SCALING LAWS FOR EXTREME PRECIPITATIONS

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Abstract

The study aims at validating the use of scaling models for the description of the spatio-temporal structure of **extreme precipitations** in North America.

By means of **scaling models**, the statistical distribution of precipitation intensity estimated at one spatial and temporal scale is related to the distributions at other scales.

It is therefore possible to assess extreme precipitation distribution at spatial/temporal scales which are partially or not sampled, and a consistent and parsimonious method to **construct IDF and IDAF curves** follows directly.

1. Main questions and Objectives

1. Estimating and validating scaling laws for extreme precipitation intensity.
2. Analysing spatial patterns and the variation over several duration intervals of scaling laws for North America precipitations.
3. Constructing IDF curves bsed on scaling laws [future work].

2. Scaling Models

Scale Invariance of Precipitations:

- Hierarchical structure of meteorological systems
- Chaotic nature of climate [Lovejoy and Schertzer, 1985]

Some statistical features of precipitation intensity X do not change if the observational scale is changed.

Simple Scaling (SS) Models

[Gupta and Waymire, 1990; Menabde et al, 1999]:

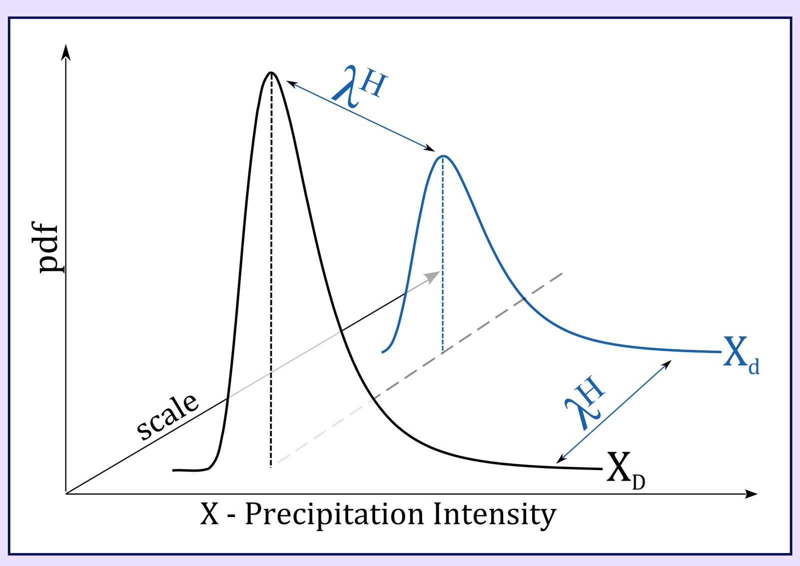


Fig. 1: SS Model: relationship between X probability distributions at scales D and d.

X observed at two different scales D and d is such that:

$$X_D \sim \lambda^H X_d$$

where

$\lambda = D/d$ scale ratio, $H \in \mathbb{R}$ scaling exponent

Scaling exponent [H]: adimensional measure of extreme variability through scales (duration and/or measuring area). H depends on the geographical and climatic characteristics of the study region. [e.g., Borga et al., 2005; Ceresetti et al., 2010]

Scaling IDF [temporal SS] and IDAF [spatio-temporal SS]

robust and parsimonious estimation:

- All observations used, not only some particular quantiles.
- Based on pooled samples [all scales].
- Consistent with parametric assumptions for X pdf [ex. GEV] and their statistical estimation [ex. Maximum Likelihood or L-Moments].

