\frac{1}{k} \sum_{i=1}^{k} \left(\frac{s_{CCOV} - e_{CCOV}}{e_{CCOV}}\right)^2} \quad (16)$$

where s_{CCOV} and e_{CCOV} denote simulated and empirical conditional covariances, respectively, and k is the number of estimations.

[24] However, these criteria do not test if the improvement in the conditional covariance estimation among different models is statistically significant or not. To address this issue, the DM statistic is applied. The DM test [*Diebold and Mariano*, 1995] is a common test in financial time series modeling to compare different models with a basic model to evaluate if their outputs are different from the basic model or not [*Mohammadi and Su*, 2010]. The DM test is applied in this study in order to evaluate if the conditional covariances estimated by diagonal VECH and diagonal BEKK models are statistically different from the simulated conditional covariance.

[25] Having $e_{1,t}$ and $e_{2,t}$, $t = 1, \ldots, n$, as the errors between simulated and estimated covariances, respectively, and $g(e_{1,t})$ and $g(e_{2,t})$ as their loss differential, and $d_t = g(e_{1,t}) - g(e_{2,t})$ as the loss differential, *Diebold and Mariano* [1995] defined the following statistic:

$$B = \frac{\bar{d}}{\sqrt{\frac{2\pi f_d(0)}{n}}} \qquad \sim \mathrm{N}(0, 1) \quad (17)$$

where $\bar{d}$ is the sample mean, $2\pi f_d(0)$ is the weighted sum of the autocovariance of loss differential, and n is the number of observations, and the numerator of above equation is the variance of the loss differential. The null hypothesis of zero mean loss differential or the equal of conditional covariance of different models is rejected if the test statistic is negative and statistically significant.

4. Testing Procedures

4.1. Test for Stationarity

[26] Stationarity is one of the basic assumptions for a number of hydrologic modeling approaches and further tests such as linearity/nonlinearity testing. The stationarity test is carried out in this study for drought and atmospheric indices and the conditional variance-covariance between them using the following two tests: augmented Dickey Fuller (ADF) test and Phillips-Perron (PP) test [*Dickey and Fuller*, 1979; *Phillips and Perron*, 1988]. The null hypothesis (H0) of the ADF test is the existence of the unit root in the time series (i.e., nonstationary time series). The null hypothesis (H0) of the PP test is stationarity around a deterministic trend (trend stationarity) and stationarity around a fixed level (level stationarity).

4.2. Test for Nonlinearity

[27] Natural systems, such as atmospheric processes, are commonly perceived as nonlinear. The nonlinear mechanism acting on drought and atmospheric circulations and their link is investigated in this study using the Brock-Dechert-Scheinkman (BDS) test [*Brock et al.*, 1996] which has its roots in the chaos theory. It is based on the m-dimensional correlation integral where m represents the embedding space in the new series $\{X_t\}$, $X_t = (x_t, x_{t-\tau}, \ldots, x_{t-(m-1)\tau})$, which is generated from a scalar time series $\{Y_t\}$ of length N, and then we have

$$C_{m,M}(r) = \binom{M}{2}^{-1} \sum_{1 \le i < j \le M} H(r - X_i - X_j \quad (18)$$

where $M = N - (m-1)\tau$ is the number of embedded points in m-dimensional space, r is the radius of a sphere centered on X_i and $H(u)$ is the Heaviside function [*Abramowitz and Stegun*, 1972, p. 1020]. Therefore, the BDS statistic for $m > 1$ is defined as

$$BDS_{m,\ M}(r) = \sqrt{M} \frac{C_m(r) - C_1^m(r)}{\sigma_{m,M}(r)} \quad (19)$$

where σ is the standard deviation of the points in the embedded m-dimensional space. Under the null hypothesis, $\{X_t\}$ is an i.i.d. process, and the BDS statistic converges to a unit normal as $M \to \infty$. This convergence requires large samples for values of embedding dimension much larger than $m = 2$. Therefore, m is usually restricted to a range from 2 to 5 [*Wang et al.*, 2006]. Therefore, the null hypothesis (H0) of the BDS test is that the time series under investigation have a linear variation in time.

5. Applications

5.1. Data Description

5.1.1. Drought Index

[28] Different indices have been developed for drought analysis among which the Standardized Precipitation Index (SPI) introduced by *McKee et al.* [1993] has received widespread applications. The SPI can quantify the precipitation deficit for different time scales and therefore, is a flexible index to show the impact of drought on different types of

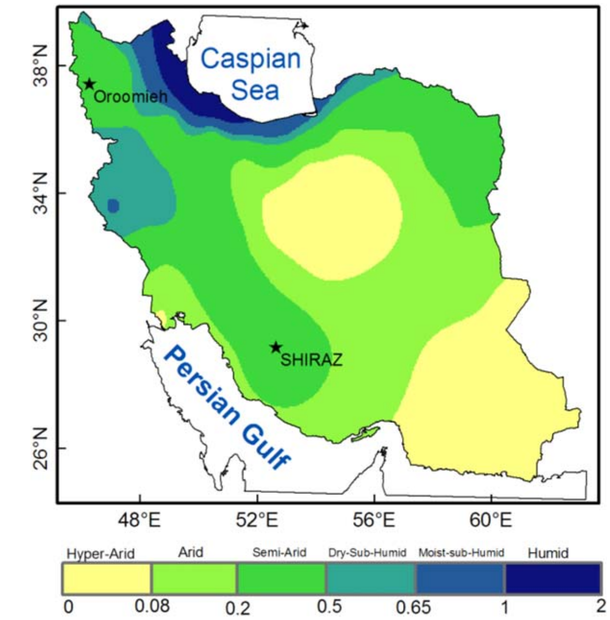

Figure 2. Aridity index map of Iran and the location of selected stations.

water resources systems in space and time. The SPI has also the advantage of statistical consistency and the ability to describe both short- and long-term drought impacts on water resources [*Hayes et al*., 1999]. Therefore, many studies on modeling drought characteristics such as severity, duration, and frequency, drought forecasting, and drought link to atmospheric and climatic indices have applied SPI as a drought index [e.g., *Bordi and Sutera*, 2001; *Vicente-Serrano*, 2005; *Ozger et al*., 2012].

[29] The drought data set in this study includes the 3 month and 12 month Standardized Precipitation Index (SPI3 and SPI12, hereafter) time series for two stations in the northwestern and southwestern territories of Iran, namely, Oroomieh and Shiraz stations.

[30] The SPI time series are selected, rather than their untransformed counterpart (rainfall), because of the importance of drought consequences on the agricultural and water resources systems of Iran. It is therefore important to investigate the temporal effect of atmospheric circulation on drought occurrence in Iran. The two above stations are located in the western territories of Iran where it is believed to be more related to global climate than other parts of the country.

[31] The SPI time series are calculated based on fitting a Gamma distribution to the continuous monthly rainfall time series during 1954–2010. The complete formulation of the SPI calculation can be found in the paper of *Loukas and Vasiliades* [2004].

[32] The location of two stations is illustrated in Figure 2 together with the climate zones of Iran classified based on the United Nations Environment Program aridity index [*Raziei and Pereira*, 2012]. The SPI time series of these stations are illustrated in Figure 3. The SPI3 and SPI12 time series are selected in order to compare the heteroscedastic characteristics of both short- and long-term drought indices.

5.1.2. Atmospheric Indices

[33] In this study, two major atmospheric indices which are widely believed to influence the precipitation of Iran, especially over the western and southwestern territories [*Nazemosadat and Ghasemi*, 2004; *Raziei et al*., 2009], the Southern Oscillation Index (SOI) and North Atlantic Oscillation (NAO) are used for drought time series modeling. These time series are obtained from National Weather Service and are standardized index calculated based on the mean of 1950–2000 period. The monthly time series for

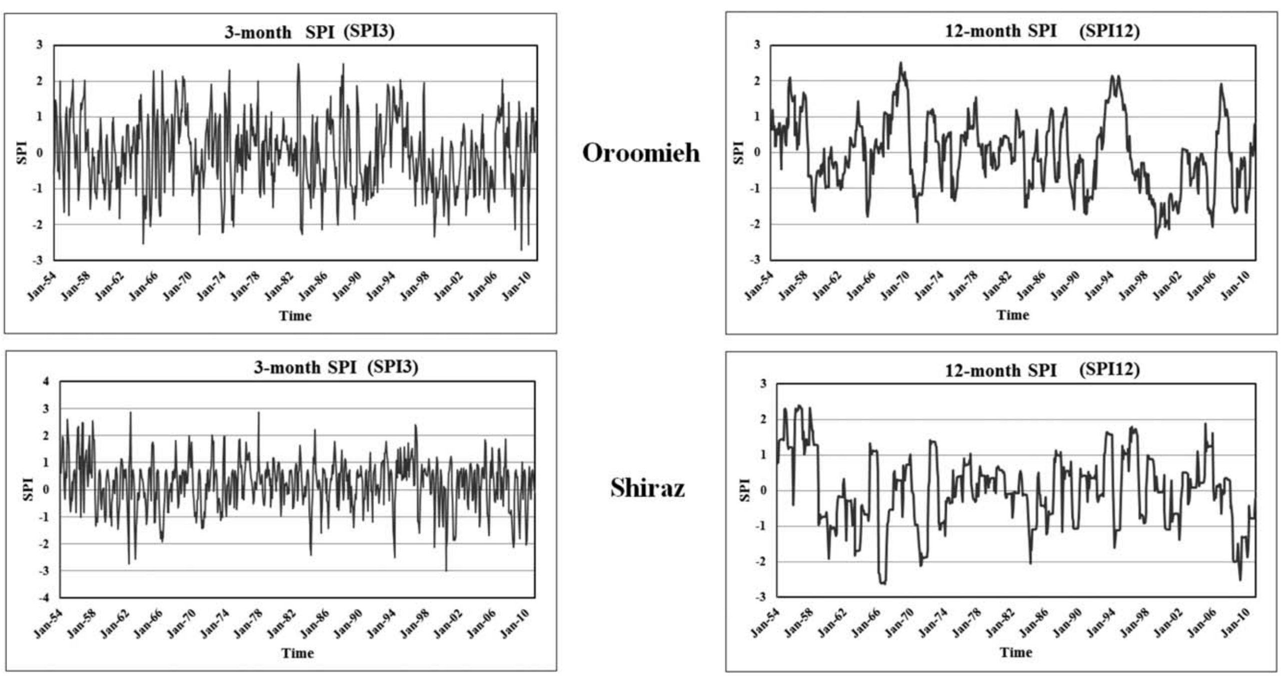

Figure 3. SPI time series for 1954–2010.

