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## Three-dimensional simulation of horseshoe vortex and local scour around a vertical cylinder using an unstructured finite-volume technique

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### ABSTRACT

A Large Eddy Simulation model is developed to simulate the hydrodynamics and scour process around a circular cylinder. The Navier-Stokes solver is based on the projection method and a second-order unstructured finite-volume method. A sigma-coordinate system is used to obtain an accurate representation of the evolution of the sediment-water interface. Bed erosion is simulated by solving the sediment continuity equation using a mass-conserving sand-slide algorithm and a bedload transport rate, which is based on a description of physical processes (Engelund & Fredsøe, 1976). Simulations of flow around a vertical cylinder for free-slip bed, rigid bed, and live-bed cases are done. The mean velocity profile and shear stress validate the accuracy of this model. Horseshoe vortex and lee-wake vortex shedding structure are simulated, and the results are thoroughly discussed in depth. The formation and the temporal development of the scour hole and other topographic bed features are successfully reproduced. The current paper reports the first known investigation of both scour evolution and coherent structure using large-eddy simulation.

Key words: Non-hydrostatic flows, Horseshoe vortex, Wake region, Scouring, Bedload transport.

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## 1. Introduction

One well-known difficulty with regard to scour studies is the local description of flow properties close to singularities (Baker, 1979; Dargahi, 1990; Kirkil et al., 2008; Kirkil & Constantinescu, 2010, 2015; Nishioka & Sato, 1974; Park, et al., 1998; Williamson, 1989). Indeed, due to the presence of obstacles, the flow forms a Horseshoe Vortex (HV) system upstream and a boundary layer detachment as well as complicated vortex shedding at the wake of the structure. These features of the problem significantly affect the scour as it evolves around the cylinder.

Due to the progress in the availability of computational power, numerical studies on scour around a vertical cylinder are now well-documented. However, the development of accurate and efficient models for the scour process is still a complicated task. First, hydrodynamic and sediment transport models are coupled such that a closure equation for the bedload estimation is required. Second, the sand-slide processes play an important role during the formation of a scour hole, thus, the local bed slope and friction angle of the bed material should be incorporated in the model. Third, a mesh adaptation around the cylinder boundary is required to capture the bed deformation (Khosronejad et al., 2011, 2012). Finally, turbulence models are necessary to simulate the coherent structures of flow fields with the presence of a HV system. These factors must be overcome if better models are to be obtained.

Roulund et al. (2005) presented a major advance in simulating local scour around a vertical cylinder by using a structured finite volume code to solve the Reynolds Averaged Navier-Stokes (RANS) equations with a  $k-\omega$  turbulence closure (where  $k$  is the turbulent kinetic energy and  $\omega$  is the specific dissipation rate). Overall, their numerical results coincide with their experimental

24 data and the study is frequently referenced in many studies (Baykal et al., 2014, 2017; Stahlmann,  
25 2013; Zhou, 2017). The work of Kirkil et al. (2008) and Kirkil & Constantinescu (2010, 2015)  
26 applied a Large-Eddy Simulation (LES) to analyze the coherent structure of flow fields with the  
27 presence of a HV system upstream and a wake region behind a vertical cylinder. However, their  
28 simulations started from an equilibrium hole obtained from experimental results such that no  
29 scour evolution was analyzed. Khosronejad et al. (2011, 2012) developed a Fluid-Structure  
30 Interaction Curvilinear Immersed Boundary (FSI-CURVIB) method with a  $k-\omega$  closure model.  
31 Their work has shown that the bluntness of the pier significantly influences the predictive  
32 capabilities of models. Link et al. (2012) used a Detached-Eddy Simulation (DES) to simulate  
33 scour hole evolution around circular and rectangular piers. More recently, Nagel (2018) used a  
34 Eulerian two-phase model to study the live bed erosion around a cylinder.

35 In the current paper, a 3D numerical study is presented on local scour around a vertical cylinder  
36 using the Engelund and Fredsøe (1976) bedload formula applied to the simulations done by  
37 Roulund et al. (2005), which is classified as a live bed case (Melville & Chiew, 1999). A second-  
38 order unstructured finite-volume model combined with a sigma-coordinate system is applied to  
39 describe the dynamic shape of the sediment-water interface. In order to prevent the bed slope  
40 from exceeding the angle of repose for the sediment material, a mass conservating sand-slide  
41 model is developed. Large Eddy Simulation is applied to simulate the HV-system structure at the  
42 base of a vertical cylinder. Large-scale coherent structures in the presence of an HV system in  
43 the near-bed region as well as vortex shedding in the wake region are observed and discussed  
44 here. Table 1 provides highlights of the current study by giving an overview of different terms  
45 handled in the previously cited studies. The current paper reports the first known investigation of  
46 both scour evolution and coherent structures using a finite volume method.

47 The current paper is organized as follows. The second section introduces the hydrodynamic  
 48 model. The third section presents the morphodynamic model in a sigma-coordinate system  
 49 including the bedload estimation and sand-slide model. The fourth section summarizes the  
 50 numerical techniques used in the simulation. The fifth section presents the numerical  
 51 simulations. Finally, the last section gives the conclusions and offers suggestions for future  
 52 research.

## 53 2. The hydrodynamic and SubGrid-Scale modeling

54 The Navier-Stokes equations are filtered with an implicit spatial filter of characteristic space size.  
 55 The resulting equations are given by:

$$56 \quad \frac{\partial \bar{u}_i}{\partial x_i} = 0, \quad (1)$$

$$57 \quad \frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j} \bar{u}_i \bar{u}_j = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \nu \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \right] - \frac{\partial \tau_{ij}^{sg}}{\partial x_j}, \quad (2)$$

58 where  $x_i$  ( $i=1,2,3$ ) are  $(x,y,z)$ , respectively, are the Cartesian coordinates, the bar over the  
 59 variables denotes filtering,  $\bar{u}_i$  are the resolved velocity components of the velocity vector,  $\bar{p}$  is  
 60 the resolved pressure, and

$$61 \quad \tau_{ij}^{sg} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j, \quad (3)$$

62 is the Sub-Grid Stress (SGS) tensor used to take into account the effect of unresolved length  
 63 scales.

64 As suggested by Smagorinsky (1963), the smallest turbulent eddies are almost isotropic and the  
 65 Boussinesq eddy viscosity assumption can be used to provide an accurate approximation of the

66 effects of these unresolved smallest eddies. According to the Boussinesq assumption, the  
 67 momentum transfer caused by turbulent eddies can be modeled with an eddy viscosity, and the  
 68 relations between the eddy viscosity and the sub-grid stress tensor can be described as:

$$69 \quad \tau_{ij}^{sg} = -2\nu_t \overline{S_{ij}} + \frac{1}{3} \tau_{ii}^{sg} \delta_{ij}, \quad (4)$$

70 where  $\delta_{ij}$  is the Kronecker delta,  $\overline{S_{ij}} = \frac{1}{2} \left( \frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right)$  is the resolved rate-of-strain tensor, and  $\nu_t$   
 71 is the SGS viscosity:

$$72 \quad \nu_t = l_{sg}^2 |\overline{S}|, \quad (5)$$

73 where  $|\overline{S}| = (2\overline{S_{ij}}\overline{S_{ij}})^{1/2}$ , and  $l_{sg}$  is the sub-grid length scale. In the near wall region, the length  
 74 scale of the sub-grid scale motions cannot be described with a constant value but will decrease as  
 75 the wall is approached. Thus, a wall damping function must be implemented in addition to the  
 76 standard Smagorinsky SGS model to capture this boundary layer effect. In the current study, the  
 77 near wall damping model of Mason and Thomson (1992) is used to achieve a modified length  
 78 scale, described by:

$$79 \quad \frac{1}{l_{sg}^n} = \frac{1}{(C_s \Delta)^n} + \frac{1}{(\kappa l_w)^n}, \quad (6)$$

80 where  $n$  is the Mason wall matching power, taken as 2,  $C_s$  is the Smagorinsky constant,  $\Delta$  is the  
 81 average spacing  $\Delta = V^{1/3}$  with  $V$  being the volume of the element, and  $\kappa$  is the von Karman  
 82 constant, and  $l_w$  is the distance from the center of a control volume to the wall. The value of  $C_s$   
 83 = 0.1 previously proposed for channel flow (Deardorff, 1970) is used in the current study,  
 84 where strong anisotropic turbulence occurs in the near-wall region.

85 In the current study, non-dimensional governing equations for the flow are derived using the  
 86 mean velocity,  $U$ , and the cylinder diameter,  $D$ , as velocity and length scales, respectively. The  
 87 Reynolds number based on the cylinder diameter is defined as  $Re_D = UD/\nu$ .

### 88 **3. The bed morphodynamic model**

89 The current study is focused on the numerical simulation of bedload transport and the suspended  
 90 load is negligible. The free surface also is ignored because small Froude numbers are accounted  
 91 for in the proposed model (Roulund et al., 2005). The bed morphodynamic model includes the  
 92 Exner-Polya equation, the bedload estimation, and the sand-slide model.