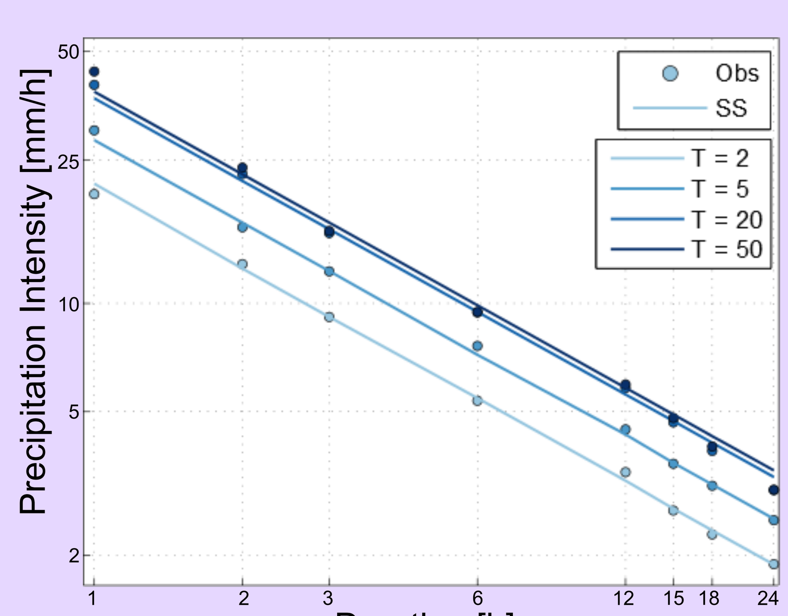


Fig. 2: Distribution-free SS IDF curves and empirical quantiles - P.E.T. Montreal Int. Airport Station, QC.

3. Data

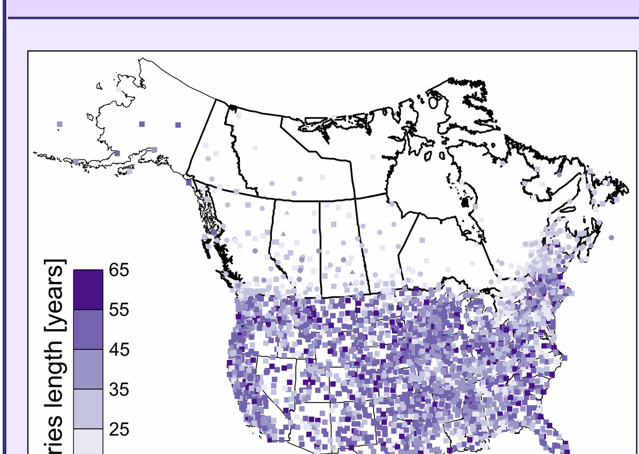


Fig. 3: Station locations and series length.

- 2170 stations [EC, MDDELCC, NOAA]: ~1940-2011 data collected at durations from 5 min to 24 h.
- Precipitation Intensity X_d [mm/h] Annual Maxima Series (AMS).

Table 1: Precipitation series type and number of stations available.

Daily Maxima**	Daily Maxima** & Hourly	Hourly	15 min series*
8	236	2351	125

** Series available for duration 5, 10, 15, 30 min and 1, 2, 6, 12 h. Only Canada.
* Only North-East US

4. Temporal Scaling for IDF

1. Estimation and validation

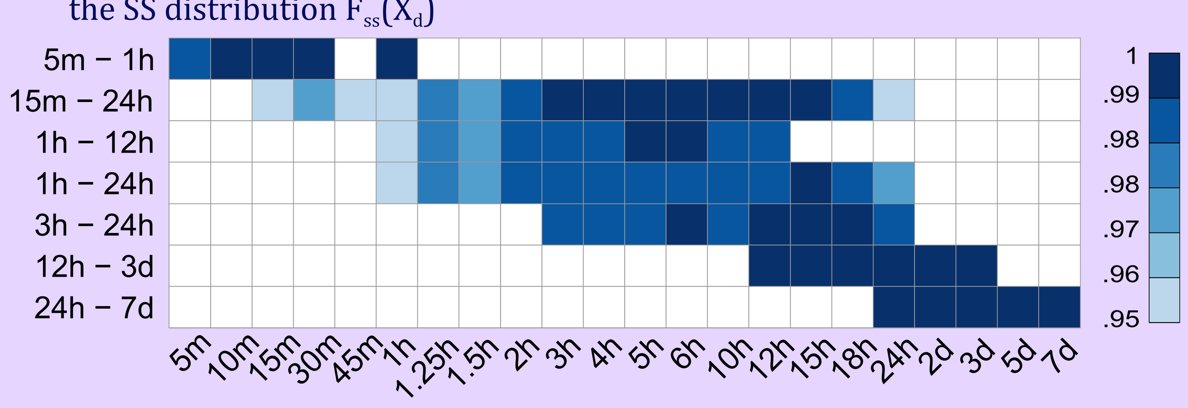
Durations (temporal scales) and SS ranges: definition of various duration ranges over $\mathcal{D}$ = [5, 10, 15, 30, 45 min, 1, 1.25, 1.5, 2, 3, 4, 5, 6, 10, 12 , 24 h, 2, 3, 5, 7 days].

