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A stochastic model for deriving the basic statistics of *J*-day averaged streamflow

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Abstract. This study provides the reader with a methodology for directly deriving basic streamflow statistics (mean, variance, and correlation coefficient) from long-term recorded daily rainfall data. A daily streamflow sequence is considered as a filtered point process where the input is a storm time sequence that is assumed to be a marked point process. The mark is the storm magnitude that is constructed from a daily rainfall time series, and the correlation of the daily rainfall during the storm is considered. The number of storms is a counting process represented by either the binomial, the Poisson, or the negative binomial probability distribution, depending on its ratio of mean versus variance. As a pulse-response function for a filtered point process, the model of three serial tanks with a parallel tank is adopted to describe the physical process of rainfall-runoff. Thus the basic statistics (mean, variance, and covariance function) of J-day averaged streamflows can be estimated in terms of the constants expressing stochastic properties of a rainfall time series and the tank model's parameters representing the causal relationship between rainfall and runoff. The method is used to derive the streamflow statistics of an actual dam basin, the Sameura Dam basin, located in Shikoku island, Japan. The resulting computed means and variances of 5-day averaged streamflows show a good correspondence with observed ones.

1. Introduction

Basic statistics (mean, variance, and correlation coefficient) of daily or J-day averaged streamflows are imperative in the planning and management of water resources. However, many basins lack long-term recorded daily streamflow data for deducing the streamflow statistics. Although some basins have long records of daily streamflows, streamflow statistics may be unreliable because the homogeneity of a daily streamflow series is violated owing to dam construction, urbanization, development, and so forth. In such cases it may be advantageous to directly derive the statistics of daily streamflows from long-term recorded daily rainfall time series, which are generally more homogeneous than daily streamflow sequences. In the past three decades several attempts have been made to derive streamflow statistics from rainfall processes on the basis of hydrological reasoning. Examples include works by *Weiss* [1977], *Kanda* [1983], *Koch* [1985], *Bierkens and Puente* [1990], and *Puente et al*. [1993]. These studies described a rainfall occurrence process by Poisson or Neyman-Scott arrivals and a basin-response system by a deterministic conceptualization model such as single linear reservoir or two parallel linear reservoirs.

A Poisson distribution is a limiting form of the binomial distribution. Daily rainfall processes are not always Poissonian [*Smith and Schreiber*, 1973; *Waymire and Gupta*, 1981]. Furthermore, a conceptual rainfall-runoff model such as one reservoir and two parallel linear reservoirs may be too simple to approach a generally nonlinear hydrological system. Considering these reasons, *Yue et al*. [1996] developed a stochastic streamflow model that describes a continuous streamflow sequence as a filtered point process. In this model a daily rainfall time series is assumed to be a marked point process. The mark is the daily rainfall amount that is assumed to be a mutually independent random variable. The daily rainfall occurrence process is represented by either the binomial, the Poisson, or the negative binomial distribution, depending on the ratio of mean versus variance of the occurrence number. As a basin-response model, taking into account that streamflow mainly consists of surface, rapid and delayed subsurface, and groundwater runoffs in the basin of study (the Sameura Dam basin, located in Shikoku island, Japan) [*Water Resources Developing Bureau* (*WRDB*), 1975], they developed a tank model, the model of three serial tanks with a parallel tank to substitute for the models used by *Weiss* [1977], *Koch* [1985], and *Bierkens and Puente* [1990]. The first two cumulants and the covariance function of J-day averaged streamflows are deduced on the basis of the characteristic function of a filtered point process by *Snyder* [1975]. The model was used for deriving the basic statistics of streamflows of the Sameura Dam basin. The resulting computed variances of streamflows fitted observed ones well during periods of no flooding. On the other hand, during flood periods (July–September), especially in August, computed variances of streamflows were much smaller than observed ones. Yue et al. think the main reason for this is that the daily rainfall process was assumed to be an independent random process. In Japan, during flood periods, one of the most evident properties of the precipitation phenomena is the cluster nature that rain continues to fall over a few days because of

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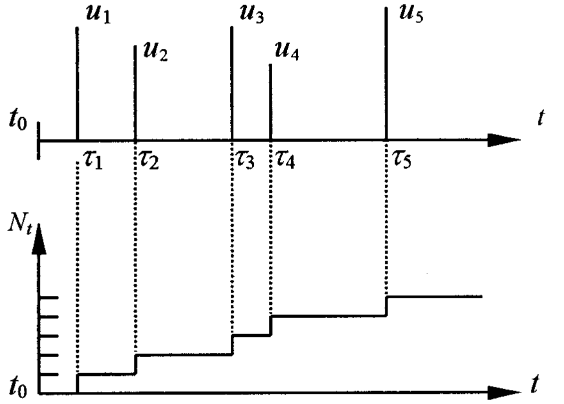

Figure 1. Schematic description of a marked point process.

typhoons and standing rainy fronts. This rain over a few days is one storm event. If the characteristics of a daily rainfall time series are modeled as a daily unit, even though a heavy rainstorm continues to fall for 2 or 3 days, the daily rainfalls within the storm are considered to be independent: the storm is split into two or three independent storms. Therefore, computed variances of the rainfall amount are smaller than actual ones, and theoretical variances of streamflows estimated from these rainfall properties are much smaller than observed ones.

This study is an extension of the previous work by *Yue et al.* [1996]. A daily rainfall time series appears to have clustered nature during rainy seasons or flooding seasons in some regions in the world, such as in Japan. Thus it is important to develop the streamflow statistics from this property of rainfall processes. We formulate the basic statistics of streamflow on the basis of a storm sequence that is assumed to be a marked point process. The properties of a storm sequence are described by storm occurrence number, storm duration, and autocorrelation of daily rainfall during the storm. On the basis of this method, computed means and variances of streamflows of the Sameura Dam basin show a good correspondence with observed ones.

For ease of understanding the proposed method, we organize this paper as follows. Section 2 introduces the general stochastic streamflow model by *Yue et al.* [1996, 1999] that is used for deriving streamflow statistics from a daily rainfall series; section 3 describes a conceptual rainfall-runoff model, the model of three serial tanks with a parallel tank that is used as the response function of a filtered point process; section 4 provides an extension form of the streamflow model introduced in section 2, in which correlation of the daily rainfall during a storm is considered; section 5 presents a practical application of the proposed model to an actual basin; and section 6 summarizes the application results.

2. Streamflow As a Filtered Point Process

2.1. Definition

Let a daily rainfall time series $\{x_t;\ t \geq t_0\}$ be a marked point process. Denote the nth rainfall occurrence time (day) and rainfall amount (mark) by τ_n and u_n, respectively. Let the number of daily rainfall occurrences $\{N_t;\ t \geq t_0\}$ be a counting process that counts points independent of their marks, and let the marks $\{u_n\}$ be mutually independent, identically distributed variables as shown in Figure 1. Streamflow as a filtered

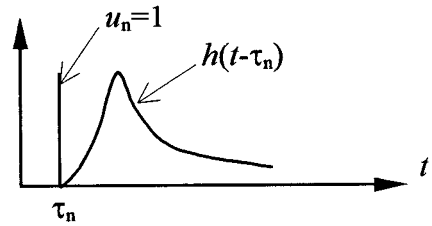

Figure 2. Schematic description of unit response function.

point process can be expressed by

$$y_t = \sum_{n=1}^{N_t} u_n h(t - \tau_n), \tag{1}$$

where $h(t - \tau_n)$ is the basin response function for a unit pulse (unit daily rainfall amount), which represents the causal relation between rainfall and runoff (Figure 2), and $(t - \tau_n)$ is the time lag since the pulse occurred at time τ_n.