#### 93 *3.1. The Exner-Polya equation*

94 The bed evolution caused by bedload transport is given by the Exner-Polya equation (Yalin,  
 95 1977):

$$96 \quad (1-\eta)\frac{\partial z_b}{\partial t} + \nabla \cdot q_b = 0. \quad (7)$$

97 Where  $z_b(x,y,t)$  is the local bed level,  $\eta$  is sediment material porosity, and  $q_b$  is the bedload  
 98 transport rate depending on the hydraulic and sediment variables. The porosity of the granular  
 99 material correlates to the particle arrangement (close and random) and monodispersity of  
 100 materials. For really monodisperse material of identical beds, the solid volume fraction could  
 101 vary between 0.54 and 0.74 depending on the cubical arrangement or maximum hexahedral  
 102 packing. With preparation of the sediment bed in water, the value usually is 0.6 and porosity  $\eta$  is  
 103 taken as 0.4 (Pham Van Bang et al., 2008). The model to calculate  $q_b$  is described in the next  
 104 section.

105 3.2. *Bedload estimation*

106 A two-dimensional bedload formula proposed by Engelund and Fredsøe (1976) is applied. The  
107 bedload transport rate,  $q_b$ , is given by:

$$108 \quad q_b = \frac{1}{6} \pi d^3 \frac{p_{EF}}{d^2} U_b, \quad p_{EF} = \left[ 1 + \left( \frac{\frac{1}{6} \pi \mu_d}{\theta - \theta_c} \right)^4 \right]^{-1/4}, \quad U_b = a U_f (1 - 0.7 \sqrt{\theta_c / \theta}), \quad (8)$$

109 where  $d$  is the grain size,  $p_{EF}$  is the percentage of particles in motion on the bed surface,  $U_b$  is  
110 the mean velocity of a moving sediment particle,  $U_f$  is the friction velocity,  $\mu_d = 0.51$  represents  
111 the dynamic friction coefficient corresponding to value  $27^\circ$  for the angle of repose of sediment  
112 material,  $\theta$  is the Shields parameter associated with the skin friction,  $\theta_c$  is the critical value of  $\theta$   
113 for the initiation of sediment motion on the bed, and  $a$  is an empirical constant. Experimental  
114 predictions are generally good for  $a = 10$  and a suitable choice of  $\theta_c$  (Fernandez Luque & van  
115 Beek, 1976; Meland & Norrman, 1966). The critical Shields parameter is calculated by:

$$116 \quad \theta_c = \theta_{c0} \left( \cos \beta \sqrt{1 - \frac{\sin^2 \alpha \tan^2 \beta}{\mu_s^2}} - \frac{\cos \alpha \sin \beta}{\mu_s} \right), \quad (9)$$

117 where  $\theta_{c0}$  is the critical Shields parameter for a horizontal bed, taken as 0.05,  $\mu_s$  is the static  
118 friction coefficient, taken as  $\mu_s = 0.63$  for sand (Roulund et al., 2005),  $\beta$  is defined as the angle of  
119 steepest decent calculated from the elevation gradient of the longitudinal bed, and  $\alpha$  is the angle  
120 between the flow-velocity vector,  $U_b$ , at the top of the bedload layer and the steepest bed slope.

121 The Shields parameter in Eq. (8) is defined by

$$122 \quad \theta = \frac{\|U_f\|^2}{(s-1)gd}, \quad (10)$$



123 where  $s=\rho_s/\rho$  is the specific gravity of the sediment grains, and  $g$  is the acceleration of gravity.  
 124 The friction velocity is determined utilizing a method developed by Nikuradse (1933), which is  
 125 based on bed roughness in a logarithmic velocity profile:

$$126 \quad \frac{\langle U \rangle}{U_f} = 2.5 \ln \left( \frac{z}{z_0} \right), \quad (11)$$

127 where  $z$  is the distance to the wall, and  $z_0$  is the distance from the boundary at which point the  
 128 idealized velocity given by the Wall Law goes to zero, and  $\langle \rangle$  represents time-averaged (Rodi et  
 129 al., 2013). According to Nikuradse (1933),  $z_0$  is equal to  $k_s/30$ , in which  $k_s$  is the Nikuradse  
 130 equivalent sand roughness.

131 In the current study, the bedload is assumed to move in the same direction as the tangential shear  
 132 stress on the bed. The present bedload model reduces the algorithm to simple calculation as  
 133 shown by Zhou (2017).

### 134 3.3. The sand-slide model

135 Without a sand-slide model, an unrealistic bed-slope which is larger than the physical value of  
 136 the angle of repose will occur, and large mesh distortion especially will appear, especially around  
 137 the vertical cylinder. As a consequence, the sigma-transformation will lose its ability to simulate  
 138 scour holes. In order to prevent the bed slope from exceeding the sediment angle of repose, a  
 139 mass-conservation-based algorithm for sand-slide has been applied successfully (Khosronejad et  
 140 al., 2011, 2012). The bed slope is defined by the elevation gradient between point  $p$  and any  
 141 point  $i$  ( $i=1, 2, 3$ ) of the neighboring cell centers, see Fig. 1(a). If the slope angle exceeds the  
 142 material angle of repose, the sediment particles will slide down to the angle of repose, see Fig.  
 143 1(b). The correction to bed elevations is calculated by:

$$144 \quad \frac{(z_{bp} + \Delta z_{bp}) - (z_{bi} + \Delta z_{bi})}{\Delta l_{pi}} = \tan \phi, \quad (12)$$

145 where  $\phi$  is the material angle of repose,  $z_{bp}$  and  $z_{bi}$  are the bed elevations at points  $p$  and its  $i^{th}$   
 146 neighbor;  $\Delta z_{bp}$  and  $\Delta z_{bi}$  are the corresponding corrections, and  $\Delta l_{pi}$  is the horizontal distance  
 147 between these two points. The bed elevation corrections are obtained by mass conservation as  
 148 follows:

$$149 \quad A_{hp} \Delta z_{bp} - \sum_{i=1}^3 A_{hi} \Delta z_{bi} = 0, \quad (13)$$

150 where  $A_{hp}$  and  $A_{hi}$  are the area projection of cells  $p$  and  $i$ , respectively. Since the mesh points  
 151 move only in the vertical direction, the bed cells projected onto the horizontal plane have the  
 152 same cell area.

#### 153 **4. Numerical techniques**

154 The hydrodynamic model solves the three-dimensional Navier-Stokes equations using a  
 155 projection method combined with a sigma-transformation and a second-order unstructured finite  
 156 volume method.

##### 157 *4.1. The projection method*

158 In this paper, the projection method developed by Chorin (1968) and Temam (1968) is used to  
 159 decouple the pressure and the velocity field in solving the Navier-Stokes equations. In Cartesian  
 160 coordinates, the velocity field is obtained by solving a convection-diffusion equation and then  
 161 the provisional velocity is projected as

$$162 \quad \frac{\bar{u}_i^* - \bar{u}_i^n}{\Delta t} + \frac{\partial(\bar{u}_i^* \bar{u}_j^n)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \bar{u}_i^*}{\partial x_i} - \tau_{ij}^n \right), \quad (14)$$

$$163 \quad \frac{\bar{u}_i^{n+1} - \bar{u}_i^*}{\Delta t} = -\frac{1}{\rho} \frac{\partial \bar{p}^{n+1}}{\partial x_i}, \quad (15)$$

$$164 \quad \frac{\partial \bar{u}_i^{n+1}}{\partial x_i} = 0, \quad (16)$$

165 where  $n$  and  $n+1$  represent time  $t^n$  and  $t^{n+1}$ , respectively,  $\Delta t$  is the time step, and (\*) is purely  
 166 symbolic; designating the velocity values obtained after the first projection step. The  
 167 combination of the continuity and momentum equations yields the Poisson equation. The sigma-  
 168 transformation is applied to the projection equations in Cartesian coordinates.