SS estimation: Moment Scaling Analysis for various duration ranges. Log-log linear regression between sample moments $E[X_d]$ and scales d: $E[X_D^q] = \lambda^{Hq} E[X_d^q]$.

SS validity check: For each d, Goodness-Of-Fit (GOF) tests for H_0: $F_{emp}(X_d) = F_{ss}(X_d)$ ⟶ Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) tests.

Validity of scale invariance property over several duration ranges for at least 95% of stations having the range.

Fig. 4: KS and AD tests (both test condition) - For each duration, non-rejected stations have empirical X_d distribution not significantly different from the SS distribution $F_{ss}(X_d)$



SS predictive ability: RMSE between empirical and SS quantiles of X_d cdfs.

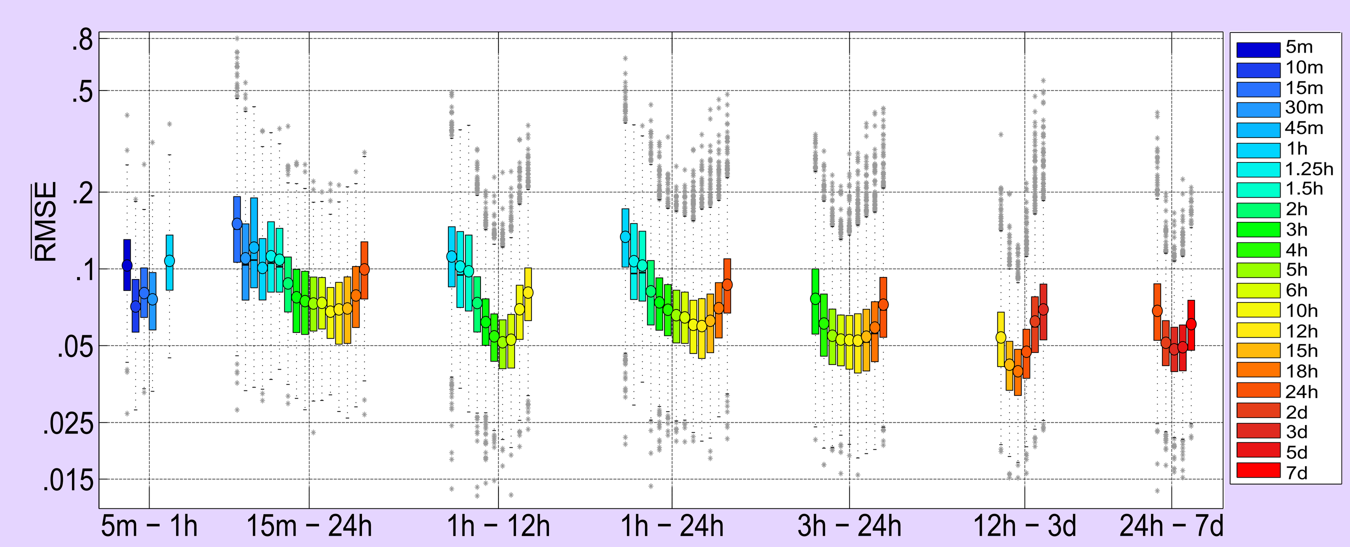


Fig. 5: Normalized RMSE of estimated SS distribution quantiles vs empirical quantiles (Cunnane plotting position) for valid series (according to GOF tests). For each d, RMSE is normalized with the mean intensity $\overline{x}_d$ of the observed AMS.

Small relative error in approximating X_d cdf by the SS distribution.

