

21 **Abstract**

22 Change point detection methods have an important role in many hydrological and
23 hydraulic studies of river basins. These methods are very useful to characterize changes
24 in hydrological regimes and can, therefore, lead to better understanding changes in
25 extreme flows behavior. Flood events are generally characterized by a finite number of
26 characteristics that may not include the entire information available in a discharge time
27 series. The aim of the current work is to present a new approach to detect changes in
28 flood events based on a functional data analysis framework. The use of the functional
29 approach allows taking into account the whole information contained in the discharge
30 time series of flood events. The presented methodology is illustrated on a flood analysis
31 case study, from the province of Quebec, Canada. Obtained results using the proposed
32 approach are consistent with those obtained using a traditional change point method, and
33 demonstrate the capability of the functional framework to simultaneously consider
34 several flood features and, therefore, presenting a comprehensive way for a better
35 exploitation of the information contained in a discharge time series.

36 **Keywords:** Functional data analysis, Change point detection, Hydrology, Flood.

37

38 **Introduction**

39 Detection of changes in hydrological data is of interest to better understand
40 hydrological regimes, and separate events. Changes in a series can occur in numerous
41 ways, gradually or abruptly, and can affect the mean, median, variance, autocorrelation,
42 or any other aspect of the data. In the future, regions that are relatively sheltered from
43 wind storms, heat waves, droughts and floods, may no longer be in a warmer climate
44 (Goudie 2006). Detection of changes in long time series of hydrological data is an
45 important and difficult issue, of increasing interest. Change point detection in hydrology
46 are essential to characterize the impacts of the climate disturbances on hydrological
47 regimes (Kingston et al. 2011). It is then very important, particularly where we observe
48 changes in the frequency and/or in the intensity of various forms of extreme weather
49 events. Detection of eventual changes in collected data of hydrologic time series sets is
50 thus obviously an important step before performing any descriptive or predictive analysis.

51 Literature abounds with studies on change point testing in scalar or vector time
52 series. For example, Kundzewicz and Robson (2004) gave a general guidance on the
53 methodology for change detection in hydrological records. Wong et al. (2006) proposed a
54 relational method for discrete data. Change point analysis is addressed in both classical
55 and Bayesian statistics. Methods in classical statistics usually consist of performing
56 several kinds of tests to either confirm or reject the hypothesis of change. Most of them
57 address slope or intercept change in linear regression models (Solow 1987, Easterling and
58 Peterson 1995, Vincent 1998). Bayesian statistics methods are performed to obtain a
59 statistical distribution for the change point and eventually a distribution for the other
60 model parameters. The inference on parameters was performed using Monte-Carlo

61 Markov Chain algorithms (MCMC). Seidou and Ouarda (2007) proposed a Bayesian
62 method of multiple change point detection in multiple linear regression. This method is
63 numerically efficient and does not involve the time-consuming Monte-Carlo Markov
64 Chain simulations as opposed to other Bayesian change point methods. The procedure
65 was initially designed to detect a change in the relationship between a set of explanatory
66 variables and the dependent variables. Using the time variable as an explanatory variable,
67 this approach can detect the change point in a given time series.

68 The flood event is an integration of spatial and temporal variations in water input,
69 storage and transfer processes within a catchment (Hannah et al. 2000). Particularly,
70 discharge (rate of flow) time series is the main source of information for studying flood
71 events. Arguably, the hydrograph of a flood event as a graph showing the discharge
72 versus time has been the cornerstone of statistical hydrology, as it is directly related to the
73 design of hydraulic infrastructures. In spite of considerable progress in the development
74 of new statistical tools for change point analysis, researchers' previous efforts have been
75 mainly focused on a single or few characteristics of the flood hydrograph ignoring the
76 continuous behaviour of the flood event in time. Classical change point detection
77 approaches involve a substantial simplification of the overall extreme hydrological event,
78 through focusing on a single or few characteristics of the flood event such as the peak or
79 the volume, and, therefore, fail to account for the whole information stored in flood
80 hydrographs presented as continuous curves. Despite the extensive literature on change
81 point methods, little recognition appears to have been given to a more general approach
82 considering the entire information contained in the discharge time series. The overall
83 objective of this paper is to present a new approach that attempts to handle this concern

84 by considering the discharge time series of the flood event as a continuous curve using a
85 functional data analysis (FDA) framework.

86 The first application of FDA to the hydrological context refers to Chebana et al.
87 (2012) introducing an exploratory analysis and outlier detection of hydrographs. Chebana
88 et al. (2012) showed that FDA is more general, flexible and representative of the real
89 hydrological phenomena. For classification of flood events, Ternynck et al. (2016)
90 showed that obtained classes using functional approaches are more representative than
91 those obtained using a traditional multivariate hierarchical classification method.
92 Masselot et al. (2016) adapted a functional regression model for streamflow forecasting.
93 Suhaila and Yusop (2017) employed the functional framework to study the spatial and
94 temporal variability of precipitation in Peninsular Malaysia. More recently, Requena et
95 al. (2018) proposed a functional multiple regression for flow duration curves estimation
96 while Larabi et al. (2018) developed a stepwise multicriteria for rainfall-runoff model
97 calibration defined on the basis of FDA.