In (1) it is assumed that the basin response function $h(t - \tau_n)$ is linear. Thus the deterministic linear rainfall-runoff model constructed for transferring rainfall to runoff should approximate a nonlinear hydrological system.

In practice, one is often interested in some averaged value of streamflow such as the J-day averaged streamflows. Streamflow averaged over a period J can be defined as

$$Y_t = J^{-1} \int_{t-J}^{t} y_\sigma \, d\sigma. \tag{2}$$

Substituting (1) into (2) gives

$$Y_t = \sum_{n=1}^{N_t} u_n h_J(t - \tau_n), \tag{3}$$

where

$$h_J(t - \tau_n) = J^{-1} \int_{t-J}^{t} h(\sigma - \tau_n) \, d\sigma. \tag{4}$$

2.2. Distribution of the Number of Daily Rainfall Occurrences

As daily rainfall occurrence processes $\{N_t;\ t \geq t_0\}$ are not always Poissonian, we describe the number of rainfall occurrences by either the binomial, the Poisson, or the negative binomial probability distribution, depending on the ratio of the mean $E(N_t)$ to the variance $V(N_t)$ of N_t.

If N_t follows a binomial distribution, then

$$f(N_t = n) = \frac{k!}{n!(k-n)!} p^n (1-p)^{k-n}, \qquad E(N_t) = kp \qquad V(N_t) = k[p(1-p)]. \tag{5a}$$

Thus

$$\frac{E(N_t)}{V(N_t)} = \frac{1}{1-p} > 1 \qquad E(N_t) > V(N_t).$$

If N_t follows a Poisson distribution, then

$$f(N_t = n) = \frac{\lambda^n}{n!} \exp(-\lambda), \qquad E(N_t) = \lambda \qquad V(N_t) = \lambda. \tag{5b}$$

Thus

$$\frac{E(N_t)}{V(N_t)} = 1 \qquad E(N_t) = V(N_t).$$

If N_t follows a negative binomial distribution, then

$$f(N_t = n) = \frac{(n + k - 1)!}{n!(k - 1)!} p^k (1 - p)^n,$$
$$E(N_t) = \frac{k(1 - p)}{p} \qquad V(N_t) = \frac{k(1 - p)}{p^2}. \tag{5c}$$

Thus

$$\frac{E(N_t)}{V(N_t)} = p < 1 \qquad E(N_t) < V(N_t).$$

In (5a)–(5c), k and p are the parameters of the binomial and negative binomial distributions, and λ is the parameter of the Poisson distribution.

2.3. Cumulants for *J*-Day Averaged Streamflows

On the basis of the theory of a filtered point process proposed by *Snyder* [1975], the cumulants and the covariance function of the J-day averaged streamflows are obtained as follows [*Yue et al.*, 1996, 1999]:

$$\gamma_1(Y_t) = E(u) \int_{t_0}^{t} \lambda_\tau h_J(t - \tau)\, d\tau, \tag{6a}$$

$$\gamma_2(Y_t) = E(u^2) \int_{t_0}^{t} \lambda_\tau h_J(t - \tau)^2\, d\tau + \varepsilon \frac{\gamma_1(Y_t)^2}{k}, \tag{6b}$$

$$\mathrm{Cov}\,(Y_t, Y_{t+o}) = E(u^2) \int_{t_0}^{t} \lambda_\tau h_J(t - \tau) h_J(t + o - \tau)\, d\tau + \varepsilon \frac{\gamma_1(Y_t)^2}{k}. \tag{6c}$$

In the above formulas, if N_t follows a binomial distribution, then $\varepsilon = -1$; if N_t follows a negative binomial distribution, then $\varepsilon = +1$; if N_t follows a Poisson distribution, then $\varepsilon = 0$, and in this particular case the formulas are the same as those given by *Parzen* [1962] and *Snyder* [1975]; $E(u^i)$ is the ith moment about the origin of daily rainfall amount u.

From these expressions it can be seen that the cumulants of the discretized streamflows are functions of the constants (k and $E(u^i)$, $i = 1, 2, 3$) describing the stochastic properties of a daily rainfall series, the response function ($h(t)$ or $h_J(t)$) representing the causal relation between rainfall and runoff, and the period J over which the streamflow process is averaged. It can also be seen that different distributions of rainfall occurrence processes will lead to rather different streamflow cumulants.

3. Pulse Response Function for a Filtered Point Process

As the response function for a filtered point process must be linear, the conceptual linear rainfall-runoff model constructed for transferring rainfall to runoff should be sufficient to approach a generally nonlinear hydrological system. In the basin of study (the Sameura Dam basin), base flow (the sum of rapid subsurface flow, delayed subsurface flow, and groundwater flow) drains out of three different aquifers (rapid subsurface flow from shallow aquifer, delayed subsurface flow from mid-deep aquifer, and groundwater from deep aquifer) [*WRDB*, 1975]. *Yue et al.* [1996] compared the two types of linear rainfall-runoff models for the basin of study: (1) the model of three serial tanks and (2) the model of three serial tanks with one tank in parallel. Model 1 represents base flow well, but it cannot sufficiently express surface runoff, as computed streamflows are much smaller than observed ones during flood periods. Model 2 can represent both base flow and surface runoff adequately. The following section will give a detailed explanation of model 2.

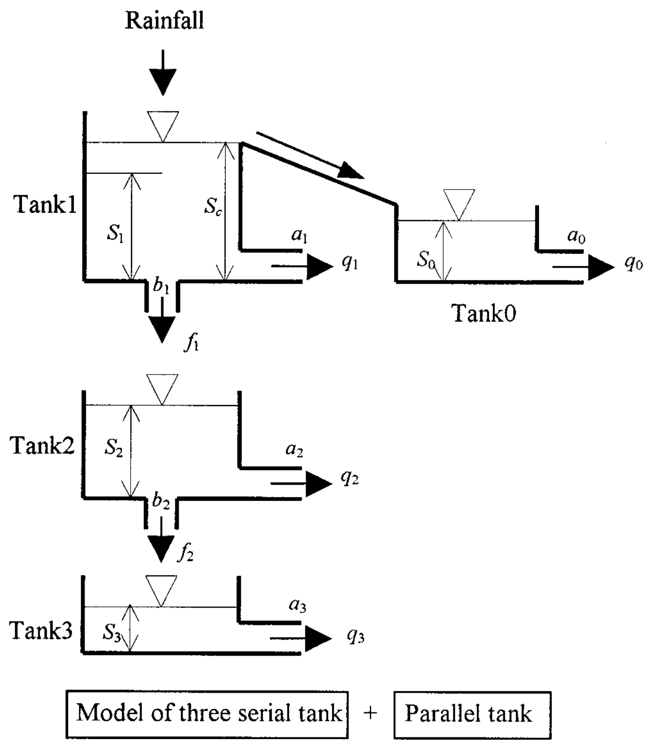

Figure 3. Schematic illustration of the model of three serial tanks with a parallel tank.