#### 169 4.2. Sigma-transformation

170 In order to accurately represent bottom geometry, the sigma-coordinate system developed by  
 171 Phillips (1957) is applied in the current study. This method is applied in order to link the  
 172 irregular physical domain to a rectangular computational one (see Fig. 2). The conventional  
 173 sigma-coordinates map the total water depth which is measured from the mobile bottom to the  
 174 surface of the water onto a fixed range  $\sigma = [0, 1]$  in the computational domain as follows:

$$175 \quad x^* = x, y^* = y, t^* = t, \sigma = \frac{z+h}{h}, \quad (17)$$

176 where  $h(x^*, y^*, t^*)$  is the total water depth, and  $z = [-h, 0]$  is the vertical coordinate in the physical  
 177 domain with the origin at  $z_s = 0$  on the still water surface level, and  $z_b = -h$  on the bed level. The  
 178 partial differentiation of a variable  $\varphi$  in the physical domain is transformed as follows:

$$179 \quad \frac{\partial \varphi}{\partial t} = \frac{\partial \varphi}{\partial t^*} + \sigma_t \frac{\partial \varphi}{\partial \sigma}, \quad (18)$$

$$180 \quad \frac{\partial \varphi}{\partial x} = \frac{\partial \varphi}{\partial x^*} + \sigma_x \frac{\partial \varphi}{\partial \sigma}, \quad \frac{\partial \varphi}{\partial y} = \frac{\partial \varphi}{\partial y^*} + \sigma_y \frac{\partial \varphi}{\partial \sigma}, \quad \frac{\partial \varphi}{\partial z} = \sigma_z \frac{\partial \varphi}{\partial \sigma}, \quad (19)$$

181 where

$$182 \quad \sigma_t = \frac{1}{h}(1-\sigma) \frac{\partial h}{\partial t^*}, \quad (20)$$

$$183 \quad \sigma_x = \frac{1}{h}(1-\sigma) \frac{\partial h}{\partial x^*}, \quad \sigma_y = \frac{1}{h}(1-\sigma) \frac{\partial h}{\partial y^*}, \quad \sigma_z = \frac{1}{h}. \quad (21)$$

184 The foregoing transformation can then be applied to all the terms in the mass and momentum  
185 equations and further details can be found in the work of Uh Zapata et al. (2014).

#### 186 4.3. Finite volume discretization

187 A second-order unstructured finite volume method (UFVM) is chosen to discretize the Navier-  
188 Stokes equations using a cell-center collocated grid with auxiliary vertex points. The 3D domain  
189 in sigma-coordinates is discretized into prisms: triangular elements in the horizontal axis and  
190 layers in the vertical axis. Thus, each prism-shaped control volume,  $V$ , has five faces  $S_l$   
191 ( $l=1, \dots, 5$ ).

192 A UFVM for the Poisson equation has already been developed and tested by the authors of this  
193 study in a previous work (Uh Zapata et al., 2014). The implicit Runge-Kutta time-advancement  
194 scheme for the momentum equation of any velocity component in the projection method can be  
195 written as follows:

$$196 \quad \int_V \bar{u}_i^* dV + \Delta t \sum_{l=1}^5 \int_{S_l} \left[ \bar{u}_i^* \bar{u}_j^n - \nu \frac{\partial \bar{u}_i^*}{\partial x_i} + (\tau_i^{sg})^n \right] n_l dS = \int_V \bar{u}_i^n dV, \quad (22)$$

197 where  $n_l$  is the outward normal unit vector at each face  $S_l$ . By the requirements of the LES  
 198 technique and to preserve second order accuracy, advection terms are handled by a centered  
 199 scheme. The Momentum Interpolation Method (Rhie & Chow, 1983) is applied to prevent a  
 200 checkerboard pattern in the pressure field by using a collocated grid. The discretization of the  
 201 diffusive fluxes involves knowledge of the derivatives at the interface between two neighboring  
 202 control volumes. In the current study, these derivatives are approximated as an average over a  
 203 prism-shape, delimited by the two cell-centered points and the vertex of the interface. Thus, the  
 204 finite volume discretization of the diffusion term relates an unknown value not only with its  
 205 surrounding cell-centered values, but also with the vertex values of a control volume. The value  
 206 at any vertex is obtained by averaging over all surrounding cell-centered nodal values.

207 The morphodynamic model, given by Eq. (7), is integrated on each triangular element. After  
 208 application of Green's theorem, the integral form of the right-hand side becomes:

$$209 \quad \iint_A \frac{z_b^{n+1} - z_b^n}{\Delta t} dS = -\frac{1}{(1-\eta)} \sum_{j=1}^3 \int_{L_j} q_b \cdot n_j dl, \quad (23)$$

210 where  $A$  indicates a triangular element, while  $L_j$  and  $n_j$  ( $j=1,2,3$ ) represent an edge and the  
 211 corresponding unit normal vector of the triangular element.

212 Boundary conditions are integrated into the problem by updating the ghost cell-centered values.  
 213 In other words, the total number of unknowns includes not only the values at the cell-centered  
 214 points inside the domain, but also the ghost points outside of the domain. In the case of Dirichlet  
 215 boundary conditions, known values are directly applied at vertex points and included at the cell-  
 216 centered points. In the case of Neumann conditions, approximations can easily be applied for the  
 217 cell-centered points using a central approximation. However, there is no direct way to apply this

218 condition at vertex values due to the unstructured grid. The vertex values are update exclusively  
219 using an interpolation technique.

220 The current code is fully parallelized using the domain decomposition scheme and Message  
221 Passing Interface (MPI) (Uh Zapata et al., 2016), and the code is fully tested on Électricité de  
222 France (EDF)-France ATHOS high performance computers (Intel Xeon E 22000 CPUs).

## 223 5. Numerical simulations

224 A sketch of the computational geometry for the numerical simulations around a circular cylinder  
225 is shown in Fig. 3. The computational domain extends  $10D$  upstream,  $20D$  downstream and  $30D$   
226 lateral from the cylinder, in which  $D$  is the cylinder diameter. The origin of the coordinate  
227 system is located at the bottom of the cylinder's center, with the  $x$ ,  $y$ , and  $z$  axes corresponding to  
228 the streamwise, spanwise, and ascendant vertical direction, respectively. The calculation domain  
229 is discretized into the order of three million finite volumes with a fine resolution at the HV  
230 system region around the cylinder and close to the bed in the vertical direction. Horizontally, the  
231 first row of elements is situated at  $0.003D$  away from the cylinder surfaces corresponding to 14  
232 wall units. A detailed view of the deformed mesh close to the cylinder is shown in Fig. 4.

233 The boundary conditions for the flow field are defined as follows: at the inlet, transverse and  
234 vertical velocities are specified as zero. The inflow velocity is given by a Poiseuille profile with  
235 a unit mean non-dimensional value and boundary layer thickness  $\delta/D = 0.5$ . At the outlet, zero-  
236 gradient conditions were applied for all variables. The boundary in the spanwise direction is set  
237 as Neumann conditions. As was pointed out in the previous section, a rigid surface is used on  
238 the top. Finally, no-slip wall boundary conditions are applied for the cylinder surface and the  
239 bottom bed.

240 In the current paper, a numerical study similar to Roulund et al. (2005) is done. The diameter of  
241 the cylinder is  $D = 0.1\text{m}$ . The water depth is set to be  $h = 4D$ . The undisturbed mean flow  
242 velocity is  $U = 0.46\text{m/s}$ . The sand size is  $d = 0.26\text{mm}$ . The equivalent roughness height for the  
243 rough wall function is set to be  $k_s = 2.5d$ , which determines the skin friction velocity in turn. In  
244 the current study, three kinds of calculations are done: a free slip bed with Reynolds number  $Re_D$   
245  $= 3,900$  is done in order to validate the results with experimental data; rigid bed calculations with  
246  $Re_D = 46,000$  is applied to investigate the flow features only; and a live bed scour with  $Re_D =$   
247  $46,000$  is applied to test the morphological model. The initial condition for live bed scour is  
248 derived from the final results of the rigid bed simulation. The conditions in the last two cases are  
249 summarized in Table 2.

### 250 *5.1. Free slip bed simulation results using $Re_D = 3,900$*

251 In order to validate the proposed model, a comparison between numerical and experimental  
252 results is presented for a free slip bed at  $Re_D = 3,900$  and the bottom boundary condition is set as  
253 free-slip. The vertical mean streamwise ( $u/U$ ) and spanwise ( $v/U$ ) velocity components at five  
254 different stations,  $x = 1.06, 2.02, 3.0, 4.0,$  and  $5.0$ , are plotted in Fig. 5. A U-shaped profile is  
255 observed near the wake region. The experimental data LS93 and OW93 were obtained from  
256 Lourenco (1993) and Ong and Wallace (1996), respectively. LES B16 corresponds to numerical  
257 results from Bai et al. (2016). It can be seen that the numerical results largely correlate with the  
258 experimental data, except at  $x = 1.06$ , where the uncertainties in the measurements are high. For  
259 the purpose of demonstrating this discrepancy, the numerical result obtained by Tremblay et al.  
260 (2000) using Direct Numerical Simulation (DNS) is also included in Fig. 5(b). It can be seen that  
261 the current numerical results actually are very close to the DNS prediction.

262 5.2. Rigid bed flow simulation results using  $Re_D = 46,000$

263 5.2.1. Shear stress validation

264 Figure 6 shows the bed shear-stress iso-values, obtained from the mean flow simulation and  
265 compared with the results of Roulund et al. (2005). The model reproduces most of the  
266 characteristics observed by Roulund et al. (2005) in the vicinity of the cylinder but with very  
267 slight discrepancies far away from it. The average distribution of the shear stress is fairly  
268 axisymmetric. High values are found for shear stress on the two sides close to the cylinder,  
269 approximately corresponding to the position where the velocity is highest, just before the flow  
270 separation line.