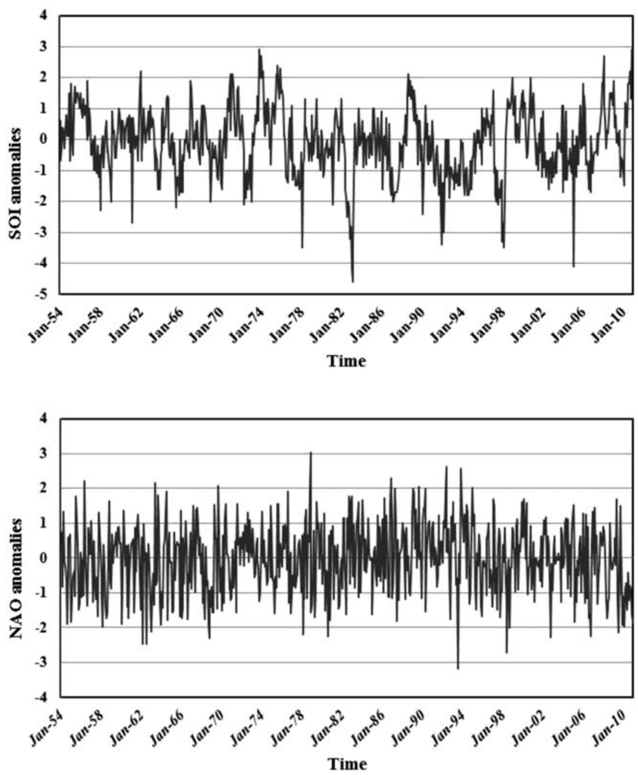

Figure 4. SOI and NAO anomalies time series for 1954–2010.

atmospheric indices during 1954–2010 applied in this study are given in Figure 4.

5.2. Preliminary Data Analysis

[34] The link between two atmospheric indices (i.e., SOI and NAO) and drought conditions for the selected stations is first investigated by unconditional correlation coefficients and cross-correlation coefficients in different lag times. The (unconditional) monthly correlation coefficients between oscillations and SPI time series are given in Figure 5. Figure 5 indicates a negative correlation between SOI and SPI for both stations and for both SPI time series. However, the negative correlation is stronger for the autumn (September, October, and November) and winter (December, January, and February) seasons than the other seasons. Some positive (but weak) relationships are observed for the summer season (June, July, and August). This is in agreement with the observations by *Nazemosadat and Ghasemi* [2004] who indicated a low intensity of winter drought during El Nino events and also in agreement with the weak correlation coefficients between SOI and SPI reported by *Raziei et al*. [2009].

[35] On the other hand, the NAO-SPI link shows both positive and negative associations for all seasons. It is observed that the NAO-SPI association is temporally irregular comparing the SOI-SPI association and does not show a strong seasonality. However, the winter drought (i.e., negative SPI) seems to be related to the negative NAO phase, while the summer drought indicates both positive and negative associations with NAO for both stations and for both drought time scales.

[36] In addition, the cross-correlation coefficients given in Figure 6a, indicate a weak lag time effect in SOI-SPI association, but it is almost insignificant for the NAO-SPI association. The association is usually weakening and becoming insignificant for lag times $k > 2$ months. This suggests a short-run memory in SOI-SPI and NAO-SPI relationship. On the other hand, the cross correlation between the squared SOI and SPI time series (Figure 6b) shows a weaker relationship than that for the original (i.e., nonsquared) data. The most interesting feature in Figure 6 belongs to SPI time series of Oroomieh station. For example, the negative insignificant correlation coefficients between the original SPI3 and SOI time series have become positive and significant for the squared time series, while this feature is not observed for SOI-SPI12 association. The same condition is observed for NAO-SPI connection at Oroomieh station where negative correlations for the original data have become positive for the squared time series. For Shiraz station, it is observed that the correlation coefficients between squared time series are always weaker than those for the original time series. This suggests that the second-order moment or the variance of atmospheric indices may have a different association to drought than the first-order moment or the mean. This phenomenon has not yet been considered in previous drought studies and is interesting to be investigated.

5.3. Conditional Variance Models

[37] The univariate GARCH model is developed for drought and atmospheric indices, and the parameters are

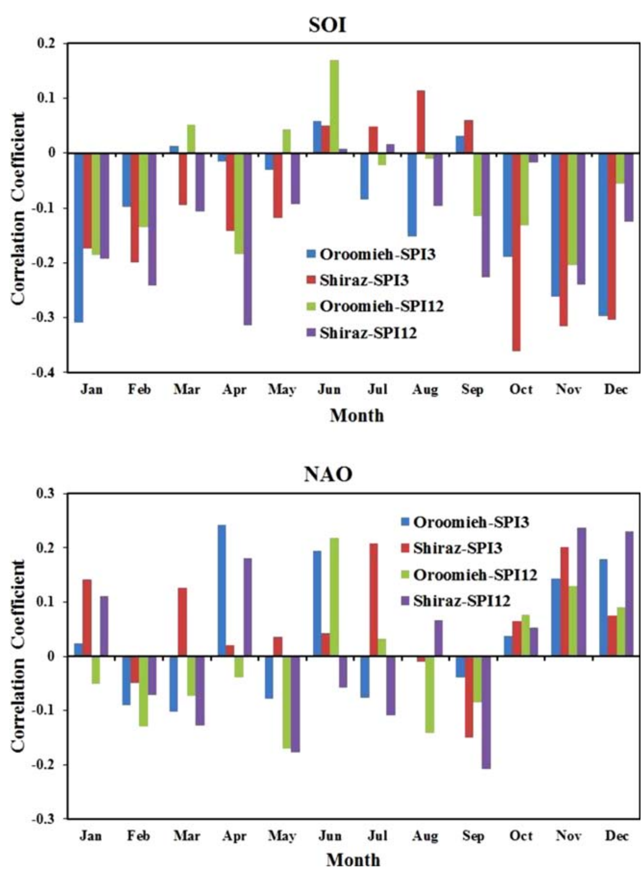

Figure 5. Monthly correlation coefficients between oscillation indices and SPI time series.

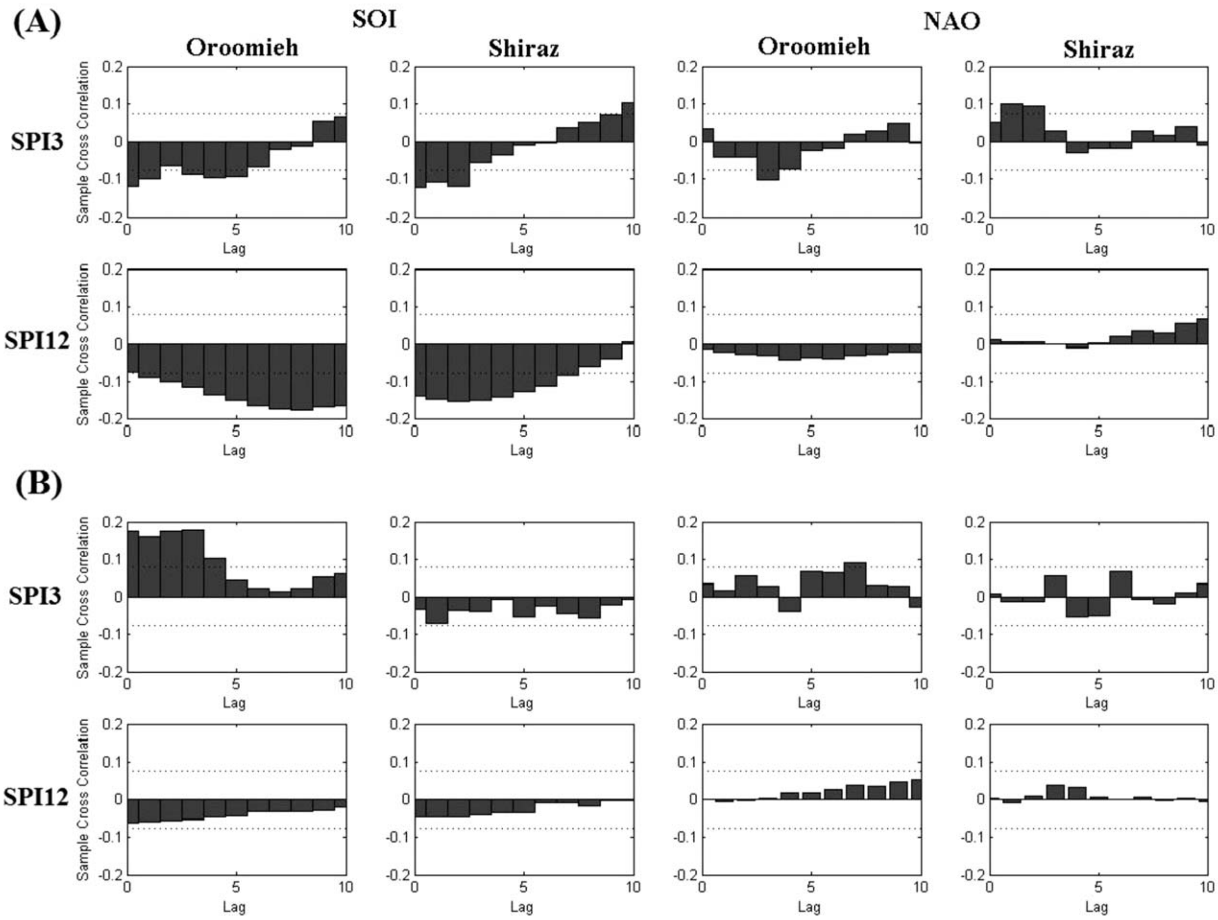

Figure 6. Cross-correlation coefficients between (a) oscillation indices and SPI and (b) squared oscillation indices and squared SPI for lag times $k = 0$–10.

estimated using the maximum likelihood method. The order of the GARCH models and their parameters are illustrated in Table 1.

[38] Table 1 shows that the conditional variances of atmospheric indices are different from each other. It can be seen that the short-run persistency of SOI is much stronger than that for NAO, while in the opposite, NAO shows a long-run persistence as the β parameter is large. One can also see that NAO has a stronger intensity of persistence and variance memory than SOI as $\alpha + \beta$ is larger for NAO. The conditional variance time series of atmospheric indices are illustrated in Figure 7. Figure 7 shows that the conditional variance of SOI is larger than that for NAO, and extreme conditional variances are observed for SOI time series. It should also be noted that no seasonal variation is observed for the conditional variances of atmospheric indices. However, extreme conditional variances are usually observed in the winter season.