3.1. Structure of the Tank Model

Since streamflow may consist of surface, rapid and delayed subsurface, and groundwater runoff which occurs from different aquifers, the model of three serial tanks with a parallel tank is used to represent physical characteristics of a watershed, as shown in Figure 3. One horizontal hole from which runoff occurs is set up on the right side at the bottom of each tank. Hole sizes for Tank 0, Tank 1, Tank 2, and Tank 3 are denoted by a_0, a_1, a_2, and a_3, respectively. In order to indicate infiltrations from Tank 1 to Tank 2 and from Tank 2 to Tank 3, the vertical holes in the bottoms of Tank 1 and Tank 2 are opened. Their sizes are denoted by b_1 and b_2, respectively. In Figure 3, q_1, q_2, and q_3 represent rapid subsurface, delayed subsurface, and groundwater runoffs occurring from Tank 1, Tank 2, and Tank 3, respectively; q_0 indicates surface runoff occurring from the parallel tank (Tank 0) when overflow from Tank 1 to Tank 0 takes place; and f_1 and f_2 are referred to as the infiltrations from Tank 1 to Tank 2 and from Tank 2 to Tank 3, respectively.

In the model, rainfall r first fills Tank 1. Rapid subsurface runoff q_1 and infiltration f_1 from Tank 1 to Tank 2 occur when $S_1 > 0$. Delayed subsurface runoff q_2 and infiltration f_2 from Tank 2 to Tank 3 occur when $S_2 > 0$. Groundwater flow q_3 occurs when $S_3 > 0$. Surface runoff q_0 occurs from Tank 0 when Tank 1 is full; that is, overflow occurs from Tank 1 to Tank 0 when $S_1 > S_c$. For the daily evapotranspiration loss, as the evapotranspiration rate can be approximated to be equal to zero during rainfall periods, for simplicity we directly subtract this loss from the daily streamflow computed from the tank model.

3.2. Unit Response Function

By assuming that q_i $(i = 0, 1, 2, 3)$ and f_i $(i = 1, 2)$ are a function of the storage height S_i in Tank i, q_i and f_i can be calculated by the following equations:

$$q_i(t) = a_i S_i(t) \qquad i = 0, 1, 2, 3, \tag{7}$$

$$f_i(t) = b_i S_i(t) \qquad i = 1, 2. \tag{8}$$

For a storage function to describe the linear system, the storage height $S_i(t)$ can be related to the rates of input $I_i(t)$ and output $Q_i(t)$ of Tank i by the following equation:

$$\frac{dS_i(t)}{dt} = I_i(t) - Q_i(t) \qquad i = 0, 1, 2, 3, \tag{9}$$

where

$$Q_i(t) = f_i(t) + q_i(t) = C_i S_i(t) \qquad C_i = a_i + b_i. \tag{10}$$

By using (7)–(10) and letting $S_i = 0$ when $t = 0$, the linear response function for the sum of runoffs occurring from the model of three serial tanks can be obtained [*Yue*, 1997]:

$$h(t) = h_1(t) = 1 + D_{11}e^{-C_1 t} + D_{12}e^{-C_2 t} + D_{13}e^{-a_3 t} \qquad 0 < t \le 1, \tag{11a}$$

$$h(t) = h_2(t) = D_{21}e^{-C_1 t} + D_{22}e^{-C_2 t} + D_{23}e^{-a_3 t} \qquad t > 1, \tag{11b}$$

where

$$C_1 = a_1 + b_1,$$

$$C_2 = a_2 + b_2,$$

$$D_{11} = \frac{b_1}{C_1}\left[\frac{(C_1 a_2 - C_2 a_3)}{(C_1 - C_2)(C_1 - a_3)} - \frac{a_1}{b_1}\right],$$

$$D_{12} = \frac{b_1(a_3 - a_2)}{(C_1 - C_2)(C_2 - a_3)},$$

$$D_{13} = \frac{-b_1 b_2}{(C_1 - a_3)(C_2 - a_3)},$$

$$D_{21} = D_{11}(1 - e^{C_1}),$$

$$D_{22} = D_{12}(1 - e^{C_2}),$$

$$D_{23} = D_{13}(1 - e^{a_3}).$$

Similarly, the response function for surface runoff occurring from the parallel tank can be derived as follows [*Yue*, 1997]:

$$h_s(t) = 1 - e^{-a_0 t} \qquad 0 < t \le 1, \tag{12a}$$

$$h_s(t) = (e^{a_0} - 1)e^{-a_0 t} \qquad t > 1. \tag{12b}$$

Thus the unit response function $u(t)$ for the total runoff can be given by the summation of the two linear unit pulse response functions $h(t)$ and $h_s(t)$:

$$h_T(t) = h(t) \qquad S_1 \le S_c, \tag{13a}$$

$$h_T(t) = h(t) + h_s(t) \qquad S_1 > S_c. \tag{13b}$$

From (11)–(13) it is obvious that the unit response function $u(t)$ is the summation of exponential functions with decreasing parameters. These exponential functions describe the responses of the different streamflow components (surface, rapid subsurface, delayed subsurface, and groundwater runoffs) to the rainfall process.

3.3. *J*-Day Averaged Response Function $h_J(t)$

The J-day averaged response function for the model of three serial tanks is derived as follows [*Yue*, 1997]:

(1) For $0 \le t < 1$,

$$h_J(t) = J^{-1}\int_0^t h_1(\sigma)\, d\sigma = E_{001} + E_{01}t + E_{11}e^{-C_1 t} + E_{12}e^{-C_2 t} + E_{13}e^{-a_3 t}. \tag{14a}$$

(2) For $1 \le t < J$,

$$h_J(t) = J^{-1}\left[\int_0^1 h_1(\sigma)\, d\sigma + \int_1^t h_2(\sigma)\, d\sigma\right] = E_{002} + E_{02}t + E_{21}e^{-C_1 t} + E_{22}e^{-C_2 t} + E_{23}e^{-a_3 t}. \tag{14b}$$

(3) For $J \le t < J + 1$,

$$h_J(t) = J^{-1}\left[\int_{t-J}^1 h_1(\sigma)\, d\sigma + \int_1^t h_2(\sigma)\, d\sigma\right] = E_{003} + E_{03}t + E_{31}e^{-C_1 t} + E_{32}e^{-C_2 t} + E_{33}e^{-a_3 t}. \tag{14c}$$

(4) For $J + 1 \le t$,

$$h_J(t) = J^{-1}\int_{t-J}^t h_2(\sigma)\, d\sigma = E_{004} + E_{04}t + E_{41}e^{-C_1 t} + E_{42}e^{-C_2 t} + E_{43}e^{-a_3 t}. \tag{14d}$$

In Figures (14a)–(14d), E_{00i}, E_{0i}, E_{i1}, E_{i2}, E_{i3} $(i = 1, 2, 3, 4)$ are functions of D_{1i} and D_{2i} $(i = 1, 2, 3)$ and are also functions of the tank model's parameters a_i and b_i, which are omitted here owing to space limitations [*Yue*, 1997].

The response function $h_{Js}(t)$ for the surface runoff occurring from the parallel tank can also be obtained using the same method [*Yue*, 1997].