271 5.2.2. Horseshoe vortex system

272 The HV system is attributable to the separation of the incoming boundary layer induced by  
273 adverse pressure gradients, which are generated by the cylinder obstruction. The turbulence  
274 driven by the resulting flows will result in a number of necklace-like structures around the  
275 upstream side of the cylinder. Because of the lateral pressure gradients, these structures stretch  
276 when they fold around the cylinder (Kirkil et al., 2008). Their legs are approximately parallel to  
277 the direction of the incoming flow. Figure 7 shows the main coherent structures in an  
278 instantaneous flow associated with a HV system on a rigid bed with  $h = 4D$  and  $Re_D = 46,000$ .  
279 Clearly two primary necklace vortices and two bottom-attached vortices are present. A U-shaped  
280 Primary Vortex (PV1) wraps around the upstream part of the cylinder, along which a small, but  
281 very coherent, junction vortex appears at the base of the cylinder (JV). Another U-shaped  
282 Primary Vortex (PV2) is observed upstream of PV1. The formation of Bottom-Attached Vortices,  
283 BAV1 and BAV2, are induced upstream by the presence of the primary vortices PV1 and PV2.  
284 Secondary Vortices (SV1) are observed towards PV2.



285 The location, size, and intensity of the turbulent HV greatly vary in time. Figure 8 shows four  
286 instances of a cycle of oscillation. The main vortex cores occur in different positions. Clockwise-  
287 rotating primary vortices (PV1, PV2) and counter-rotating bottom-attached vortices (BAV1,  
288 BAV2) appear at all times during the oscillation cycle, exhibiting a relatively stable behavior.  
289 Smaller secondary vortices (SV) shed randomly from the separation region of the incoming  
290 boundary layer. These SV are convected toward PV2 and can interact and merge with it. The  
291 direction of the oscillation centers of PV1 and PV2 are found to be oppositely positioned to one  
292 another: at  $t = T/2$  they move closer, and at  $t = T/4$  and  $t = 3T/4$  they move far away from each  
293 other. The amplitude of oscillation in the direction of flow is about  $0.1D$ . The structure of the HV  
294 system, observed in the results of current model using  $h = 4D$  and  $Re_D = 46,000$ , is similar to the  
295 ones of Kirkil et al. (2008) using  $h = 1.12D$  and  $Re_D = 16,000$ .

### 296 5.2.3. Near wake flow

297 Figure 9 shows mean streamlines on a longitudinal plane located behind the cylinder in the wake.  
298  $S$ ,  $F$ , and  $N$  denote the saddle points, the centers of foci, and nodal points, respectively. The  
299 arrows represent the direction of the flow. It is noticed that a nodal point of attachment  $N_l$  is  
300 present, corresponding to the merging point of shear layers emanating from both sides of the  
301 vertical cylinder to form a spanwise-oriented vortex. As a consequence, fluid particles situated  
302 close to the bed are first entrained into the core of this vortex, and then from there toward the  
303 surface by an upwelling anti-clockwise vortex, shown by a foci point,  $F_l$ . This counter-clockwise  
304 rotating vortex is responsible for the scour mechanisms downstream of the cylinder.

305 Figure 10 shows mean flow streamlines on different spanwise cross sections located at four  
306 different stations:  $x/D = 0.5, 1.0, 1.5,$  and  $2.0$ . The streamline patterns are almost symmetric,  
307 except for the lee wake just behind the cylinder. A pair of vortices is present near the bed

308 corresponding to the primary vortex legs. It can be seen that even at  $x = 2D$ , these legs are still  
309 present. The shear layers, emanating from the side edges of the cylinder, roll up to form vortices  
310 in the lee wake of the cylinder. These vortices engender upwelling motion of fluid particles  
311 toward the surface. The upwelling motion inside the wake region is compensated by  
312 downwelling motion on the outside.

313 Figure 11 shows mean flow streamlines on horizontal planes at  $z/D = 0.05, 1.0, \text{ and } 2.0$  in the  
314 wake region. The flow separation points are located at  $\varphi = 90^\circ, 105^\circ, \text{ and } 110^\circ$  at  $z/D = 0.05, 1.0$   
315 and  $2.0$ , respectively. It can be seen that these separation points move downstream as  $z/D$   
316 increases. This observation is also noted by Kirkil and Constantinescu (2015). Moreover, Figure  
317 11 shows that the wake region becomes bigger as it rises to the surface. Saddle points,  $S_2$  and  $S_3$   
318 indicate the end of the wake region at planes  $z/D = 0.05$  and  $1.0$  and the corresponding  
319 detachment lengths are  $0.9D$  and  $1.9D$ , respectively. Foci points  $F_4 (x = 0.55, y = 0.13)$ ,  $F_5 (x =$   
320  $1.24, y = 0.2)$  and  $F_6 (x = 2.05, y = -0.22)$  show the position of upwelling vortices  $V_1, V_2, \text{ and } V_3$ .

### 321 5.3. Live bed scour simulation results at $Re_D = 46,000$

#### 322 5.3.1. Horseshoe vortex system

323 Figure 12 shows the main necklace vortices inside the scour hole by using the  $Q$  criterion.  
324 Similar to the rigid bed case, U-shaped necklace vortices also appear in the live bed case. They  
325 contain two Primary Vortices (PV1 and PV2), a Bottom-Attached Vortex (BAV), and a Joint  
326 Vortex (JV) (Dey & Raikar, 2007). The HV system is closely related to the scour process.  
327 According to Baker (1979) and Kirkil et al. (2008), the number of these necklace vortices and  
328 their extent in the polar direction may change with the Reynolds number. Necklace-like  
329 structures detach from the incoming bottom boundary layer over a certain range of polar angles

330 and then interact with other secondary vortices or with the primary one. Some of these vortices  
331 merge with the main or another secondary necklace vortex; others will lose their coherence  
332 rapidly. Consequently, the intensity of the overall HV system varies substantially over time. In  
333 many cases, the interaction with another necklace structure takes place only over a limited area  
334 of the total length of two vortices.

335 Figure 13 provides more details on the temporal evolution of the streamlines in the scour hole  
336 upstream from the cylinder. The current model successfully simulates the dynamics of the initial  
337 stages of erosion by the unsteady coherent structures of the HV system. As the scour hole gets  
338 deeper and extends, the HV system grows in size but decreases in strength until reaching the  
339 conditions for equilibrium. Such a state of equilibrium is reached when the shear stresses are  
340 reduced down to a local threshold value for sediment-particle entrainment. Once the scour hole  
341 has formed, the HV system becomes more stable. During the scour process, PV1 still oscillates  
342 around at  $x = 0.5D$  upstream the cylinder, as for the rigid bed case, and slips down into the  
343 middle of the hole to adapt to a new position in the bed. PV2 grows bigger and increases in  
344 coherence. With the development of PV1 and PV2, a BAV is generated and grows between them.  
345 The vortices corresponding to JV grow bigger as the scour domain develops close to the cylinder.

### 346 5.3.2. Near wake flow

347 Figure 14 shows time-averaged streamlines for the live bed case on a longitudinal plane behind  
348 the cylinder in the wake region. A large recirculation zone is generated due to the changes in bed  
349 topography as the flow moves downstream. As with the rigid bed cases, a nodal point,  $N_2$ , of  
350 attachment positioned at  $(x/D = 0.875, z/D = -0.1785)$  exists. The negative sign indicates that this  
351 level is below the initial bed level.  $N_2$  corresponds to the merging point of the streamlines issued  
352 from the convergence of both lateral sides of the cylinder. Thus, sediment particles situated close

353 to the bed are entrained by the flow: first in the spanwise direction and later toward the surface  
354 by an upwelling anti-clockwise vortex, indicated by foci  $F_7$ . The presence of foci and nodal  
355 points in the wake region has already been observed in experimentation using  $h = 6D$  and  $Re_D =$   
356 7000 (Sahin & Ozturk, 2009).

357 Figure 15 shows a 3D view of time-averaged streamlines around the cylinder. Clearly, a down  
358 flow can be observed at the upstream side of the cylinder which generates a primary vortex, as  
359 discussed in the preceding paragraphs. The flow structure in the wake region also is shown in  
360 this figure. Streamlines wrap the cylinder from both lateral sides. Then, flow in the spanwise  
361 direction converges at nodal point  $N_2$ . From  $N_2$ , due to an upwelling vortex ( $F_7$ ), an important  
362 patch of fluid rises up into the surface layers, entraining sediment particles situated on the bed,  
363 and bringing them out of the wake. This phenomenon induces the scour process behind the  
364 cylinder.

### 365 5.3.3. *General erosion patterns and maximum erosion depth prediction*

366 Figure 16 shows the evolution of the scour hole over time as obtained by the proposed model at  
367 four times: 30 s, 120 s, 300 s and 750 s. The pattern exhibited by the scour hole closely  
368 resembles the results observed by Roulund et al. (2005). The deepest part of the main scour hole  
369 occupies the upstream and spanwise sides of the cylinder. A maximum angle for the bed slope is  
370 fixed inside this region, equal to the prescribed angle of repose:  $32^\circ$ . The exact localization of the  
371 maximum angle corresponds to regions where the sand-slide algorithm functions and where the  
372 avalanching process is yielded by the model. Additionally, sand particles are deposited  
373 downstream from the cylinder. Some small bed changes are observed far from the main scour  
374 hole. The evolution of sand deposition downstream from the cylinder is almost symmetric with

375 the presence of sandpits. These sandpits tend to decrease with time without being fully erased at  $t$   
376  $= 750s$ .