[39] Table 1 also shows that the short-run persistence is dominant for drought time series where the GARCH parameter (β) is much smaller than the ARCH parameter (α) (except for SPI12 at Shiraz station) in the models. The ARCH parameters also indicate a stronger short-run persistency in conditional variance for SPI12 than that for SPI3 time series, implying volatility clustering in long-term drought. The conditional variance for SPI12 at both stations shows a high degree of intensity of persistency according to $\alpha + \beta$ measurement. The conditional variances of drought time series are illustrated in Figure 8. Figure 8 shows that the conditional variance of short-term drought varies (from low to high or vice versa) rapidly through time, while the conditional variance of long-term drought shows some sudden drastic increases in conditional variance (e.g., 1998–2002) interspersed by periods of relatively low fluctuation (e.g., 1974–1990). In addition, no sharp seasonality is observed for drought conditional variances.

[40] We next test the conditional variance of atmospheric and drought indices for stationarity and nonlinearity using

Table 1. Univariate GARCH Model Estimations for Selected Time Series

Data	Series	Parameters ω	α	β	Persistency Measurement $\alpha + \beta$	Order
Atmospheric index	SOI	0.34	0.39	0.34	0.73	GARCH(1,1)
	NAO	0.12	0.05	0.82	0.87	GARCH(1,1)
Oroomieh drought	SPI3	0.51	0.50	0.07	0.57	GARCH(1,1)
	SPI12	0.05	0.89	0.02	0.91	GARCH(1,1)
Shiraz drought	SPI3	0.47	0.49	0.08	0.57	GARCH(1,1)
	SPI12	0.04	0.70	0.29	0.99	GARCH(1,1)

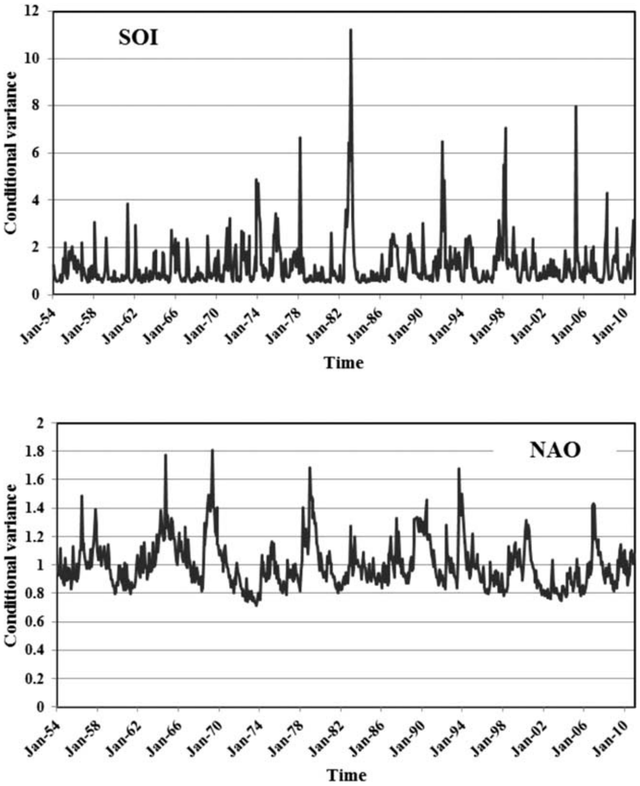

Figure 7. Conditional variance (volatility) for atmospheric indices.

the ADF, PP, and BDS tests. The results are given in Table 2. The zero p values of the ADF test indicate that the nonstationarity can be strongly rejected. The p values of the PP tests indicate that we cannot reject stationary conditional variance for all data series. It can also be seen that neither level nonstationarity nor trend nonstationarity is observed for the conditional variance of atmospheric indices. The BDS test statistic (and related p values) show that we can strongly reject the null hypothesis of linearity for the conditional variance of drought and atmospheric indices for all dimensions.

5.4. Conditional Covariance Models

[41] The bivariate model for conditional covariance between SPI and atmospheric indices is developed using two types of multivariate GARCH model diagonal specifications, the diagonal VECH and the diagonal BEKK models. The results for the two stations are given in the following sections.

5.4.1. Model Development for Oroomieh Station

[42] The estimates of the parameters of the diagonal VECH(1,1) model for Oroomieh station are given in Table 3. These parameters are estimated using maximum likelihood method. It is observed that the elements of the matrices, **W**, **A**, and **B**, are all statistically significant for SOI-SPI relationship. However, some parameters for NAO-SPI relationship in matrix **B** are not statistically significant. Based on these estimations we can write the equations for conditional covariances between drought and atmospheric indices (Table 3).

[43] The diagonal VECH models show that the conditional covariances depend greatly on the cross products of the lagged shocks rather than the lagged covariances. We can see that β parameters are negative, except for SOI-SPI12, implying that the covariance at each time step, t, has a negative association to the covariance at time step $t-1$. The highest (negative) covariance link is observed for NAO-SPI3 with $\beta=-0.80$. The highest intensity of persistency is observed for SOI-SPI12 covariance where $\alpha+\beta=0.8$ and the lowest belongs to NAO-SPI12 covariance with $\alpha+\beta=0.08$.

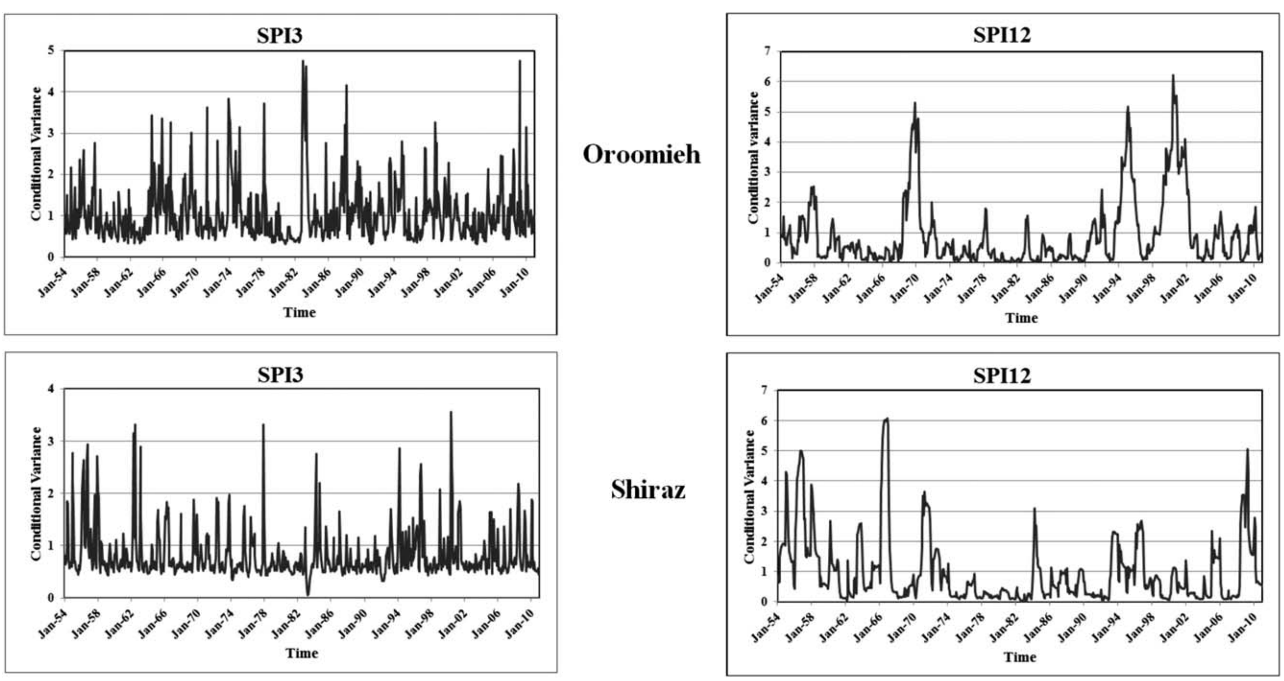

Figure 8. Conditional variance (volatility) for SPI time series.

Table 2. Results for PP and ADF Tests for Stationarity and BDS Test for Nonlinearity of the Conditional Variance

		PP Level Stationary Test		PP Trend Stationary Test		ADF Unit Root Test	
Data	Series	Results	*p* Value	Results	*p* Value	Results	*p* Value
Atmospheric indices	SOI	−12.5	>0.1	0.0001	0.29	−9.57	0
	NAO	−8.03	>0.1	−0.0002	0.28	−6.49	0
Oroomieh	SPI3	−17.37	>0.1	0.0003	0.78	−9.99	0
	SPI12	−4.13	>0.1	0.0003	0.65	−3.65	0.02
Shiraz	SPI3	−14.21	>0.1	−0.0008	0.25	−14.88	0
	SPI12	−5.73	>0.1	−0.001	0.22	−6.07	0

		BDS Test					
		$m = 2$		$m = 3$		$m = 4$	
Data	Series	Statistic	*p* Value	Statistic	*p* Value	Statistic	*p* Value
Atmospheric indices	SOI	0.06	0	0.10	0	0.12	0
	NAO	0.11	0	0.18	0	0.22	0
Oroomieh	SPI3	0.05	0	0.08	0	0.10	0
	SPI12	0.16	0	0.27	0	0.34	0
Shiraz	SPI3	0.07	0	0.10	0	0.11	0
	SPI12	0.16	0	0.27	0	0.34	0

[44] We next move to estimate the diagonal BEKK model and its seven parameters (Table 4). The diagonal matrices, **A** and **B**, are all significant indicating that both SOI and NAO influence the conditional variance of the SPI time series. The conditional covariance equations between atmospheric indices and drought at Oroomieh station using the diagonal BEKK model are given in Table 4. Similar to the diagonal VECH model, these equations also indicate that short-run persistency is much stronger than long-run persistency in the covariance structure between drought and atmospheric indices. The largest persistency is observed between SOI and SPI12 ($\alpha + \beta = 0.79$) It is also observed that the covariance between drought and NAO is negative for both short- and long-term drought time series. The only difference between BEKK and VECH estimation is the β parameter for equation (26) which is much smaller than the estimation of the VECH model.