From the foregoing section it can be seen that streamflow splits into two parts: one part is the baseflow y_b, and the other part is the surface runoff y_s. Streamflow y_t can be represented by

$$y_t = y_b + y_{s|y_t > y_c}, \tag{15}$$

$$y_{s|y_t \le y_c} = 0 \qquad y_t \le y_c \qquad P_r = P(y_t \le y_c) = 0, \tag{16a}$$

$$y_{s|y_t > y_c} = y_s \qquad y_t > y_c \qquad P_r = P(y_t > y_c) > 0. \tag{16b}$$

where P_r is the probability of surface-runoff occurrences and is estimated by

$$P_r = n_r/D_t, \tag{17}$$

where n_r and D_t are respectively the number of days for $y_t > y_c$ and the total number of days in a given period, respectively; y_c is the threshold over which surface runoff occurs and is approximately estimated as follows:

$$y_c = (a_1 + b_1)S_c. \tag{18}$$

As properties of the storms causing the surface flow are different from those of the storms causing the base flow, observed daily rainfall data are split into two parts: one is the rainfall series that produces only the base flow y_b, and the other is the rainfall series that produces both the base flow y_b and the surface runoff y_s. By subtracting the evapotranspiration loss, the modified cumulants of J-day averaged streamflows in a given period can be expressed by

$$\gamma_1(Y_t) = f_p\left[E(u)\lambda\int_0^t h_J(\tau)\,d\tau + P_rE(u_s)\int_0^t h_{Js}(\tau)\,d\tau\right] - \frac{f_E E_p(D_t - M_N)}{D_t}, \tag{19a}$$

$$\gamma_2(Y_t) = f_P^2\left[E(u^2)\lambda\int_0^t h_J(\tau)^2\,d\tau + P_rE(u_s^2)\int_0^t h_{Js}(\tau)^2\,d\tau\right] + \varepsilon\,\frac{\gamma_1(Y_{1t})^2}{k}, \tag{19b}$$

$$\mathrm{Cov}\,(Y_t, Y_{t+o}) = f_P^2\left[E(u^2)\lambda\int_0^t h_J(\tau)h_J(o+\tau)\,d\tau + P_rE(u_s^2)\cdot\int_0^t h_{Js}(\tau)h_{Js}(o+\tau)\,d\tau\right] + \varepsilon\,\frac{\gamma_1(Y_{1t})^2}{k}, \tag{19c}$$

where E_P is the total evapotranspiration amount during a given period (D_t); $E(u^i)$ $(i = 1, 2)$ is the ith order moment of the daily rainfall amount on rainy days when surface runoff does not take place; $E(u_s^i)$ $(i = 1, 2)$ is the ith order moment of the daily rainfall amount on rainy days when surface runoff occurs $(y_t > y_c = (a_1 + b_1)S_c)$; M_N is the mean of the number of rainy days in a given period; $\gamma_1(Y_{1t})$ is the first cumulant (mean) of J-day averaged streamflows for $P_r = 0$; and f_P and f_E are the correction parameters for modifying daily rainfall amount and daily evapotranspiration, respectively, that cannot sufficiently represent the areal mean values of a given basin. If observed daily rainfall amount and evapotranspiration are sufficient to represent the areal mean values of a given basin, then these two correction parameters should not be used anymore; that is, $f_P = f_E = 1$.

Equations (19a)–(19c) were used to derive the statistics of 5-day averaged streamflows of the Sameura Dam basin [*Yue et al.*, 1996]. Computed variances of streamflows fitted observed ones well during periods of no flooding. On the other hand, during flood periods (July–September), especially in August, computed variances of streamflows were much smaller than observed ones. The main reason for this might be that strong correlation or cluster nature of the daily rainfall processes during rainy seasons is not considered, as stated in section 1. Therefore, in section 4 we construct the cumulants of J-day averaged streamflows concerning this property of rainfall processes.

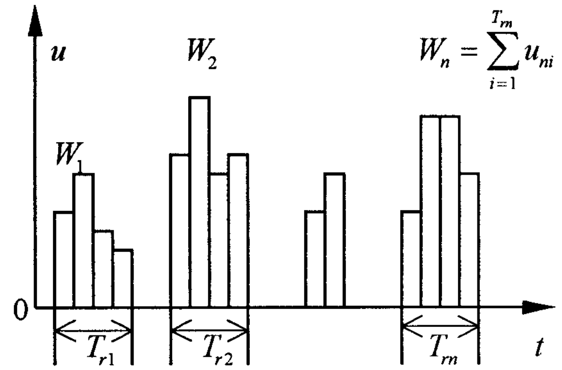

Figure 4. Illustration of a storm sequence.

4. An Extension of the Stochastic Streamflow Model Introduced in Section 2

The daily rainfall time series appears to have strong correlation or clustered nature during rainy seasons or flooding seasons in some regions in the world, such as in Japan. It is necessary to construct the streamflow statistics using this property of rainfall processes. In this section we formulate the basic statistics of streamflows on the basis of a storm sequence which is assumed to be a marked point process. The properties of a storm sequence are described by storm occurrence number, storm duration, and correlation of daily rainfall during the storm.

4.1. Definition of a Storm Sequence

As the rain that continues to fall over a few days is one storm event due to typhoons and standing rainy fronts in some regions, such as in Japan, and as historical rainfall data is often in the form of averages over a period of 1 day, one storm is defined as continuous daily rainfalls, as shown in Figure 4. Denoting the duration (continuous rainy days) and the total rainfall amount of the nth storm by T_{rn} and W_n, respectively, the total storm amount W_n can be given by

$$W_n = \sum_{i=1}^{T_{rn}} u_{ni}, \tag{20}$$

where u_{ni} is the daily rainfall amount of the ith day for the nth storm.

4.2. Streamflow as a Filtered Point Process

As stated in section 2.1, let a storm cluster sequence be a marked point process in which the mark of the process is a storm amount W_n, and let the number of storms $\{N_c; t \geq t_0\}$ be a counting process that counts points independent of their marks. Streamflow y_t as a filtered point process can be represented as follows:

$$y_t = \sum_{n=1}^{N_c} W_n h(t - \tau_i) = \sum_{n=1}^{N_c}\sum_{i=1}^{T_{rn}} u_{ni} h(t - \tau_i). \tag{21}$$

The distribution of the number of storm occurrences is represented in the same manner as that used in describing the number of daily rainfall occurrences (section 2.2).

4.3. Cumulants of *J*-Day Averaged Streamflows

On the basis of the theory of a filtered point process by *Snyder* [1975], the general cumulants of J-day averaged streamflows are given as follows:

$$\gamma_1(Y_t) = \lambda \int_{t_0}^{t} E[g_J(t-\tau; W)]\, d\tau, \tag{22a}$$

$$\gamma_2(Y_t) = \lambda \int_{t_0}^{t} E[g_J^2(t-\tau; W)]\, d\tau + \varepsilon \frac{\gamma_1(Y_t)^2}{k}, \tag{22b}$$

$$\mathrm{Cov}\,(Y_t, Y_{t+o}) = \lambda \int_{t_0}^{t} E[g_J(t-\tau; W) g_J(t+o-\tau; W)]\, d\tau + \varepsilon \frac{\gamma_1(Y_t)^2}{k}, \tag{22c}$$

$$g_J(t-\tau; W) = \sum_{i=1}^{n} u_i h_J(t-\tau_i), \tag{22d}$$

In (22a)–(22d), $\tau = \tau_1 < \tau_2 < \tau_3 < \cdots < \tau_n$; $\tau_i = \tau_{i-1} + 1$; $i = 2, 3, \ldots, n = T_r$.