377 Figure 17 shows the time evolution of the scour depth at the upstream and downstream sides of  
378 the cylinder compared with the measurements of Roulund et al. (2005). At the beginning of the  
379 scour, both numerical models slightly over-predict the depth at the upstream side while under-  
380 predicting the depth at the downstream side compared to the experimental data. Roulund et al.  
381 (2005) explained the reasons for the downstream discrepancy between simulation and  
382 experimental results. One reason is that the suspended load process is not included in the model,  
383 therefore, the model scour depth remains rather small during this stage. Another reason is that  
384 the vortex shedding in the lee wake of the cylinder is ignored, which will decrease the predicted  
385 scour depth downstream of the cylinder (Sumer et al., 1988). Here, the second factor has been  
386 taken into consideration in the proposed model. Clearly, the current numerical results on the hole  
387 evolution are in a better agreement with the experimental results both in the upstream and  
388 downstream directions, compared with those of Roulund et al. (2005). The simulation results  
389 from Roulund et al. (2005) have reached the equilibrium condition at about 1000 s, but with  
390 scour depth values are smaller than the experimental results. The current results are closer to the  
391 experimental results in both upstream and downstream locations.

## 392 **6. Conclusions and future work**

393 A Navier-Stokes solver based on the projection method and a second-order unstructured finite-  
394 volume mesh, using LES is proposed to simulate the hydrodynamics and the scour process  
395 around a circular cylinder. A sigma-coordinate system is applied to follow the sediment-water  
396 interface. Bed erosion is simulated by solving the sediment continuity equation in the bedload

397 layer using a mass-conservation-based algorithm for sand-slide and a bedload transport rate  
398 based on a description of the physical processes (Engelund & Fredsøe, 1976). The proposed  
399 model has been rigorously validated for the free slip bed case and the rigid bed case through  
400 comparison with previous studies. Then, the simulation of the scour process around a cylinder  
401 for the live bed case with  $h = 4D$  and  $Re_D = 46,000$  is then done. The current study focused on  
402 the coherent structure of the flow fields at the beginning stage of the scour process.

403 In the free slip bed case, mean streamwise and spanwise velocity profiles on a rigid bed at  
404 different streamwise positions are in good agreement with measurements. In the rigid bed case,  
405 the proposed model reproduces shear stress observed by Roulund et al. (2005) in the vicinity of  
406 the cylinder but with a very slight discrepancy far away from it. The HV system is relatively  
407 stable and under an oscillating cycle. It is composed of primary, bottom-attached and secondary  
408 necklace vortices and a small coherent junction vortex. For  $h = 4D$  and  $Re_D = 46,000$ , there are  
409 two primary and two bottom-attached vortices in the instantaneous flows. During an oscillation  
410 cycle, the two primary vortices move in opposite directions, and in the end, they return to  
411 approximate their initial positions. The oscillating distance is about  $0.1D$ . The legs of the  
412 primary vortices are nearly parallel to the incoming flow and they are still present downstream  
413 from the cylinder until the position  $x = 2D$ . Indeed, the HV structure in the documented case has  
414 almost the same structure as that for Kirkil et al. (2008) for  $h/D = 1.12$ .

415 In the live bed case, the HV system is closely related to the scour process. It is unstable at the  
416 initial stage of scour and then becomes more stable as the scour hole is formed. The obstruction  
417 caused by the cylinder generates the local redistribution of pressure and induces down flows at  
418 the upstream face of the cylinder generating primary vortices. Streamlines wrap the cylinder  
419 from both lateral sides, converging at a nodal point downstream. From there, due to an upwelling

420 vortex, fluids rise up to the surface, entraining the sediment particles situated on the bed,  
421 bringing them downstream to deposit somewhere else. This generates the scour process behind  
422 the cylinder.

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532

533 **Figure legends:**

534 **Fig. 1.** Sand-slide algorithm: (a) an unstructured triangular bed mesh, and (b) definition of  
535 quantities used to adjust the computed bed slope.

536 **Fig. 2.** Sigma-transformation links irregular domain to a rectangular one.

537 **Fig. 3.** Sketch of the geometry of the computational domain and specification of the boundary  
538 conditions.

539 **Fig. 4.** Detailed view of the deformed mesh close to the cylinder with scouring.

540 **Fig. 5.** Mean velocity on a free slip bed at different streamwise positions ( $x/D$ ) using  $Re_D =$   
541 3,900 (a) streamwise ( $u/U$ ) and (b) spanwise ( $v/U$ ) velocities.

542 **Fig. 6.** Non-dimensional bed shear stress distribution at  $Re_D=46,000$ . (a) Results from Roulund  
543 et al. (2005) and (b) current numerical simulation.

544 **Fig. 7.** Detailed view of the coherent structure with a HV system on a rigid bed.

545 **Fig. 8.** Instantaneous streamlines on the longitudinal section upstream of the cylinder at five  
546 instants in time for the rigid bed case.

547 **Fig. 9.** Mean flow streamlines on the longitudinal section behind the cylinder for the rigid bed  
548 case. The direction of the flow is indicated by the arrows.

549 **Fig. 10.** Mean flow streamlines on the vertical cross sections behind the cylinder for the rigid  
550 bed case.

551 **Fig. 11.** Mean streamlines on different horizontal planes for the rigid bed case.

552 **Fig. 12.** Visualization of the main necklace vortices inside the scour hole for the live bed case.

553 **Fig. 13.** Horseshoe vortex system in the scour hole development around and in front of the  
554 cylinder for the live bed case.

555 **Fig. 14.** Time-averaged streamlines on the longitudinal section behind the cylinder for the live  
556 bed case.

557 **Fig. 15.** 3D time-averaged streamlines of the mean flow for the live bed case.

558 **Fig. 16.** Instantaneous images of the bed elevation, showing the evolution of the scour hole  
559 around the cylinder.

560 **Fig. 17.** Numerical (Num.) and experimental (Exp.) results of the scour depth evolution at the (a)  
561 upstream and (b) downstream side of the cylinder for the live bed case.

562

563

564 **Table**565 **Table 1.** List of methods used in scour simulations in different references

Reference	Numerical model	Turbulence model	Sand slide model	Water-sediment interface	$Re_D$	Physical phenomenon
Roulund (2005)	Structured finite volume method (FVM)	RANS (k- $\omega$ )	Updated particle velocity	Multigrid mesh	$4.6 \times 10^4$	Scour hole evolution (Live bed erosion)
Kirkil (2008, 2010, 2015)	Structured finite volume method (FVM)	LES	Fixed bed (no)	Fixed bed (no)	$1.6 \times 10^4$	HV system No scour evolution
Khosronejad (2012)	Unstructured finite volume method (UFVM)	RANS (k- $\omega$ )	Mass-conservating	FSI-CURVIB	$4.95 \times 10^4$	Scour hole evolution (Live bed erosion)
Link et al. (2012)	Structured finite volume method (FVM)	DES	No	Lagrangian model	$3.15 \times 10^4$	Scour hole evolution (Clear-water erosion)
Baykal et al. (2014, 2017)	Structured finite volume method (FVM)	RANS (k- $\omega$ )	Updated particle velocity	Multigrid mesh	$1.7 \times 10^4$	Scour hole evolution (Clear-water erosion)
Zhou (2017)	Structured finite volume method (FVM)	RANS (k- $\omega$ )	Mass-conservating	Dynamic mesh deformation	$4.6 \times 10^4$	Scour hole evolution (Live bed erosion)
Nagel (2018)	Structured finite volume method (FVM)	RANS (k- $\omega$ )	No	Two-phase model	$4.6 \times 10^4$	Scour hole evolution (Live bed erosion)
Current paper (2019)	Unstructured finite volume method (UFVM)	LES	Mass-conservating	Sigma transformation	$4.6 \times 10^4$	HV system Scour evolution (Live bed erosion)

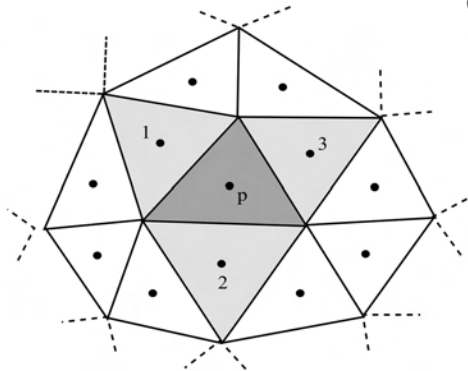
566 Note:  $Re_D$  = Reynolds number567 **Table 2.** Test conditions around a cylinder in the rigid bed and live bed scour cases.