Table 3. Diagonal VECH(1,1) Estimates for Oroomieh Station[a]

	Parameters			
	W	A	B	AIC
(a) SOI				
SPI3	$\begin{bmatrix} 0.37 & -0.08 \\ & 0.44 \end{bmatrix}$	$\begin{bmatrix} 0.37 & 0.48 \\ & 0.42 \end{bmatrix}$	$\begin{bmatrix} 0.32 & -0.13 \\ & 0.15 \end{bmatrix}$	5.49
SPI12	$\begin{bmatrix} 0.31 & 0.004 \\ & 0.04 \end{bmatrix}$	$\begin{bmatrix} 0.42 & 0.56 \\ & 0.85 \end{bmatrix}$	$\begin{bmatrix} 0.33 & 0.24 \\ & 0.12 \end{bmatrix}$	4.58
(b) NAO				
SPI3	$\begin{bmatrix} 0.11 & 0.01 \\ & 0.48 \end{bmatrix}$	$\begin{bmatrix} 0.04 & 0.10 \\ & 0.50 \end{bmatrix}$	$\begin{bmatrix} 0.84 & 0.80 \\ & 0.03 \end{bmatrix}$	5.56
SPI12	$\begin{bmatrix} 0.04 & 0.02 \\ & 0.05 \end{bmatrix}$	$\begin{bmatrix} 0.09 & 0.18 \\ 0.86 & \end{bmatrix}$	$\begin{bmatrix} -0.13 & 0.10 \\ & 0.05 \end{bmatrix}$	4.86

[a]Entries in bold are significant at the 10% level and less. Substituted coefficients SOI: $H_{SOI\_SPI3} = -0.08 + 0.48\varepsilon_{1,t-1}\varepsilon_{2,t-1} - 0.13H_{SOI\_SPI3,t-1}$; $H_{SOI\_SPI12} = 0.004 + 0.56\varepsilon_{1,t-1}\varepsilon_{2,t-1} + 0.24H_{SOI\_SPI12,t-1}$. Substituted coefficients NAO: $H_{NAO\_SPI3} = 0.01 + 0.10\varepsilon_{1,t-1}\varepsilon_{2,t-1} - 0.80H_{NAO\_SPI3,t-1}$; $H_{NAO\_SPI12} = 0.02 + 0.18\varepsilon_{1,t-1}\varepsilon_{2,t-1} - 0.10H_{NAO\_SPI12,t-1}$.

Table 4. Diagonal BEKK(1,1) Estimates for Oroomieh Station[a]

	Parameters			
	W	A	B	AIC
(a) SOI				
SPI3	$\begin{bmatrix} 0.37 & -0.08 \\ & 0.49 \end{bmatrix}$	$\begin{bmatrix} 0.64 & \\ & 0.68 \end{bmatrix}$	$\begin{bmatrix} 0.53 & \\ & -0.21 \end{bmatrix}$	5.49
SPI12	$\begin{bmatrix} 0.32 & 0.004 \\ & 0.04 \end{bmatrix}$	$\begin{bmatrix} 0.64 & \\ & 0.88 \end{bmatrix}$	$\begin{bmatrix} 0.58 & \\ & 0.38 \end{bmatrix}$	4.57
(b) NAO				
SPI3	$\begin{bmatrix} 0.11 & 0.02 \\ & 0.45 \end{bmatrix}$	$\begin{bmatrix} 0.21 & \\ & 0.71 \end{bmatrix}$	$\begin{bmatrix} 0.91 & \\ & -0.26 \end{bmatrix}$	5.56
SPI12	$\begin{bmatrix} 0.13 & 0.02 \\ & 0.05 \end{bmatrix}$	$\begin{bmatrix} 0.19 & \\ & 0.93 \end{bmatrix}$	$\begin{bmatrix} -0.91 & \\ & 0.21 \end{bmatrix}$	4.85

[a]Entries in bold are significant at the 10% level and less. Substituted coefficients SOI: $H_{SOI\_\_SPI3} = -0.08 + 0.41\varepsilon_{1,t-1}\varepsilon_{2,t-1} - 0.11H_{SOI\_SPI3,t-1}$; $H_{SOI\_SPI12} = 0.004 + 0.57\varepsilon_{1,t-1}\varepsilon_{2,t-1} + 0.22H_{SOI\_SPI12,t-1}$. Substituted coefficients NAO: $H_{NAO\_SPI3} = 0.02 + 0.15\varepsilon_{1,t-1}\varepsilon_{2,t-1} - 0.23H_{NAO\_SPI3,t-1}$; $H_{NAO\_SPI12} = 0.02 + 0.18\varepsilon_{1,t-1}\varepsilon_{2,t-1} - 0.19H_{NAO\_SPI12,t-1}$.

[45] In order to verify and select one multivariate GARCH model among diagonal models, we apply a simulation procedure to simulate the conditional covariance between SPI and atmospheric indices. The criteria values for performance evaluation of multivariate GARCH models for conditional covariance estimation are given in Table 5. The NBIAS indicates that the diagonal VECH model performs relatively better than the diagonal BEKK model for estimating the conditional covariance. However, the NRMSE shows a better performance for the diagonal BEKK model to estimate the SOI-SPI3 and NAO-SPI12 conditional covariances. It is also observed that the uncertainty of the covariance estimation is relatively higher for the NAO-SPI relationship than that for SOI-SPI link according to both criteria. This may be due to the weak covariance structure between NAO and drought at both stations.

[46] On the other side, the DM statistics reveal that the estimated and simulated conditional covariances are statistically different among the models, except for the covariance between SOI and SPI12. In other words, the

Table 5. Criteria Estimates for Conditional Covariance at Oroomieh Station

	Diagonal VECH		Diagonal BEKK		
Covariance Series	NBIAS	NRSME	NBIAS	NRSME	DM Statistic
SOI-SPI3	0.42	7.97	0.64	3.86	−4.19[a]
NAO-SPI3	0.82	12.94	−1.5	20.94	−17.4[a]
SOI-SPI12	−0.48	6.1	0.61	8.3	3.16
NAO-SPI12	−0.63	17.08	−0.8	8.7	−7.31[a]

[a]Significant at 5% level and better.

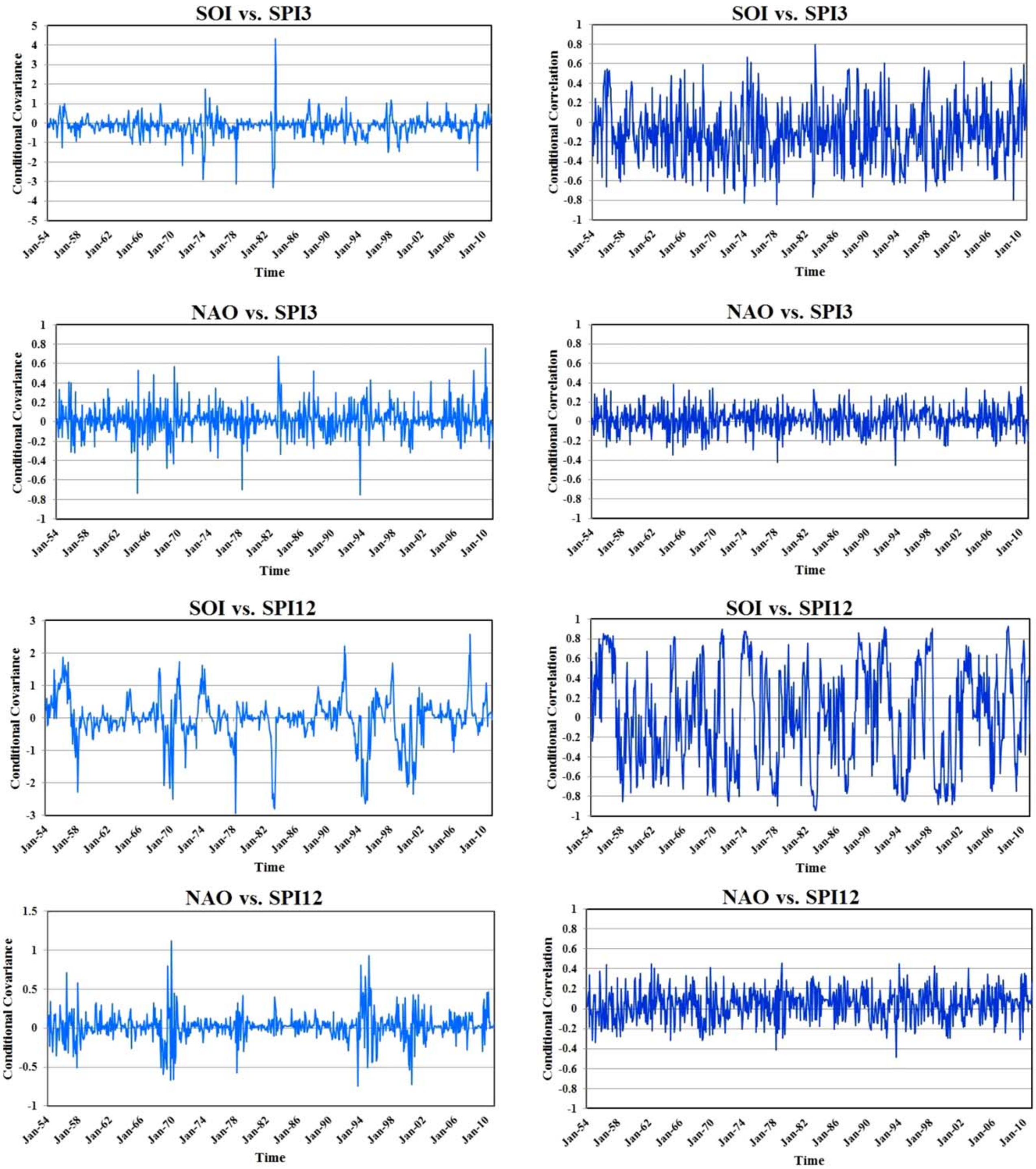

Figure 9. (left) Estimated conditional covariance and (right) conditional correlation between atmospheric indices and SPI for Oroomieh station.

difference between diagonal VECH estimation and simulation (e_1) as well as the difference between diagonal BEKK estimation and simulation (e_2) is statistically significant. Therefore, there is a significant difference between the two models for estimating the conditional covariance and the VECH model seems to give a better covariance estimation.

[47] According to simulation results, the diagonal VECH model is used to plot the time-varying conditional covariances and correlations between drought and atmospheric indices for Oroomieh station (Figure 9). Figure 9 illustrates that SOI has a larger covariance link with drought than the NAO for both short- and long-term drought time series. The conditional correlation between drought and SOI is much stronger than that between drought and NAO. The conditional correlation between SOI and SPI is usually falling within ± 0.40, while they are usually falling within ± 0.2 for NAO. It should be noted that the correlation coefficients outside ± 0.075 are statistically significant at 5%.