By using the following formulas and omitting the higher-order (>1) items of P_r,

$$\rho = \frac{E\{[u_i - E(u)][u_{i+1} - E(u)]\}}{V(u)}, \tag{23a}$$

$$\rho_s = \frac{E\{[u_{s,i} - E(u_s)][u_{s,i+1} - E(u_s)]\}}{V(u_s)}, \tag{23b}$$

$$E(u_i, u_{i+1}) = \rho V(u) + E(u)^2, \tag{23c}$$

$$E(u_{s,i}, u_{s,i+1}) = \rho_s V(u_s) + E(u_s)^2, \tag{23d}$$

the explicit expressions of (22a)–(22c) are deduced and presented in the appendix (equations (A1)–(A3)).

In the above expressions, $E(u)$, $V(u)$, ρ, and $E(u_i, u_{i+1})$ are the mean, variance, correlation coefficient, and product-moment, respectively, of the daily rainfall time series that produces only the base flow y_b; $E(u_s)$, $V(u_s)$, ρ_s, and $E(u_{s,i}, u_{s,i+1})$ are the mean, variance, correlation coefficient, and product-moment, respectively, of the daily rainfall time series that produces both the base flow y_b and the surface runoff y_s.

5. Application

In order to verify the applicability and validity of the proposed model, we make use of daily rainfall-streamflow data (from 1953 to 1989) observed at an actual dam basin, the Sameura Dam basin, located in Shikoku island, Japan. The basin is a mountainous forest watershed with an area of 472 km^2. Base flow roughly consists of rapid subsurface, delayed subsurface, and groundwater flows which drain out of three different aquifers (shallow aquifer, middeep aquifer, and deep aquifer, respectively) [*WRDB*, 1975].

Table 1. Values of Correction Parameters f_E and f_P

	Season			
	May–June	July–September	October–November	December–April
f_E	0.37	0.91	0.52	0.54
f_P	0.73	0.89	0.76	0.76

5.1. Evapotranspiration

The daily evapotranspiration amount is calculated using the Hamon method based on monthly mean temperature. The correction parameter f_E is used to modify the difference between calculated total amount and the observed one in a given period of 1 month. In the basin of study, values of f_E in different seasons (May–June, July–September, October–November, and December–April) [*WRDB*, 1975; *Tokushima Meteorological Observatory Administration* (*TMOA*), 1991] are estimated and listed in Table 1.

5.2. Areal Precipitation

Because only observed daily precipitation data at Motoyama located just downstream from the Sameura Dam is available, the correction parameter f_P is used to modify daily precipitation error caused by substituting point precipitation for areal precipitation in the Sameura Dam basin. Values of f_P are estimated on the basis of the relationship between areal precipitation and point precipitation in this basin [*WRDB*, 1975; *TMOA*, 1991] and are also presented in Table 1.

5.3. Identification of Parameters for the Model

5.3.1. Computed streamflow. A daily streamflow time series is computed as follows:

$$y(t) = y_t - f_E e_t, \tag{24}$$

where y_t is the streamflow value computed using (3), in which the period is equal to 1 day ($J = 1$) and the daily rainfall amount u_n is the rainfall amount modified by the correction parameter f_P; e_t is the daily evapotranspiration amount estimated by the Hamon method.

5.3.2. Objective function. In this study the following χ^2 criterion [*Nagai et al.*, 1980] is used to choose suitable parameter values:

$$F(a_0, S_c, a_1, a_2, a_3, b_1, b_2) = \frac{1}{n} \sum_{t=1}^{n} \left\{ \frac{[Q(t) - y(t)]^2}{Q(t)} \right\}, \tag{25}$$

where $Q(t)$ is the observed daily streamflow and n is the number of days in a given period.

5.3.3. Identification method. Parameter optimization methods for conceptual rainfall-runoff models may be broadly classified into two major groups: the descent methods and the direct search methods [*Kowalik and Osborne*, 1968]. Works by *Johnston and Pilgrim* [1976], *Pickup* [1977], and others have demonstrated that direct search procedures are superior to the descent procedures in parameter identification for rainfall-runoff models. In this study we use a direct search method, the Simplex method [*Nelder and Mead*, 1965], to search appropriate parameters that give a minimum value of the objective function.

5.3.3.1. Constraints: In order to prevent parameters from taking unrealistic values and ensure that the true parameter values will be reached, the following constraints are set on

Table 2. Identified Parameters for the Tank Model

	s_c, mm	a_0, 1/day	a_1, 1/day	b_1, 1/day	a_2, 1/day	b_2, 1/day	a_3, 1/day
1978	22.614	3.869	0.239	1.527	0.087	0.042	0.019
1979	22.974	4.738	0.298	1.532	0.088	0.034	0.022
1980	22.404	4.002	0.300	1.520	0.104	0.039	0.021
1981	22.982	3.975	0.280	1.600	0.100	0.042	0.024
1982	22.378	4.517	0.249	1.507	0.098	0.035	0.019
1983	20.430	3.880	0.298	1.580	0.085	0.041	0.020
1984	22.946	4.644	0.275	1.656	0.105	0.036	0.022
1985	23.111	4.145	0.261	1.538	0.085	0.038	0.025
1986	21.639	4.642	0.243	1.540	0.086	0.040	0.023
1987	22.161	4.200	0.260	1.510	0.090	0.037	0.020
1988	20.298	3.950	0.240	1.490	0.101	0.038	0.021
1989	22.696	3.995	0.290	1.540	0.103	0.036	0.020
Mean	22.219	4.213	0.269	1.545	0.094	0.038	0.021
Max	23.111	4.738	0.300	1.656	0.105	0.042	0.025
Min	20.298	3.869	0.239	1.490	0.085	0.034	0.019
SD	0.920	0.315	0.023	0.148	0.008	0.003	0.002
Cv	0.041	0.075	0.085	0.096	0.083	0.071	0.092

SD, standard deviation; Cv, skewness coefficient.

the basis of parameters' denotations, catchment characteristics, and the authors' experience on parameter identification of the tank model.

Constraint 1 is the relationship between parameters:

$$a_0 > a_1 > a_2 > a_3 > 0, \quad 1 - (a_1 + b_1) > 0, \quad b_1 > b_2 > 0.$$

Constraint 2 is the boundary conditions:

$$0 \le a_0 \le 10, \quad 0 \le a_1 \le 1, \quad 0 \le a_2 \le 0.5, \quad 0 \le a_3 \le 0.1,$$

$$0 \le b_1 \le 5, \quad 0 \le b_2 \le 0.5, \quad 0 \le S_c \le 100.$$

Constraint 3 is the initial parameter values. In order to ensure an adequate searching space, the starting point for each parameter is set to be at the midpoint of the above range of the parameter; that is,

$$a_0 = 5, \quad a_1 = 0.5, \quad a_2 = 0.25, \quad a_3 = 0.05,$$

$$b_1 = 2.5, \quad b_2 = 0.25, \quad S_c = 50.0.$$

Constraint 4 is the initial storage height in tanks. The starting point of calculation is on January 1 of each year. There is little rainfall during winter seasons, so we assume that streamflow consists of only groundwater runoff; that is, runoff occurs only from the lowest tank, Tank 3. The storage height S_i in Tank i is taken to be

$$S_0 = 0, \quad S_1 = 0, \quad S_2 = 0, \quad S_3 = \frac{Q(1)}{a_3},$$

where $Q(1)$ is the observed daily streamflow on January 1.