2. Scaling exponent spatial distribution and variation over temporal scales

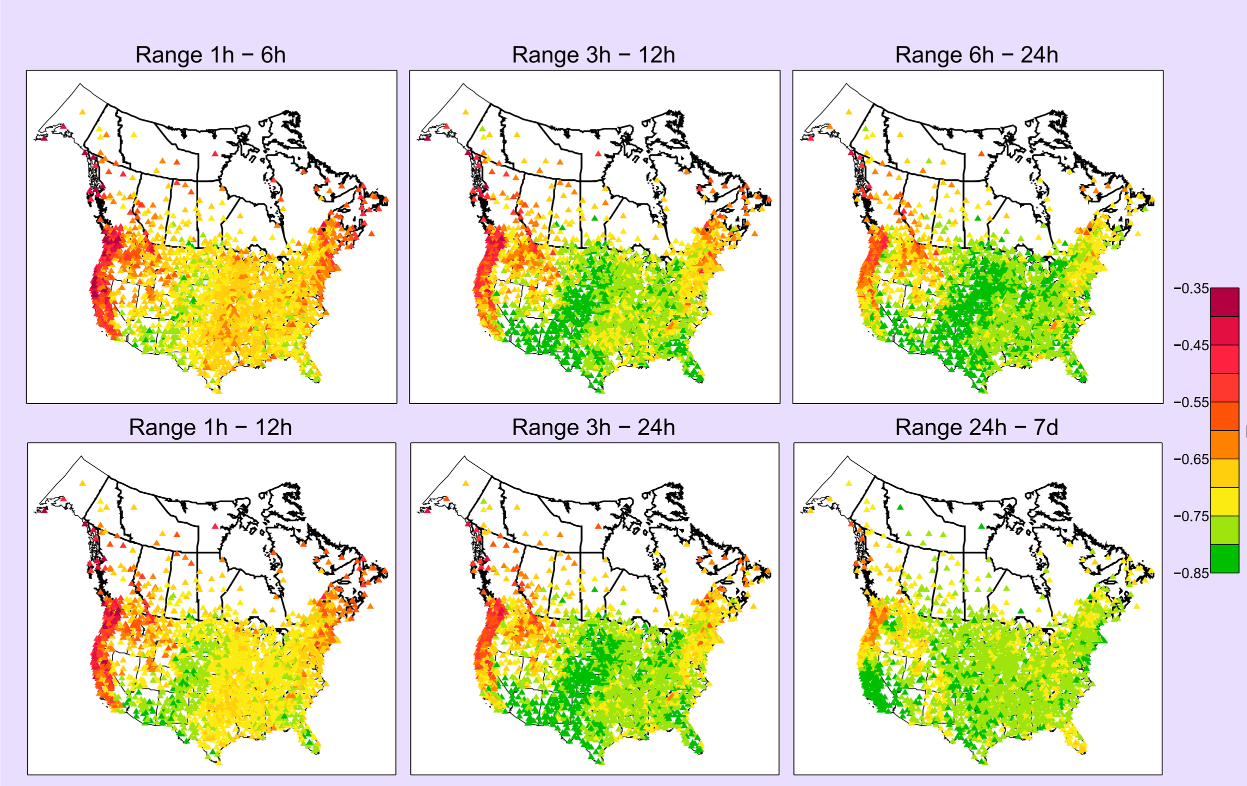


Fig. 6: Spatial distribution of the **scaling exponent** over various SS duration ranges. Higher H, in absolute value, is associated to larger variations in moment values as scale (duration) is changed.

Transition from smaller to higher H values (in absolute value): transition from homogeneous to more variable weather regimes/dynamics as the duration range is changed.

H homogeneous in space over large areas: possibility of estimating H regionally and/or to interpolate H for non-observed sites.

3. IDF construction

Distributional assumption for X. For instance, GEV Simple Scaling: if $X_d \sim GEV(\mu, \sigma, \xi)$ then $X_D \sim GEV(\lambda^H\mu, \lambda^H\sigma, \xi)$ and the IDF is

$$X(T,d) = d^H \times \{\mu^* + (\sigma^*/\xi^*)\,[\,(-\ln(p))^{-\xi^*} - 1]\}$$

where μ*, σ*, and ξ* GEV parameters for the reference d* = 1.

Uncertainty estimation available.

5. Future steps

- Temporal SS for **other observation** [radar, gridded interpolated stations, etc.] **and simulated datasets** [reanalysis data, climate models].
- Spatio-temporal SS and **IDAF construction** [De Michele, 2002]: stations as point reference precipitation and areal AMS from radar, reanalysis, etc.

Conclusions

- Validity of scale invariance property over several duration ranges for at least 95% of stations. Mean error in approximating $F(X_d)$ by SS quantiles generally less than 10%.
- The SS exponent is a climatological measure of variabiliy: H smoothly changes in space and provide information about main meteorological processes characterizing large areas.

  Possible estimation of H at a regional scale: uncertainty reduction and IDF estimation for (partially) non-sample sites.

References

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