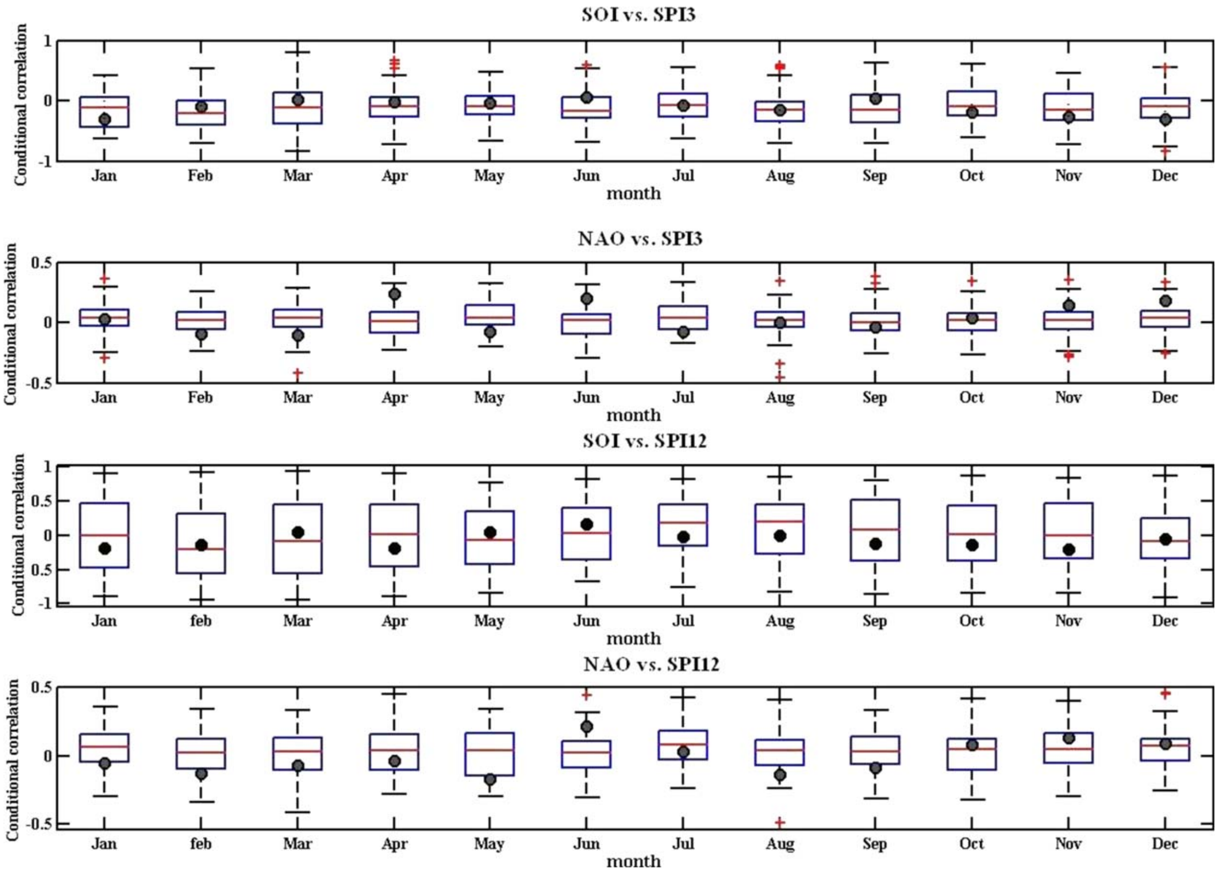

Figure 10. Monthly conditional correlation coefficient box plots for Oroomieh station. Circles show unconditional correlation coefficients.

[48] In addition, the seasonal variation in the link between drought and atmospheric indices is investigated through drawing monthly box plots of conditional correlation coefficients (Figure 10). In Figure 10, each box plot includes 57 correlation coefficients. Figure 10 indicates no sharp seasonality in the correlation coefficients. However, a few extreme (out of the 75% quantiles) correlation coefficients between NAO and SPI3 are observed from August to January. One can see that these extreme coefficients are mostly observed for the short-term drought index (SPI3), while long-term drought (SPI12) does not show extreme (positive or negative) correlation coefficients. This suggests that the link between drought and atmospheric circulation becomes stronger than normal condition, mostly for short-term drought events at Oroomieh station.

[49] The annual variation of conditional correlation between SPI and atmospheric indices is given in Figures 11 and 12 for SOI and NAO, respectively. In this figure each box plot includes 12 correlation coefficients for 12 months. This figure does not show a strong fluctuation between SPI3 and atmospheric indices, but the link between SPI12 and atmospheric indices shows a weak 3–5 years periodicity, especially for SPI12-SOI link.

[50] Finally, we come to compare the nonlinearity, stationarity, and unit root of conditional covariances using BDS, PP, and ADF tests. The results are given in Table 6. It is seen that the conditional covariances are stationary regarding *p* values of both ADF and PP test results. However, a weak trend nonstationarity in the covariance of NAO-SPI12 structure should be noticed where the *p* value of the test statistic is on the level of hypothesis rejection ($p = 0.05$).

[51] The BDS test indicates that the linearity of the conditional covariance between SPI and the atmospheric indices can be rejected as all *p* values are zero.

5.4.2. Model Development for Shiraz Station

[52] The same procedure is followed to estimate the conditional covariance and correlation between SPI and the atmospheric indices for Shiraz station. The estimated diagonal VECH models and their nine parameters are shown in Table 7. It is clear that the conditional variance of SPI depends on their own lags, lagged cross products of the shocks, and lagged conditional covariance. However, the conditional covariance parameters are not significant for NAO-SPI12 link (same as for Oroomieh station). Based on the parameters, the conditional covariances for drought at Shiraz station (Table 7) show the same covariance structure as the Oroomieh station. It is seen that β parameters are negative for drought and NAO relationship, and the persistency of the covariance structure is not very strong for NAO. However, the covariance between drought and SOI is significant and positive. The largest intensity is observed for SOI-SPI12 relationship where $\alpha + \beta = 0.74$ which is relatively high but not very strong ($\alpha + \beta < 0.90$).

[53] In the following, we look at the estimates of the diagonal BEKK models for Shiraz station (see Table 8).

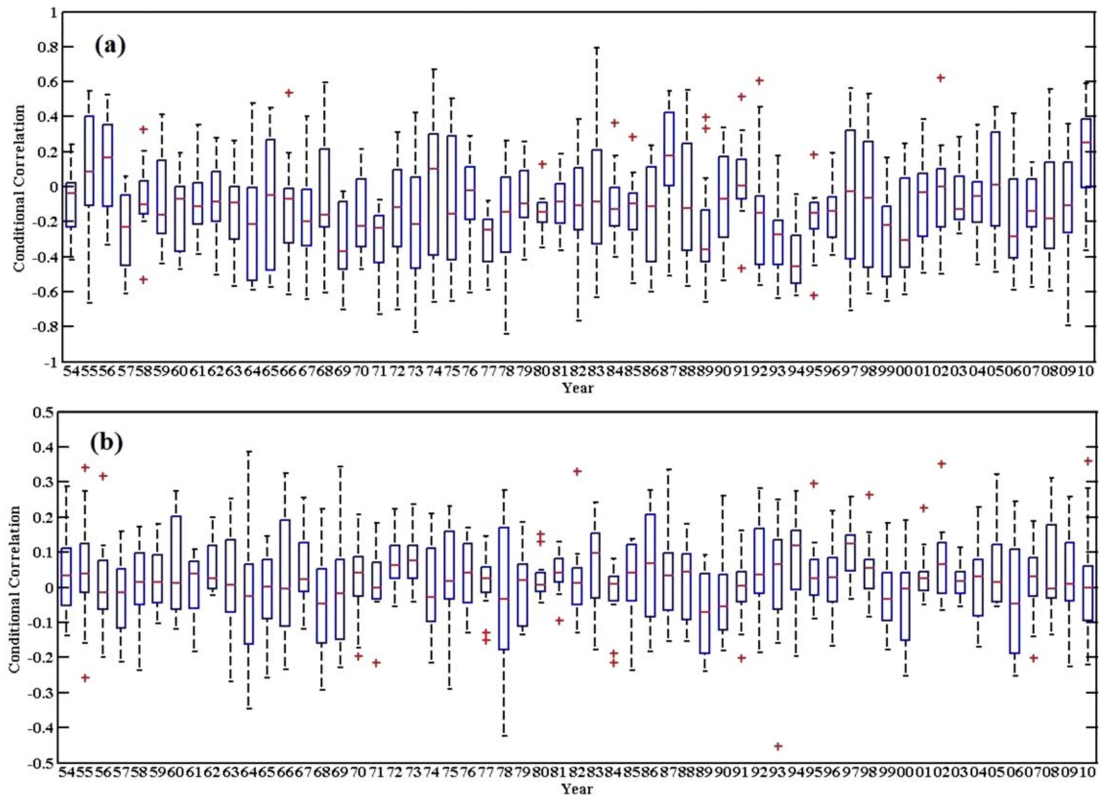

Figure 11. Annual conditional correlation box plots between (a) SPI3 and SOI as well as (b) SPI3 and NAO for Oroomieh station.