5.3.3.2. Termination criterion: The criterion for deciding that the minimum value of the objective function has been reached is

$$|F^{(i+1)}(P_{\mathrm{opt}}^{(i+1)}) - F^{(i)}(P_{\mathrm{opt}}^{(i)})| \le 0.0001$$

$$P_{\mathrm{opt}}^{(i)} = [a_{0i}, a_{1i}, a_{2i}, a_{3i}; b_{1i}, b_{2i}; S_c], \tag{26}$$

where $F^{(i)}$ and $F^{(i+1)}$ are the values of the objective function for optimization runs i and $i+1$, respectively, and $P_{\mathrm{opt}}^{(i)}$ is a set of parameter values for optimization run i.

5.3.3.3. Identification procedure: First, an optimization run is carried out until the objective function cannot be reduced any more; that is, (26) is satisfied, and a set of parameter values $P_{\mathrm{opt}}^{(i)}$ is obtained. Then, in order to avoid choosing a "local optimum" set of parameter values [*Johnston and Pilgrim*, 1976], the selected set of parameter values $P_{\mathrm{opt}}^{(i)}$ is considered to be initial parameter values, and the boundary conditions are reset on the basis of visual inspection of the degree of agreement between the computed streamflow and observed one. An optimization run is executed again. The first and second steps are reexecuted until only very minor changes occur in the parameter values $P_{\mathrm{opt}}^{(i)}$.

Observed daily rainfall-streamflow data from 1978 to 1989 are used to identify parameter values. Because there is little snowfall in the basin of study, the structure of the runoff process is considered to be invariant throughout the year; that is, the tank model's parameters are constant throughout the year. Identified parameter values for each year are listed in Table 2.

5.3.3.4. Evaluation of uncertainty: In order to demonstrate the reliability of identified parameters, the 12-year mean (Mean), maximum (Max), minimum (Min), standard deviation (SD), and skewness coefficient (Cv) for each parameter are also presented in Table 2. It can be seen that all the identified parameters are stable. In order to diminish impacts of some unknown factors on the streamflow, the 12-year mean values are finally determined as the most appropriate values for describing the catchment characteristics.

In order to demonstrate that the model of three serial tanks with a parallel tank is suitable to represent the basin of study, a daily streamflow time series for 1978 is calculated using observed daily rainfall data and mean values of the identified parameters and is presented in Figure 5. By comparing computed daily streamflows with observed ones, it is found that computed flows fit observed ones adequately. Therefore the model of three serial tanks with a parallel tank is an appropriate model to describe the physical characteristics of the basin of study.

5.4. Basic Statistics of 5-Day Averaged Streamflow

As 5-day averaged streamflow data is usually used in the planning, management, and utilization of water resources in Japan, statistics of 5-day averaged streamflows over a selected time period of 1 month are analyzed. Calculation procedures and results are summarized as follows.

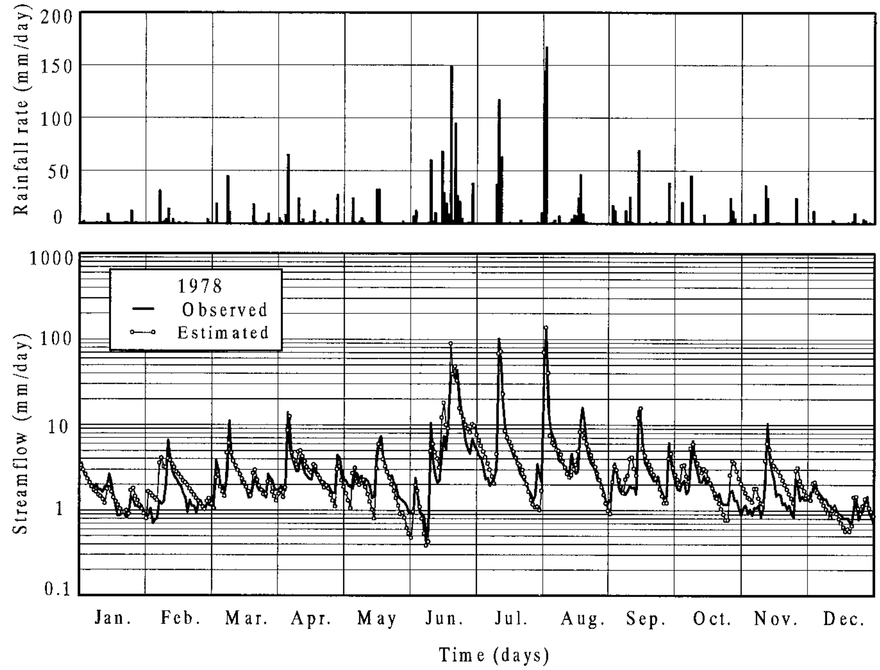

Figure 5. Observed and computed daily streamflow hydrographs (1978).

5.4.1. Observed basic statistics of 5-day averaged streamflow. On the basis of observed daily streamflow data from 1953 to 1989, mean, standard deviation, and lag-one correlation coefficient of 5-day averaged streamflows for each month are estimated and shown in Figures 6a, 6b, and 6c, respectively.

5.4.2. Characteristics of a storm sequence. Storm sequences can be understood through the number of storms, storm duration, and daily rainfall properties.

5.4.2.1. Number of storms: On the basis of observed daily rainfall data from 1953 to 1989, monthly mean and variance of the number of storms are estimated and presented in Table 3. The distribution of the number of storms is verified to follow a binomial distribution. Parameters of the distribution for each month are computed using (5) and are also provided in Table 3.

5.4.2.2. Storm duration: The distribution of storm durations for each month can be approximated by a binomial or negative binomial distribution based on observed data. Monthly mean and variance of the storm duration are calculated and presented in Table 3.

5.4.2.3. Properties of a daily rainfall series: Because the properties (occurrence number and amount) of the storms causing the surface runoff are very different from those of the storms causing the base flow, on the basis of the condition of surface runoff occurrences, observed daily rainfall data is split into two data groups: one without surface-runoff occurrences (runoff occurs only from the model of three serial tanks) and the other with surface-runoff occurrences (runoff occurs from both the model of three serial tanks and the parallel tank). In the basin of study, when surface runoff occurs, the storage height in Tank 1 is equal to or greater than 22.22 mm. At this time, daily streamflow y_t $(= (a_1 + b_1)S_c)$ is equal to 40.3 mm/d; that is, if surface runoff occurs, streamflow must be >40.3 mm/d. On the basis of observed daily rainfall and streamflow data, by using the method of regression analysis, it is found that the storm amount causing surface-runoff occurrences is ≥194.8 mm. On the basis of this storm amount, the observed daily rainfall data is split into the aforementioned two parts.

On the basis of these two daily rainfall sequences, the means $(E(u), E(u_s))$, standard variances $(V(u)^{1/2}, V(u_s)^{1/2})$, and autocorrelation coefficients (ρ, ρ_s) of a daily rainfall series without surface-runoff occurrences and with surface-runoff occurrences are estimated and presented in Table 4.