Bed	Loose sand
Water depth, $h$	0.4 m
Cylinder diameter, $D$	0.1 m
Boundary layer thickness, $\delta$	0.2 m
Mean flow velocity, $U$	0.46 m/s
Reynolds number, $Re_D$	46,000
Froude number, $F_r$	0.23
Sediment density, $\rho_s$	2,600 kg/m <sup>3</sup>
Fluid density, $\rho$	1,000 kg/m <sup>3</sup>
Grain size, $d$	0.26 mm
Sand roughness due to skin friction, $k_s$	0.65 mm

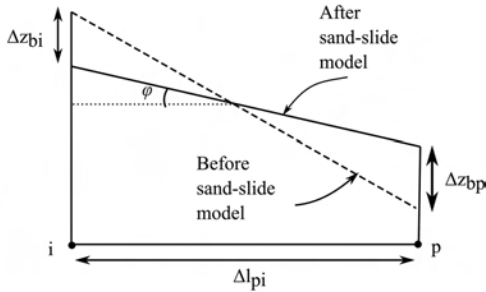
568

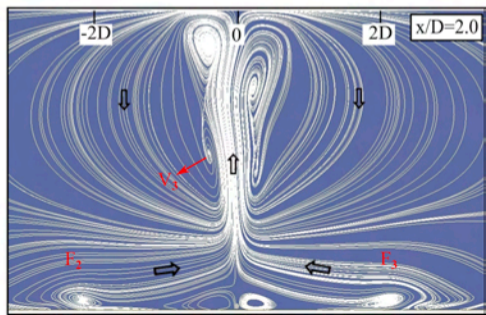
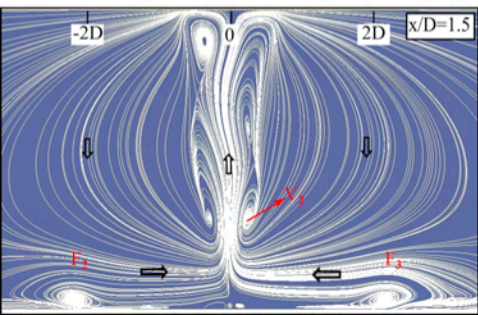
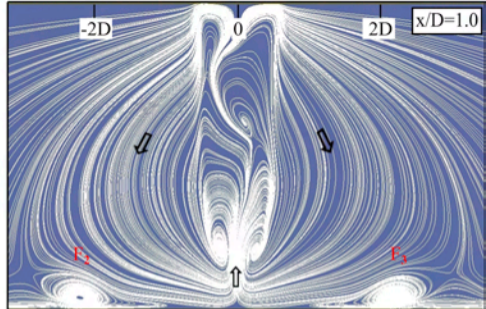
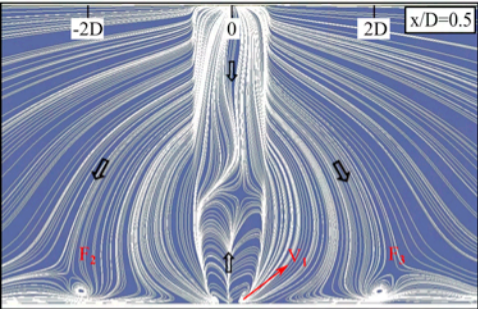
569

(a)

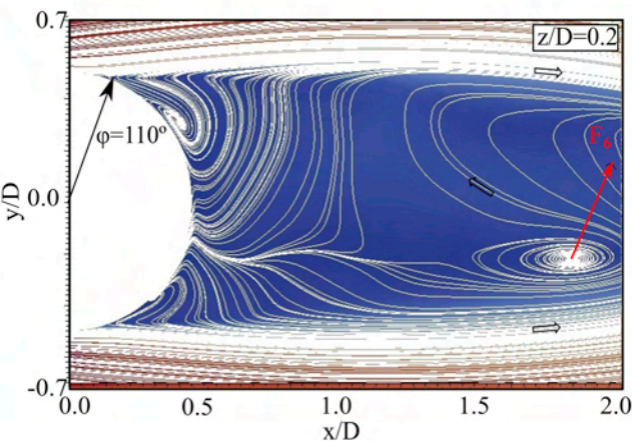
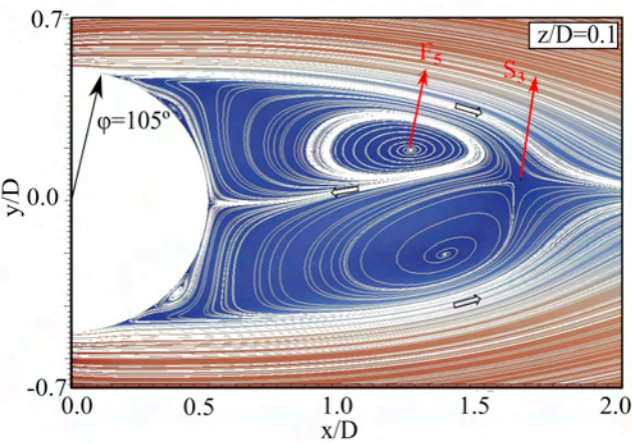
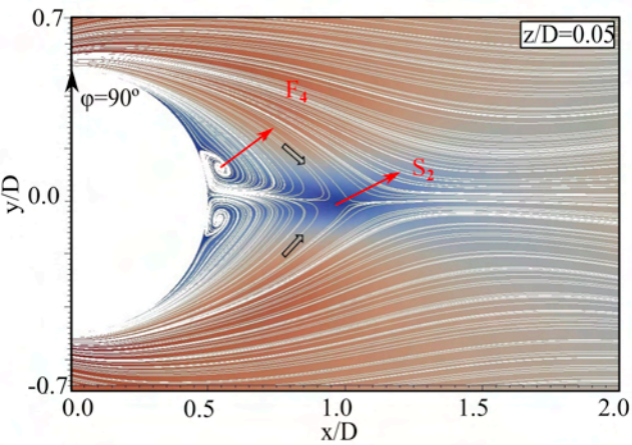


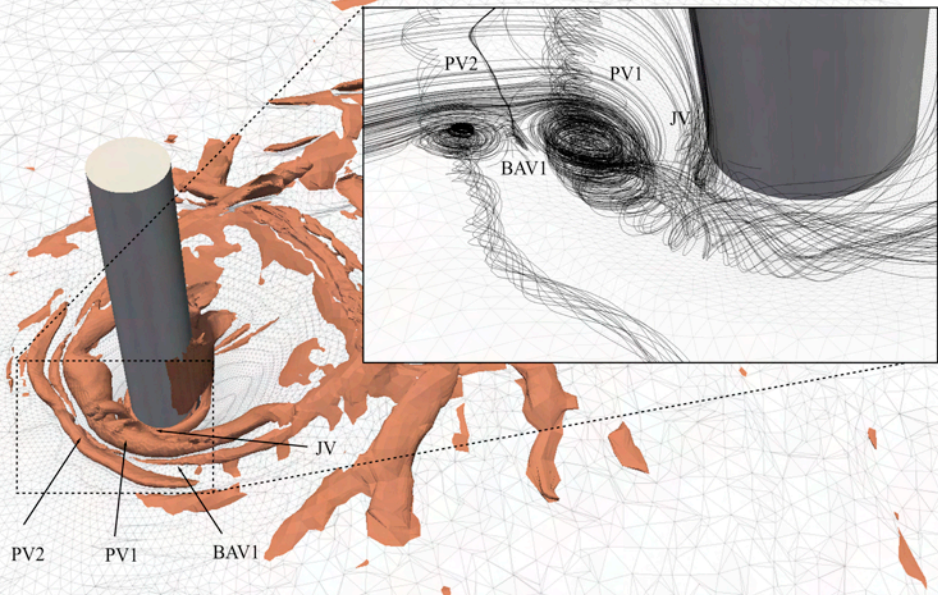
(b)