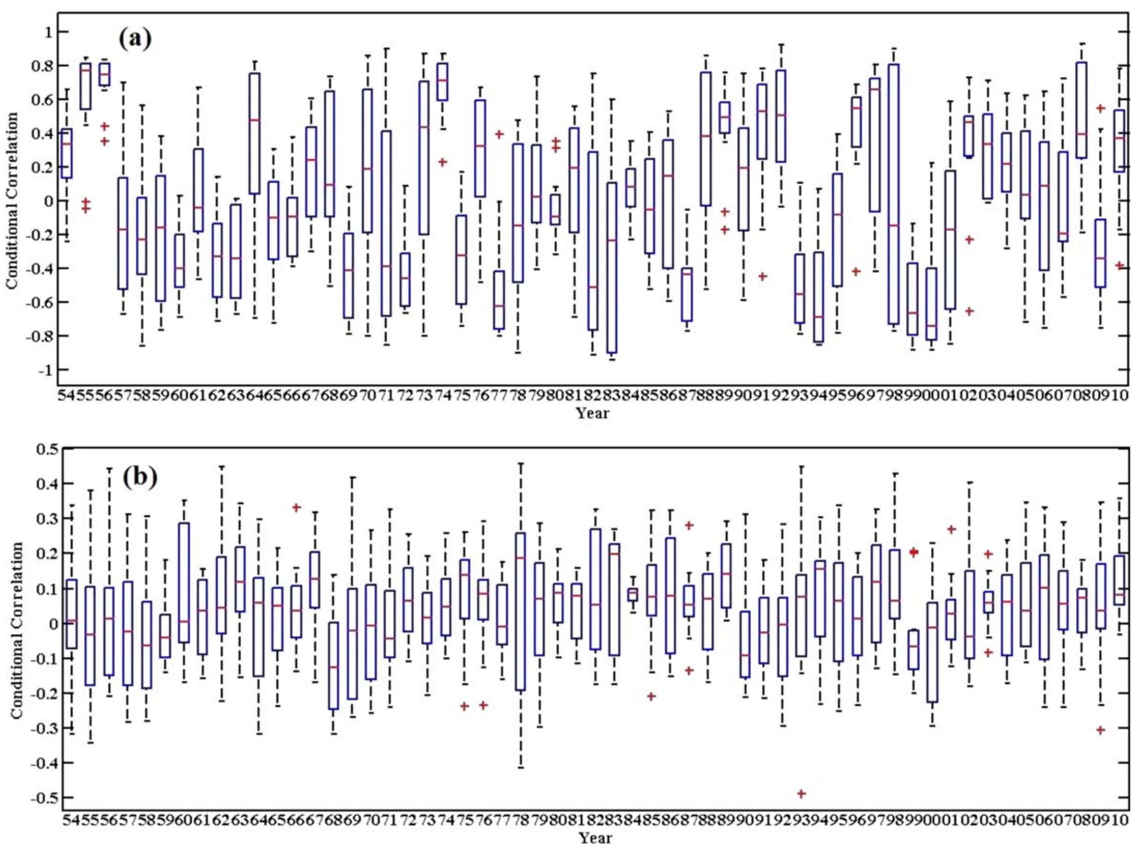

Figure 12. Same as Figure 11 but for SPI12.

Table 6. Results for PP and ADF Tests for Stationary and BDS Test for Nonlinearity of Conditional Covariances at Oroomieh Station

		PP Level Stationary Test		PP Trend Stationary Test		ADF Unit Root Test	
Indices Series	Drought Series	Results	*p* Value	Results	*p* Value	Results	*p* Value
SOI	SPI3	−17.4	>0.1	0.002	0.84	−15.1	0
	SPI12	−9.7	>0.1	−0.0006	0.47	−7.8	0
NAO	SPI3	−29.2	>0.1	0.004	0.17	−29.4	0
	SPI12	−26.1	>0.1	0.006	0.05	−26.1	0

BDS Test							
		$m = 2$		$m = 3$		$m = 4$	
Indices Series	Drought Series	Statistic	*p* Value	Statistic	*p* Value	Statistic	*p* Value
SOI	SPI3	0.04	0	0.07	0	0.09	0
	SPI12	0.10	0	0.18	0	0.22	0
NAO	SPI3	0.01	0	0.02	0	0.03	0
	SPI12	0.05	0	0.10	0	0.13	0

The 2 × 2 parameter matrices are almost all significant for both drought time series. According to the BEKK models in Table 8, a weak covariance structure between drought and atmospheric indices at Shiraz station is observed. The SOI-SPI connection is stronger than NAO-SPI link. The covariance between SOI and SPI seems to depend relatively on cross products of shocks and weakly on lagged covariances which is relatively small for SOI12 ($\beta = 0.19$) and almost nil for SPI3 ($\beta = 0.02$). The covariance structure between NAO and SPI also depends dimly on cross products of shocks. However, the lagged covariance structure between NAO and SPI is negative and nil ($\beta = -0.06$). Similar to Oroomieh station, the largest long-run persistency is observed between SOI and 12 month SPI ($\alpha + \beta = 0.72$).

Table 7. Diagonal VECH(1,1) Estimates for Shiraz Station[a]

	Parameters: W	A	B	AIC
(a) SOI				
SPI3	$\begin{bmatrix}0.38 & -0.08\\ & 0.49\end{bmatrix}$	$\begin{bmatrix}0.37 & 0.38\\ & 0.47\end{bmatrix}$	$\begin{bmatrix}0.30 & 0.06\\ & -0.08\end{bmatrix}$	5.27
SPI12	$\begin{bmatrix}0.39 & -0.03\\ & 0.04\end{bmatrix}$	$\begin{bmatrix}0.38 & 0.50\\ & 0.84\end{bmatrix}$	$\begin{bmatrix}0.28 & 0.24\\ & 0.08\end{bmatrix}$	4.53
(b) NAO				
SPI3	$\begin{bmatrix}0.15 & 0.05\\ & 0.47\end{bmatrix}$	$\begin{bmatrix}0.04 & 0.15\\ & 0.47\end{bmatrix}$	$\begin{bmatrix}0.79 & -0.37\\ & -0.06\end{bmatrix}$	5.31
SPI12	$\begin{bmatrix}0.97 & -0.01\\ & 0.05\end{bmatrix}$	$\begin{bmatrix}0.07 & 0.16\\ & 0.95\end{bmatrix}$	$\begin{bmatrix}-0.04 & -0.14\\ & -0.01\end{bmatrix}$	4.73

[a]Entries in bold are significant at the 10% level and less. Substituted coefficients SOI: $H_{SOI\_sPI3} = -0.08 + 0.38\varepsilon_{1,t-1}\varepsilon_{2,t-1} + 0.06H_{SOI\_SPI3,t-1}$; $H_{SOI\_SPI12} = -0.03 + 0.50\varepsilon_{1,t-1}\varepsilon_{2,t-1} + 0.24H_{SOI\_SPI12,t-1}$. Substituted coefficients NAO: $H_{NAO\_SPI3} = 0.05 + 0.15\varepsilon_{1,t-1}\varepsilon_{2,t-1} - 0.37H_{NAO\_SPI3,t-1}$; $H_{NAO\_SPI12} = -0.01 + 0.16\varepsilon_{1,t-1}\varepsilon_{2,t-1} - 0.14H_{NAO\_SPI12,t-1}$.

Table 8. Diagonal BEKK (1,1) Estimates for Shiraz Station[a]

	Parameters: W	A	B	AIC
(a) SOI				
SPI3	$\begin{bmatrix}0.38 & -0.08\\ & 0.44\end{bmatrix}$	$\begin{bmatrix}0.59 & \\ & 0.67\end{bmatrix}$	$\begin{bmatrix}0.56 & \\ & 0.04\end{bmatrix}$	5.27
SPI12	$\begin{bmatrix}0.39 & -0.03\\ & 0.04\end{bmatrix}$	$\begin{bmatrix}0.58 & \\ & 0.91\end{bmatrix}$	$\begin{bmatrix}0.56 & \\ & 0.34\end{bmatrix}$	4.59
(b) NAO				
SPI3	$\begin{bmatrix}0.15 & 0.04\\ & 0.43\end{bmatrix}$	$\begin{bmatrix}0.21 & \\ & 0.68\end{bmatrix}$	$\begin{bmatrix}0.89 & \\ & -0.07\end{bmatrix}$	5.31
SPI12	$\begin{bmatrix}0.11 & -0.02\\ & 0.04\end{bmatrix}$	$\begin{bmatrix}0.17 & \\ & 0.97\end{bmatrix}$	$\begin{bmatrix}0.92 & \\ & -0.06\end{bmatrix}$	4.73

[a]Entries in bold are significant at the 10% level and less. Substituted coefficients SOI: $H_{SOI\_SPI3} = -0.08 + 0.40\varepsilon_{1,t-1}\varepsilon_{2,t-1} + 0.02H_{SOI\_SPI3,t-1}$; $H_{SOI\_SPI12} = -0.03 + 0.53\varepsilon_{1,t-1}\varepsilon_{2,t-1} + 0.19H_{SOI\_SPI12,t-1}$. Substituted coefficients NAO: $H_{NAO\_SPI3} = 0.03 + 0.15\varepsilon_{1,t-1}\varepsilon_{2,t-1} - 0.06H_{NAO\_SPI3,t-1}$; $H_{NAO\_SPI12} = -0.02 + 0.17\varepsilon_{1,t-1}\varepsilon_{2,t-1} - 0.06H_{NAO\_SPI12,t-1}$.

[54] To select between VECH and BEKK models, the performance criteria of the models are examined (Table 9). It is clear from the DM statistics that the accuracy of the two models is statistically different if we compare the simulated and estimated conditional covariances. Following the DM test results, both NBIAS and NRMSE criteria show a much better performance of diagonal VECH model as compared to diagonal BEKK model for estimating the conditional covariance between drought and atmospheric indices. Therefore, the diagonal VECH model is applied to present the conditional covariance time series at Shiraz station.

[55] Using the diagonal VECH model, we give the time-varying conditional covariances and correlation coefficients for Shiraz station in Figure 13. Figure 13 shows no significant temporal variation difference between SOI-SPI3 and NAO-SPI3. However, the covariance for SOI-SPI12 link is much stronger than that for NAO-SPI12. The correlation coefficients show a much larger association between SOI and drought than that between NAO and drought at Shiraz station. Similar to Oroomieh station, the correlation coefficients outside ±0.075 are statistically significant at 5%.

[56] The monthly and annual variation of conditional correlation coefficients is also illustrated in Figures 14–16 through box plots. The monthly distribution of conditional correlation coefficients is almost identical to what we observed for Oroomieh station. However, the annual

Table 9. Criteria Estimates for Conditional Covariance at Shiraz Station

Covariance Series	Diagonal VECH NBIAS	Diagonal VECH NRSME	Diagonal BEKK NBIAS	Diagonal BEKK NRSME	DM Statistic
SOI-SPI3	0.24	8.32	0.62	17.45	−3.14[a]
NAO-SPI3	0.45	9.8	−0.41	51.87	−8.02[a]
SOI-SPI12	−0.14	12.98	−1.64	54.36	−10.7[a]
NAO-SPI12	−0.29	13.1	−2.09	12.52	−2.72[a]

[a]Significant at 5% level and better.