5.4.3. Theoretical basic statistics of 5-day averaged streamflow. In order to examine the applicability of the proposed method to the basins that have long-term recorded daily rainfall data but lack long-term observed daily streamflow data, we analyze and compare the following two cases: case 1, with both long-term daily rainfall data and long-term daily streamflow data (from 1953 to 1989), and case 2, with long-term recorded daily rainfall data (the same as that in Case 1) and without long-term observed daily streamflow data (but it is long enough to identify the reliable tank model's parameters, for example, about 10 years or even less).

In case 1, first, on the basis of observed daily streamflow data, the probability P_r of surface-runoff occurrences is estimated using (17) and is shown in Figure 6d. Then, by substituting P_r, the properties of a storm time series, and the tank model's parameters into (A1)–(A3), the mean, standard deviation, and lag-one correlation coefficient of 5-day averaged streamflow are estimated and presented in Figures 6a, 6b, and 6c, respectively.

In case 2, 12-year daily streamflow data from 1978 to 1989 are made use of, and the probability P_r is estimated and shown in Figure 6d. Then, similar to case 1, the mean, standard variance, and lag-one correlation coefficient are computed and presented in Figures 6a, 6b, and 6c, respectively.

From Figure 6 it can be seen that (1) computed means and standard deviations for both cases are almost the same as observed ones and (2) computed autocorrelation coefficients are smaller than observed ones, especially during the winter

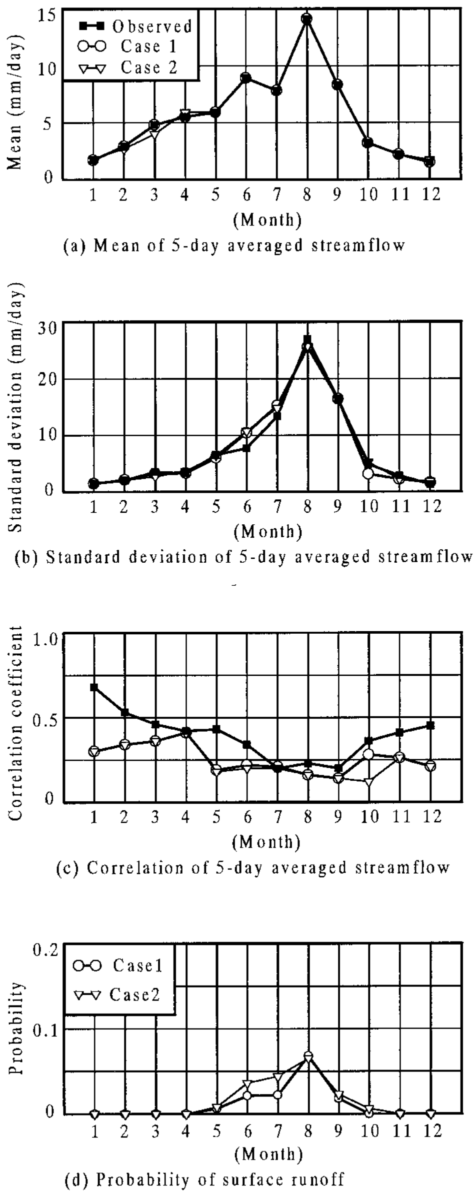

Figure 6. Observed and computed statistics of 5-day averaged streamflows.

seasons (January–March and November–December). The authors consider that the artificial water release from another hydroelectric power station, located on the upper reaches of the stream, leads to the results obtained. For the mean of streamflows the effect of this release flow on it is represented by the model's parameters as these parameters are identified on the basis of the streamflow that includes this release flow. For the variance of streamflows the release flow has no effect on it, as adding a constant to a variable cannot change its variance. For the theoretical correlation coefficient it is mainly decided by two factors: one is the autocorrelation coefficient of a daily rainfall time series, and the other is the parameter values of the tank model. The artificial water release from the hydroelectric power station can affect only the identified parameter values of the tank model and cannot affect the correlation nature of rainfall processes, while for the observed streamflows, because of the nearly constant water release from the hydroelectric power station during winter seasons, the correlation coefficient of the observed streamflows becomes much higher than that of the natural streamflows that are not artificially controlled. Thus the theoretical correlation coefficients are smaller than the observed ones during winter seasons.

6. Concluding Remarks

For the purpose of water resources planning and so forth, basic statistics (mean, variance, and autocorrelation coefficients) of J-day averaged streamflows are formulated on the basis of the theory of a filtered point process by *Snyder* [1975]. Streamflow is considered to be a filtered point process where the input is a storm sequence that is assumed to be a marked point process. The mark is the storm magnitude that is constructed from a daily rainfall time series, and the correlation of the daily rainfall during the storm is considered. The number of storms is a counting process represented by either the binomial, the Poisson, or the negative binomial probability distribution, depending on its ratio of mean versus variance. As a pulse-response function for a filtered point process, the model of three serial tanks with a parallel tank is adopted to describe the physical process of rainfall-runoff. The derived basic statistics of J-day averaged streamflows is expressed as the functions of the constants describing stochastic properties of a rainfall time series, the tank model's parameters representing the causal relationship between rainfall and runoff, and the period over which streamflow is averaged. The model is used with the data from the Sameura Dam basin. The resulting computed monthly means and

Table 3. Monthly Properties of a Storm Sequence

	Number of Storms					Storm Duration		
Month	$E(N_c)$	$V(N_c)$	Distribution	p	k	$E(T_r)$	$V(T_r)$	Distribution
January	4.8	3.1	binomial	0.354	13.6	1.7	1.1	binomial
February	5.3	2.0	binomial	0.623	8.5	1.8	1.3	binomial
March	6.3	2.9	binomial	0.540	11.7	1.9	1.3	binomial
April	6.0	3.4	binomial	0.433	13.9	2.1	1.6	binomial
May	5.9	2.7	binomial	0.542	10.9	2.1	1.4	binomial
June	5.3	2.4	binomial	0.547	9.7	2.9	4.7	negative binomial
July	5.0	2.6	binomial	0.480	10.4	2.8	4.7	negative binomial
August	5.2	3.7	binomial	0.288	18.1	2.7	4.9	negative binomial
September	5.2	2.2	binomial	0.577	9.0	2.3	2.4	negative binomial
October	5.0	2.7	binomial	0.460	10.9	1.8	0.7	binomial
November	4.7	2.9	binomial	0.383	12.3	1.5	0.5	binomial
December	4.7	3.8	binomial	0.191	24.6	1.5	0.4	binomial

Table 4. Monthly Properties of Daily Rainfall Amount During a Storm

	Without Surface Runoff			With Surface Runoff		
Month	$E(u)$, mm	$V(u)^{1/2}$, mm	ρ	$E(u_s)$, mm	$V(u_s)^{1/2}$, mm	ρ_s
January	8.9	12.3	0.022	0	0	0
February	11.0	15.3	0.042	0	0	0
March	15.4	19.3	0.090	0	0	0
April	19.1	21.9	0.122	0	0	0
May	18.1	20.1	0.071	73.8	79.1	0
June	18.1	21.4	0.065	48.6	58.2	0.103
July	16.3	22.2	0.103	54.7	74.3	0.330
August	16.8	24.3	0.133	75.0	93.5	0.262
September	16.7	23.6	0.089	89.0	100.2	0.277
October	15.2	20.6	0.046	0	0	0
November	12.8	20.1	0.105	0	0	0
December	10.3	17.6	0.018	0	0	0

variances of 5-day averaged streamflows show a good correspondence with observed ones. Therefore, even though computed correlation coefficients could not fit the observed ones well during the winter seasons because of the effect of the water release from an electric power station in the upper reaches of the stream, we can also conclude the following: if there is only enough daily streamflow data (for example, about 10 years or even less) to identify the parameters for the tank model, the basic statistics of streamflows can be successfully derived from the properties of a rainfall time series and the basin-response function using the proposed method.