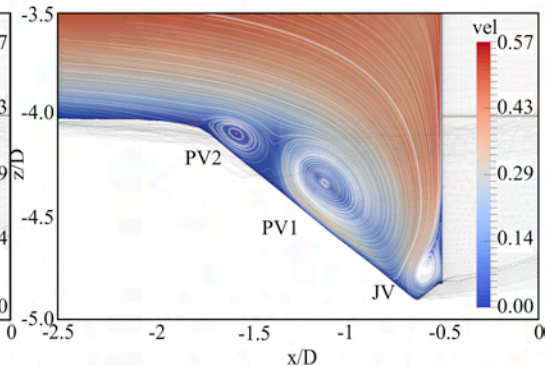
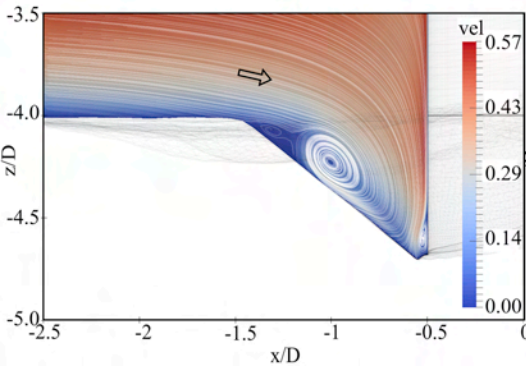
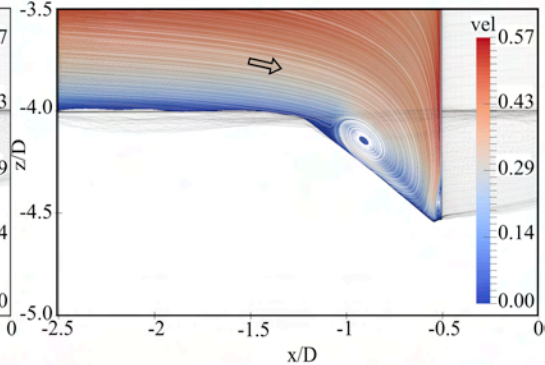
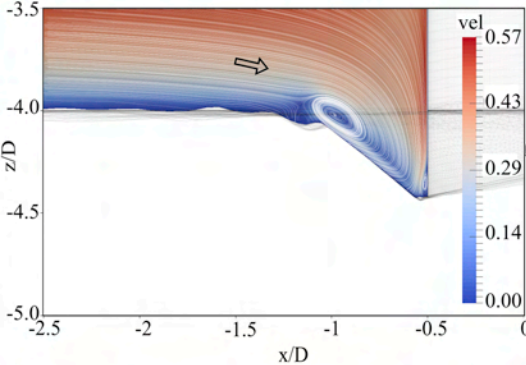


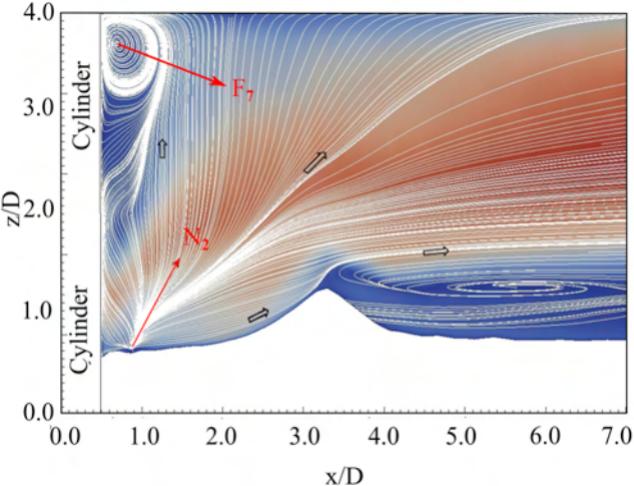


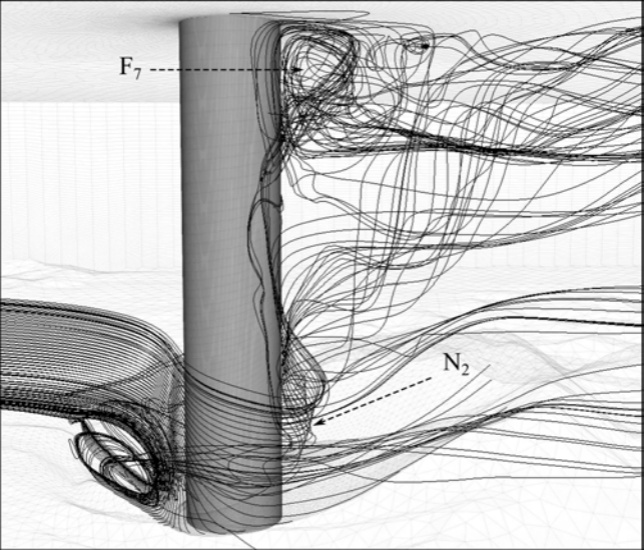


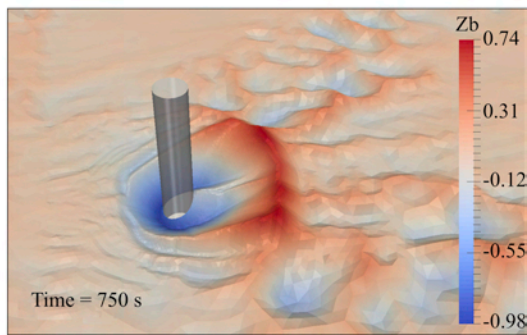
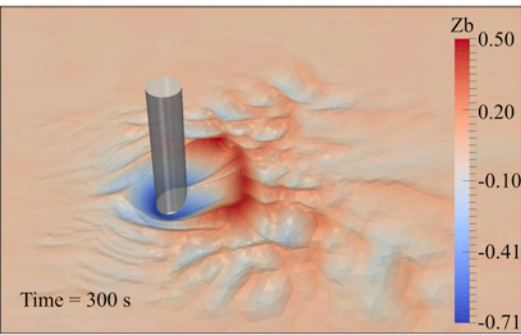
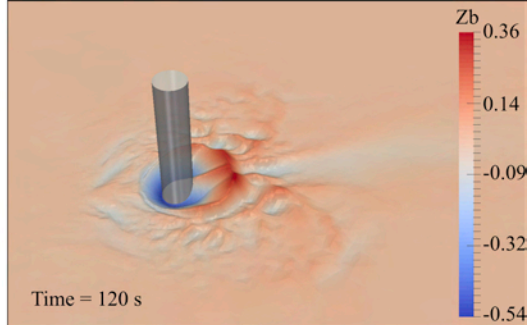
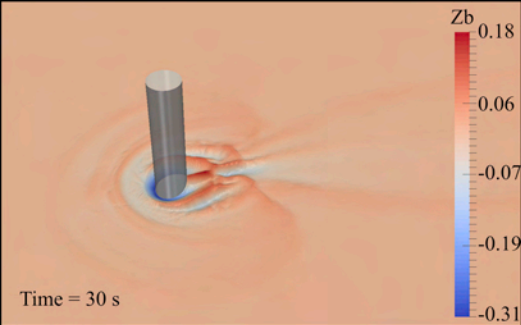


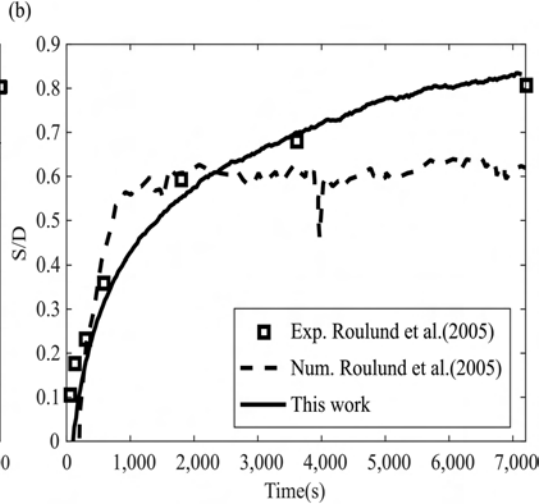
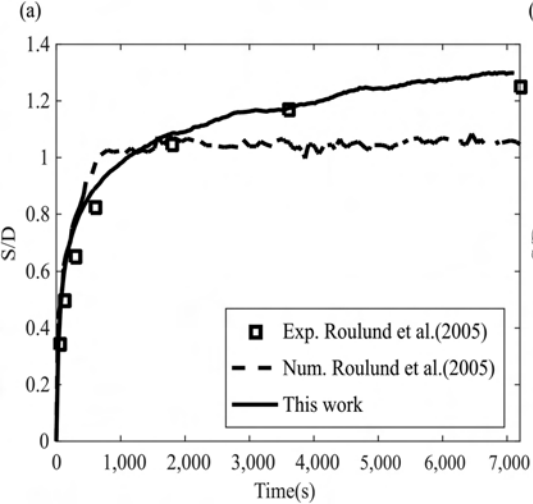


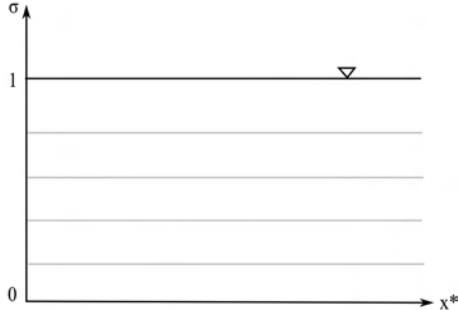
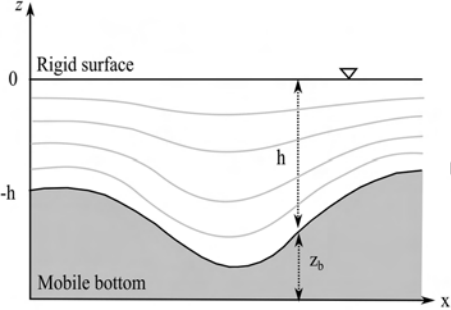