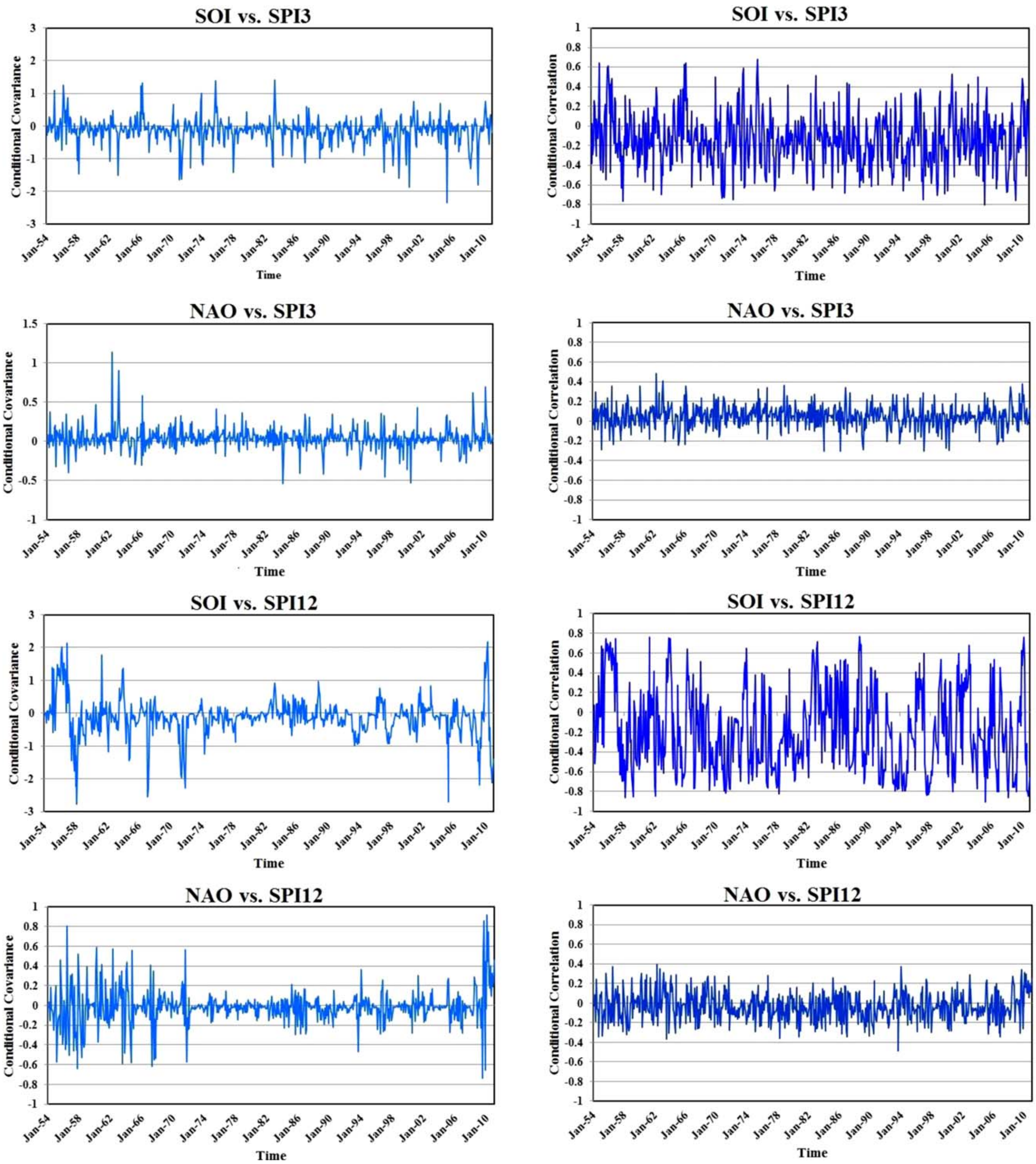

Figure 13. (left) Estimated conditional covariance and (right) conditional correlation between atmospheric indices and SPI for Shiraz station.

variation of the correlation coefficient seems to be more irregular than that for Oroomieh station. This indicates that the atmospheric circulations have more regular relationship with drought at Oroomieh station than with drought at Shiraz station.

[57] The stationarity and nonlinearity results for the conditional covariance are given in Table 10. The results indicate stationary covariances between drought and atmospheric indices regarding p values of the ADF test which rejects nonstationarity and PP test which cannot reject stationarity. However, a nonstationarity trend is observed at 5% significance level for the SOI-SPI3 and NAO-SPI12 covariances. This is perhaps due to extreme drought events which are strongly influenced by atmospheric indices.

[58] On the other hand, all covariances are nonlinear according to BDS test results as all p values are zero and

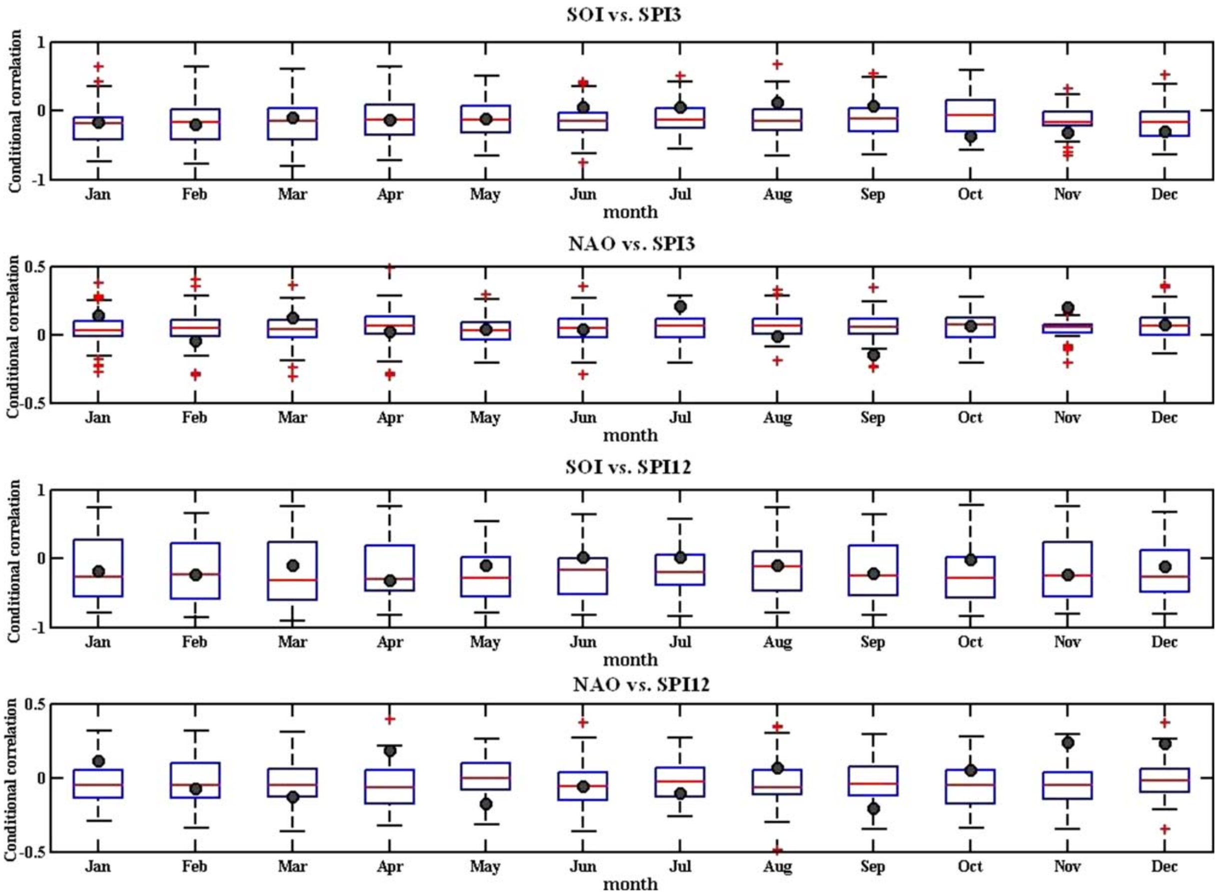

Figure 14. Monthly conditional correlation coefficient box plots for Shiraz station. Circles show unconditional correlation coefficients.

reject the null hypothesis of linearity. In contrast to Oroomieh station, it is observed that the nonlinearity is more or less similar among different covariances at Shiraz station.

6. Summary and Conclusions

[59] This paper developed and applied univariate and multivariate GARCH approaches, namely, diagonal VECH and diagonal BEKK models, to investigate the time-varying association between drought and atmospheric indices through a new conditional variance-covariance perspective. This study provides this new outlook by an example of the association of SOI and NAO to short-term and long-term SPI time series at two stations in Iran. For our case study, we showed that conditional variance of drought and atmospheric indices demonstrated different behaviors. While NAO shows a high degree of memory in the conditional variance, SOI does not show a strong long-run memory in the conditional variance. It is also observed that SPI time series have a stronger short-run persistency than long-run persistency for both short- and long-term SPI time series. In addition, the conditional variance of the atmospheric and drought indices seems to be stationary but nonlinear and suggests an inverse relationship between intensity of nonlinearity and stationarity.

[60] It was shown that the diagonal VECH model with nine parameters has less biased covariance estimations than the diagonal BEKK model with seven parameters. Both these models have reasonably fewer parameters than the full VECH model with 21 parameters, but the diagonal VECH model seems to give more accurate estimations for conditional covariance regarding the simulation experiment. Therefore, based on the diagonal VECH model outputs, a low level of covariance interaction between atmospheric circulations (SOI and NAO) and SPI time series for our two examples in Iran is observed. Both models show a weak long-run persistency link between the second-order moment of atmospheric circulations and drought. However, the short-run interaction is much stronger and indicates a significant relationship between the lagged cross products of the shocks, or random process, of atmospheric and drought indices. This implies that the variation of SOI and NAO may have a rapid and short-run influence on drought variation at the stations under investigation. This short-run interaction is also observed from conditional correlation coefficient time series which do not remain at the same level (high or low) for the long time before changing to the next (low or high) status. It was also seen that the correlation coefficients do not show a sharp seasonality and trend during 1954–2010.