As a basin-response system, the model of three serial tanks with a parallel tank is a suitable linear rainfall-runoff model for expressing the physical characteristics of the basin of study. This model has a flexible structure; that is, one can select the model's structure depending on the number of different sources or aquifers of which streamflow drains out. In this study we construct the model on the basis of the fact that the streamflow in the basin of study consists of surface, rapid and delayed subsurface, and groundwater runoffs that come from different sources. If the base flow mainly comes from two different aquifers, one should use the model of two serial tanks with a parallel tank to represent rainfall-runoff processes. Conversely, if the baseflow comes from four different aquifers, one should select the model of four serial tanks with a parallel.

In the proposed method we introduce the two correction parameters: one is the evapotranspiration correction parameter f_E for modifying the daily evapotranspiration by the Hamon method, and the other is the daily rainfall correction parameter f_P for modifying the observed point rainfall amount. If observed daily evapotranspiration and rainfall data are sufficient to represent the areal mean values of a given basin, these two correction parameters should not be used anymore.

Appendix

$$\gamma_1(Y_t) = f_P\left[D_t^{-1}E(u)H_J^{(1)} \sum_{n=1}^{D_t/2} P(N_t = n)L(n) + P_rE(u_s)H_{Js}^{(1)}L(1) \right] - \frac{f_E E_p[D_t - E(N_c)E(T_r)]}{D_t}, \tag{A1}$$

$$\begin{aligned}\gamma_2(Y_t) = f_P^2 D_t^{-1}\Bigg[& E(u^2)H_J^{(2)} \sum_{n=1}^{D_t/2} P(N_t = n)L(n) \\ & + 2V(u) \sum_{n=1}^{D_t/2} P(N_t = n) \sum_{j=1}^{n} \sum_{l_j=2}^{D_t-2j+1} P(T_j = l_j) \\ & \cdot \sum_{m=1}^{l_j-1} (l_j - m)\rho^m H_J^{(1,1)}(m) + 2E(u)^2 \sum_{n=1}^{D_t/2} P(N_t = n) \\ & \cdot \sum_{j=1}^{n} \sum_{l_j=2}^{D_t-2j+1} P(T_j = l_j) \cdot \sum_{m=1}^{l_j-1} (l_j - m)H_J^{(1,1)}(m) \Bigg] \\ & + \varepsilon \frac{\gamma_1(Y_{1t})^2}{k} + P_r f_P^2[E(u_s^2)H_{Js}^{(2)}L(1) + 2V(u_s) \\ & \cdot \sum_{l_1=2}^{D_t-1} P(T_1 = l_1) \sum_{m=1}^{l_1-1} (l_1 - m)\rho_s^m H_{Js}^{(1,1)}(m) \\ & + 2E(u_s)^2 \cdot \sum_{l_1=2}^{D_t-1} P(T_1 = l_1) \sum_{m=1}^{l_1-1} (l_1 - m)H_{Js}^{(1,1)}(m)],\end{aligned} \tag{A2}$$

$$\begin{aligned}\mathrm{Cov}\,(Y_t, Y_{t+o}) = f_P^2 D_t^{-1}\Bigg[& E(u)^2 H_J^{(1,1)} \sum_{n=1}^{D_t/2} P(N_t = n)L(n) \\ & + 2V(u) \sum_{n=1}^{D_t/2} P(N_t = n) \sum_{j=1}^{n} \sum_{l_j=2}^{D_t-2j+1} P(T_j = l_j) \\ & \cdot \sum_{m=1}^{l_j-1} (l_j - m)\rho^m H_J^{(1,1)}(o + m) + 2V(u) \\ & \cdot \sum_{n=1}^{D_t/2} P(N_t = n) \sum_{j=1}^{n} \sum_{l_j=2}^{D_t-2j+1} P(T_j = l_j) \\ & \cdot \sum_{m=1}^{l_j-1} (l_j - m)\rho^m H_J^{(1,1)}(|o - m|) + 2E(u)^2\end{aligned}$$

$$
\cdot \sum_{n=1}^{D_t/2} P(N_t = n) \sum_{j=1}^{n} \sum_{l_j=2}^{D_t-2j+1} P(T_j = l_j)
$$

$$
\cdot \sum_{m=1}^{l_j-1} (l_j - m)\rho^m H_J^{(1,1)}(o + m) + 2E(u)^2
$$

$$
\cdot \sum_{n=1}^{D_t/2} P(N_t = n) \sum_{j=1}^{n} \sum_{l_j=2}^{D_t-2j+1} P(T_j = l_j)
$$

$$
\left. \cdot \sum_{m=1}^{l_j-1} (l_j - m) H_J^{(1,1)}(|o - m|) \right]
$$

$$
+ \varepsilon \frac{\gamma_1 (Y_{1t})^2}{k} + P_r f_P^2 [E(u_s)^2 H_{Js}^{(1,1)} L(1) + 2V(u_s)
$$

$$
\cdot \sum_{l_1=2}^{D_t-1} P(T_1 = l_1) \sum_{m=1}^{l_1-1} (l_1 - m)\rho_s^m H_{Js}^{(1,1)}(o + m) + 2V(u_s)
$$

$$
\cdot \sum_{l_1=2}^{D_t-1} P(T_1 = l_1) \sum_{m=1}^{l_1-1} (l_1 - m)\rho_s^m H_{Js}^{(1,1)}(|o - m|) + 2E(u_s)^2
$$

$$
\cdot \sum_{l_1=2}^{D_t-1} P(T_1 = l_1) \cdot \sum_{m=1}^{l_1-1} (l_1 - m)\rho_s^m H_{Js}^{(1,1)}(o + m) + 2E(u_s)^2
$$

$$
\cdot \sum_{l_1=2}^{D_t-1} P(T_1 = l_1) \sum_{m=1}^{l_1-1} (l_1 - m) H_{Js}^{(1,1)}(|o - m|)], \qquad \text{(A3)}
$$

where

$$
H_J^{(i)} = \int_{t_0}^{D_t} h_J(t)^i \, dt \qquad i = 1, 2
$$

$$
H_{Js}^{(i)} = \int_{t_0}^{D_t} h_{Js}(t)^i \, dt \qquad i = 1, 2
$$

$$
H_J^{(1,1)} = \int_{t_0}^{D_t} h_J(t) h_J(t + m) \, dt \qquad i = 1, 2
$$

$$
H_{Js}^{(1,1)} = \int_{t_0}^{D_t} h_{Js}(t) h_{Js}(t + m) \, dt \qquad i = 1, 2
$$

$$
L(n) = \sum_{j=1}^{n} \sum_{l_j=1}^{D_t-2j+1} l_j P(T_j = l_j),
$$

where $E(N_c)$ and $E(T_r)$ are the mean of the number of storms and the mean of storm durations, respectively; the notations of the other symbols in the above formulas are the same as those in (17) and (19)–(23).

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