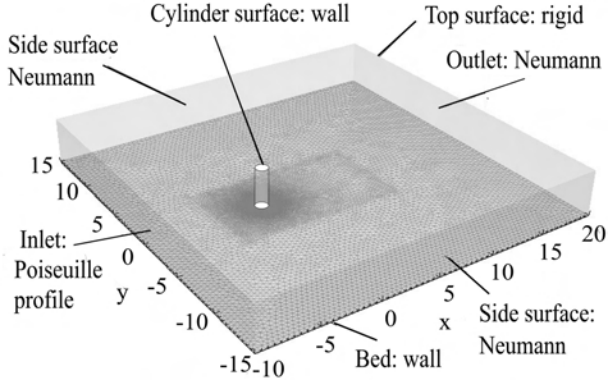


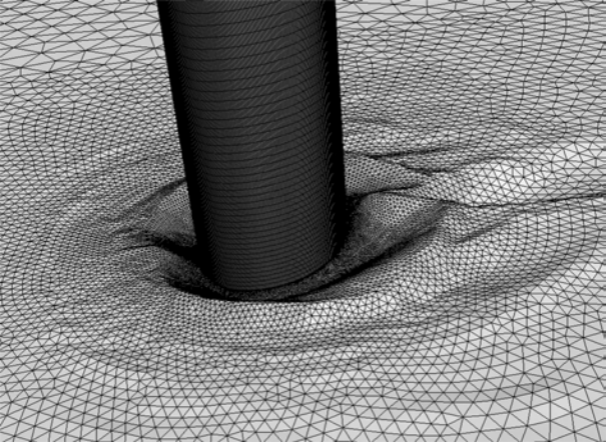




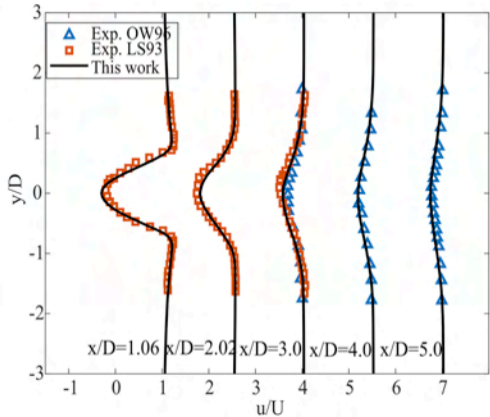




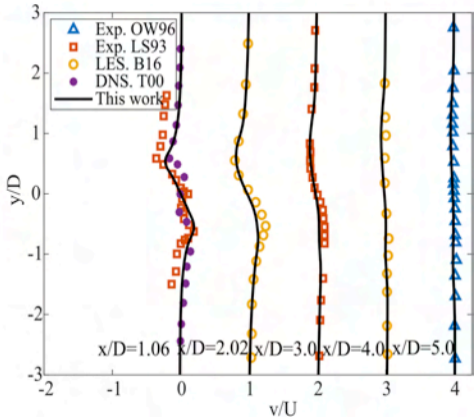


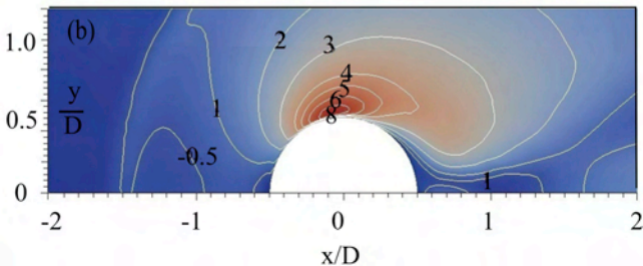
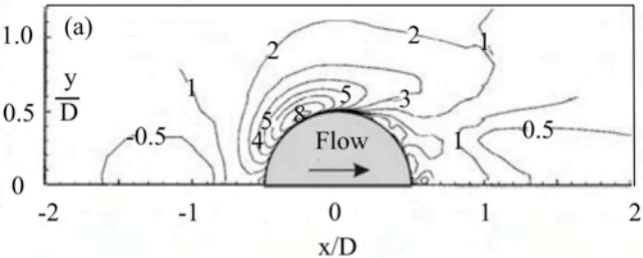


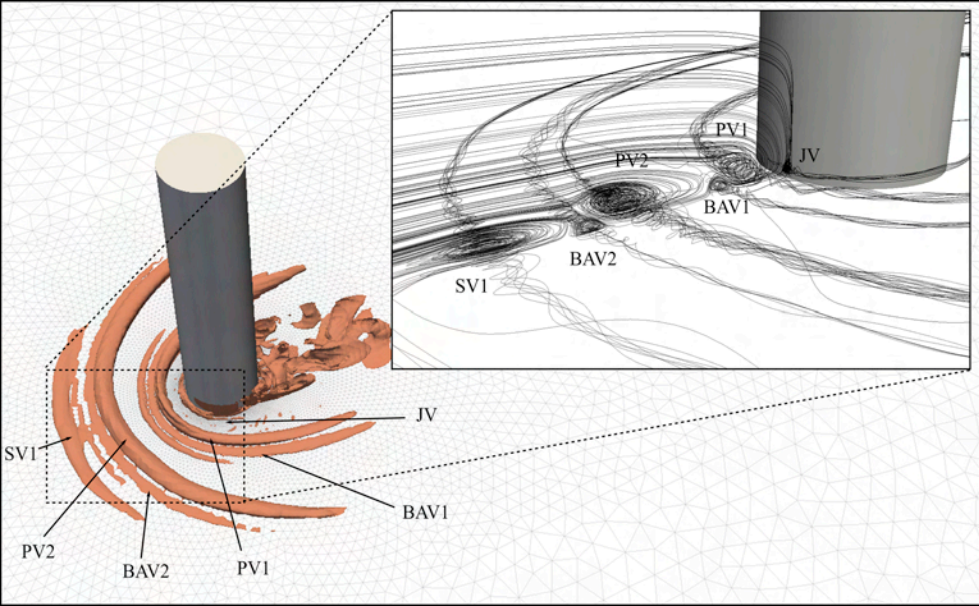
(a)

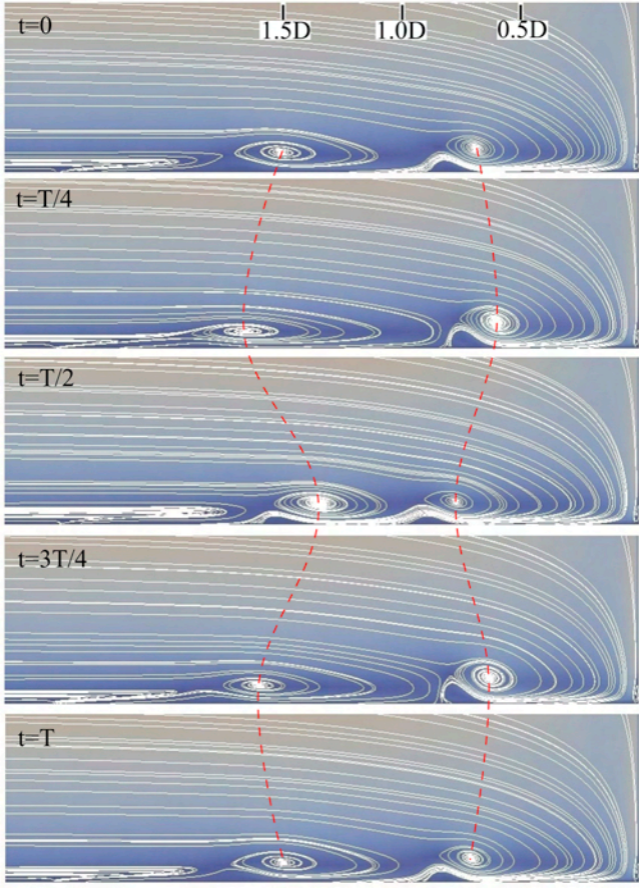


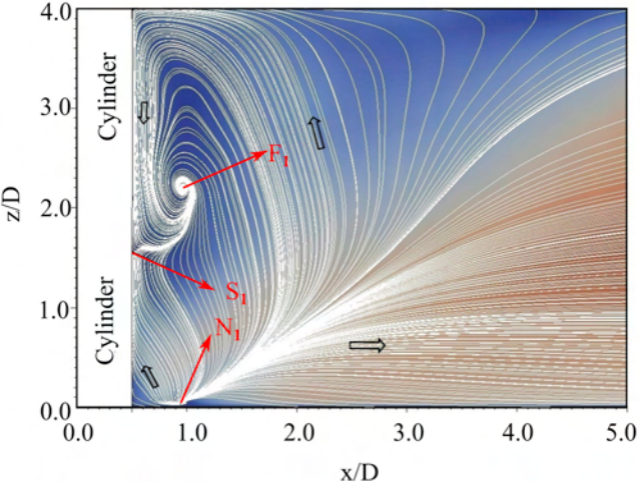
(b)











**Conflicts of interest**

**“Three-dimensional simulation of horseshoe vortex and local scour around a vertical cylinder using an unstructured finite-volume technique”**

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