[61] The results of the stationarity test for conditional covariances reveal a stationary covariance between drought and atmospheric indices for most cases during 1954–2010. It may indicate that the change in the link between

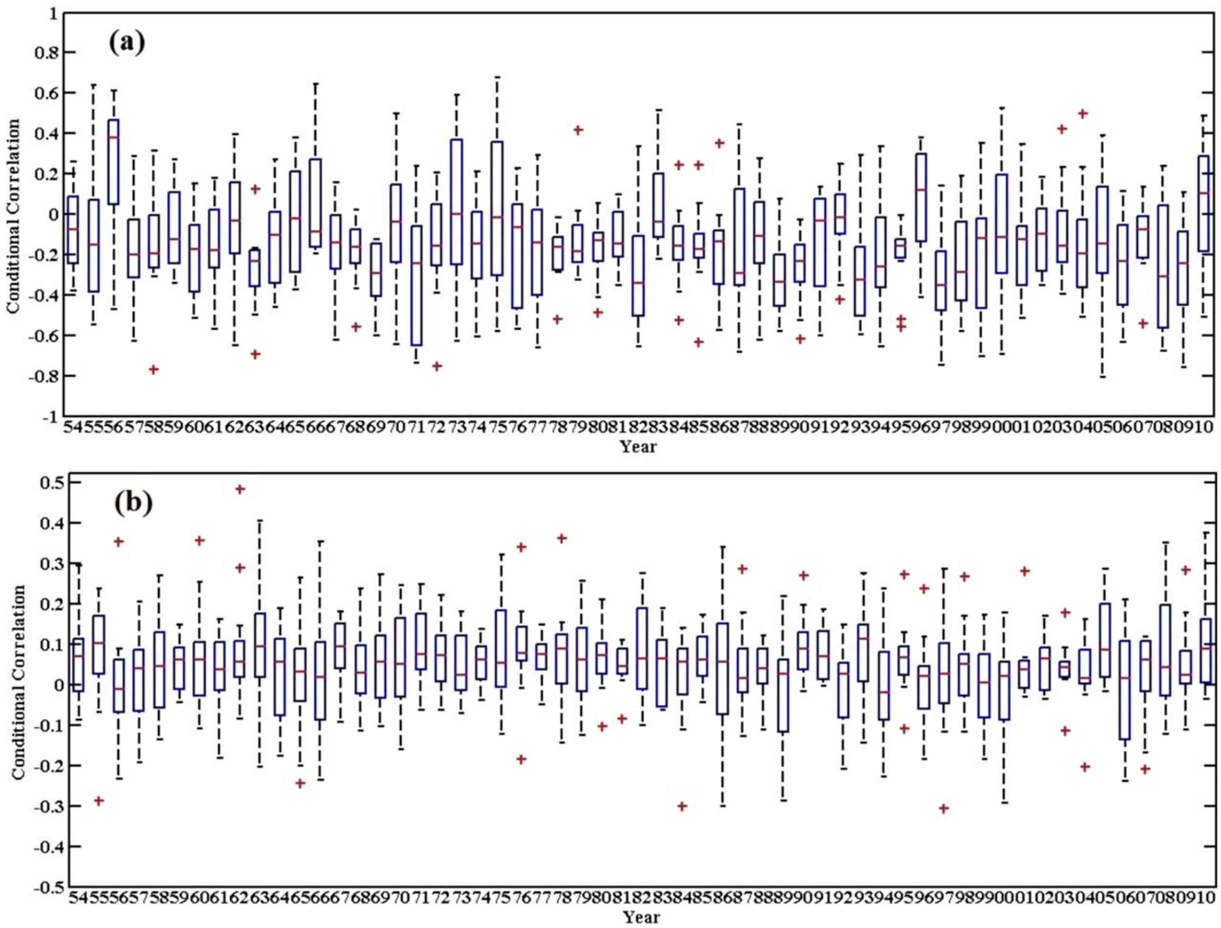

Figure 15. Annual conditional correlation box plots between (a) SPI3 and SOI as well as (b) SPI3 and NAO for Shiraz station.

atmospheric circulation and drought does not show a significant change during 1954–2010. However, some covariances cannot pass the trend stationarity test. Although the nonstationarity around a trend could be overaffected by some outlier data which makes the whole series nonstationary around such a basic trend, further tests and careful analysis are necessary to reveal the exact reason for nonstationarity around the trend for the connection between drought and atmospheric indices.

[62] It should be noted that (econometric) multivariate GARCH models show some advantages and disadvantages over hydrological models, temporal models such as time series models for the (conditional) mean and physically based models for rainfall-runoff modeling. The advantage of the proposed bivariate GARCH approach is incorporating the memory of the correlation between two time series in the previous time steps, $t-1$, $t-2, \ldots,$ to the correlation between them at time step t. This will improve our understanding of the association between two time series, i.e., drought and atmospheric indices. In other words, Figure 5 may show a weak association between drought and atmospheric indices regarding *unconditional* correlation coefficients. While, using the bivariate GARCH models shows that the *conditional* correlation between drought and atmospheric could be much stronger than what is inferred from unconditional correlation.

[63] However, the number of parameters grows rapidly with the order of the model, and parameter estimation becomes a real problem.

[64] Instead of this disadvantage, it is recommended to investigate the relationship between other atmospheric indices and other hydrologic and climatic variables such as other types of drought, rainfall, or streamflow. It is also important to examine the multivariate GARCH models for hydrologic drought and its relationship to meteorological drought or other atmospheric indices. In addition to examining the temporal behavior of the variance-covariance structure between drought and other atmospheric indices, it is important to investigate the physical rules governing this conditional variance-covariance structure in future studies. It would be interesting to apply the multivariate GARCH approach to investigate the effect of the time-varying variance of different variables such as rainfall, streamflow, temperature, wind speed, and evaporation on each other. The investigation and evaluation of the volatility and covolatility between climate and hydrologic variables in the context of climate change are also vital as the second-order moment of hydrologic variables may show a higher degree of fluctuations and nonlinearity in the future. This change in a climate variable may influence the other climatic and hydrologic variables in an exponential manner in future. Moreover, the physical variables influencing conditional covariance and the parameters

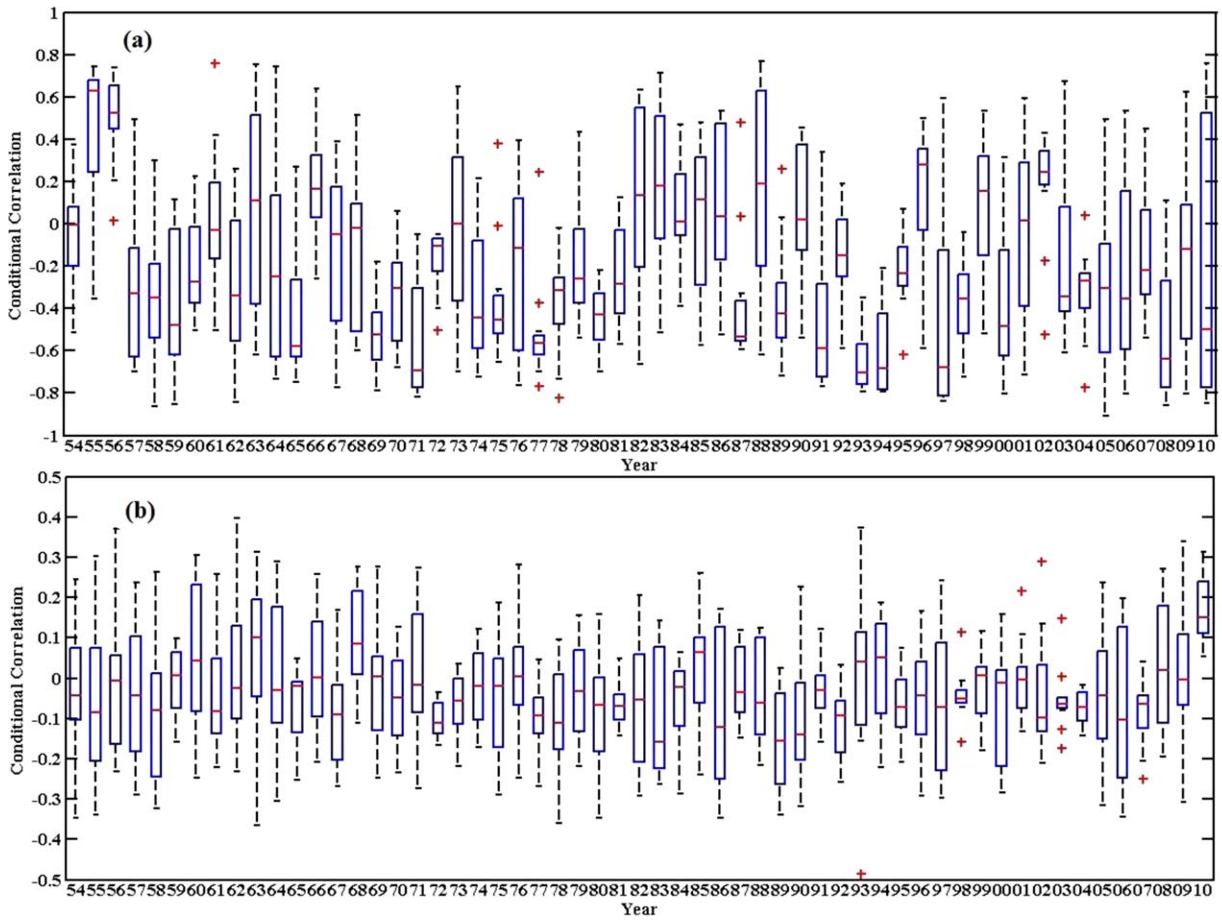

Figure 16. Same as Figure 15 but for SPI12.

of the multivariate GARCH models in hydrologic applications remain an important challenge for further studies.

[65] It is very important to mention that the above conclusion and statements of this study are based on only two stations in Iran. As the MGARH models and the stationarity and nonlinearity results are reported here for the first time for drought analysis, it is strongly recommended to continue this work for drought analysis in other parts of the world, especially the regions where are strongly influenced by NAO and SOI variation such as Australia [e.g., *Gallant et al.*, 2012]. This will give a better understanding of the conditional variance-covariance relationship between drought and atmospheric indices and provides an outlook of future climate change consequences on stationarity and nonlinearity of this relationship.

Table 10. Results for PP and ADF Tests for Stationary and BDS Test for Nonlinearity of Conditional Covariances at Shiraz Station

		PP Level Stationary Test		PP Trend Stationary Test		ADF Unit Root Test	
Indices Series	Drought Series	Results	*p* Value	Results	*p* Value	Results	*p* Value
SOI	SPI3	−17	>0.1	−0.0001	0.02	−17.3	0
	SPI12	−22.9	>0.1	−0.0003	0.25	−22.9	0
NAO	SPI3	−22.9	>0.1	−0.0003	0.25	−22.9	0
	SPI12	−23.8	>0.1	0.0008	0.02	−23.5	0

BDS Test							
		$m = 2$		$m = 3$		$m = 4$	
Indices Series	Drought Series	Statistic	*p* Value	Statistic	*p* Value	Statistic	*p* Value
SOI	SPI3	0.04	0	0.06	0	0.08	0
	SPI12	0.03	0	0.04	0	0.04	0
NAO	SPI3	0.03	0	0.04	0	0.04	0
	SPI12	0.04	0	0.09	0	0.13	0

[66] **Acknowledgments.** The authors thank the Natural Sciences and Engineering Research Council (NSERC) of Canada and the Canada Research Chair (CRC) Program for providing the financial support for this study.

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