98 A growing research area is being advanced focusing on the development of new
99 statistical tools to analyze functional data. For instance, many existing tools in the
100 univariate and multivariate statistical literature have been adapted to the functional
101 context (Dabo-Niang et al. 2010, Fischer 2010, Chebana et al. 2012). Some authors
102 investigated the change point detection method in the FDA context for testing the
103 assumption of a common functional mean for independent functional data (Aue et al.
104 2009, Berkes et al. 2009). Thereafter, Zhang et al. (2011) adapted this work to the case of
105 functional dependent data.

106 The aim of the present paper is to introduce and adapt the FDA framework to change
107 point detection of flood events. The present paper is structured as follows: a brief
108 presentation of the data set and the study area is provided in section 1, the proposed
109 functional change point detection approach is presented in section 2. Results of the
110 application of the proposed method to the case of flood events in two stations from the
111 province of Quebec, Canada, are illustrated in section 3. Discussion and conclusion of the
112 main findings are given in sections 4 and 5, respectively.

113 **1. Data Description**

114 Daily flow data recorded at two hydrological stations in the province of Quebec,
115 the Romaine River and the Moisie River stations, are considered (Figure 1). The
116 available data series for the Romaine river station covers the period from 1961 to 2000
117 recorded over a drainage area of 13000 Km^2 . For the Moisie river station, with a
118 drainage area of 19000 Km^2 , daily flow records between 1968 and 1991 are used. Given
119 the nature of most of the flooding events that characterize the area, mainly caused by
120 snow melting in spring and summer, only flood events occurring between March 1st and
121 August 31st are considered in the current analysis.

122 The selection of these two stations is mainly based on previous finding about the
123 inhomogeneity of their flood regimes (Ternynck et al. 2016). Furthermore, previous
124 results on flood event behaviour for both Romaine river and Moisie river stations
125 demonstrate an apparent change in annual maxima discharges time series (Seidou and
126 Ouarda 2007). Thus, it is expected that these two case studies may represent
127 comprehensive examples to test and validate the proposed approach.

128 While the proposed approach is general and can be applied to entire annual
129 discharge series, a prior knowledge about the season on which major flood events occur
130 can be helpful to primarily focus on possible changes in the flood event of interest. This
131 allows avoiding misleading conclusion in change point results that are due to changes
132 affecting streamflow not related to the major flood event. Although climate change might
133 shift the timing of flood events (Blöschl et al. 2017), this should not be a concern since
134 our choice of the spring-summer period is long enough to account for this fact.

135 **2. Functional change point detection method**

136 Consider n years of daily flow series recorded from March 1st to August 31st at a
137 given station corresponding to flood events occurring in the spring-summer period. Let
138 $x_i = (x_i(t_1), \dots, x_i(t_j), \dots, x_i(t_T))$, $i = 1, \dots, n$ be the set of n discrete observations where
139 each $t_j \in \mathbf{T} \subset \mathfrak{R}^+$ and $j = 1, \dots, T$ is the j^{th} record time point corresponding to the day j
140 from time subset corresponding to the \mathbf{T} from March 1st to August 31st which include the
141 set $\{1, \dots, T\}$. For instance, discrete observations x_i are daily flow within a given i^{th} year
142 for the spring-summer period with $T = 181$. For a given year i , each set of measurements
143 $(x_i(t_1), \dots, x_i(t_T))$ will be converted to a functional data denoted $\{X_i(t), t \in \mathbf{T} \subset \mathfrak{R}^+\}$ using
144 a smoothing technique.

145 In order to build functions, Ramsay and Silverman (2007) presented two main
146 basis systems namely: the Fourier system and the B-spline system. Those systems are
147 now well-established in the statistical literature of FDA. Actually, most of theoretical
148 developments have been made based on them. As suggested by Ternynck et al. (2016),

149 we use the B-spline basis system for smoothing spring and summer daily discharge data.
 150 The Fourier system is commonly used for periodic data, while the B-spline system is
 151 rather used for non-periodic data. Fourier basis functions have been used by Chebana et
 152 al. (2012) for smoothing daily streamflow that cover the entire year to obtain annual
 153 streamflow curves. Since the present application considers only the spring and summer
 154 period, the Fourier basis appears, however, to be less suited.

155 The main idea of the change point detection, here, is to test whether the mean of
 156 the functional observations X_1, \dots, X_n remains constant over time. We assume that
 157 $X_i(t) = \mu_i(t) + \varepsilon_i(t)$, $i = 1, \dots, n$ where $\mu_i(t)$ denotes the functional mean and $\varepsilon_i(t)$ is a
 158 zero-mean functional sequence. We wish to test the null hypothesis
 159 $H_0 : \mu_1(t) = \mu_2(t) = \dots = \mu_n(t)$ against the alternative H_a that there is an unknown change
 160 point k^* in the mean, i.e. $H_a : \mu_1(t) = \mu_2(t) = \dots = \mu_{k^*}(t) \neq \mu_{k^*+1}(t) = \dots = \mu_n(t)$. The
 161 change can occur at any point i and we want to test whether it occurs or not. The
 162 existence of change points means that the data can be divided into several consecutive
 163 segments, with a constant mean within each segment. Berkes et al. (2009) proposed an
 164 approach to test the assumption of a common functional mean for independent data. This
 165 approach is based on the following quantity (which measures a deviation between the
 166 mean of the functional observations X_1, \dots, X_k and that of X_{k+1}, \dots, X_n):

$$167 \quad P_k(t) = \frac{k(n-k)}{n} \{ \hat{\mu}_k(t) - \tilde{\mu}_k(t) \}, k = 1, \dots, n \quad (1)$$

168 where $\hat{\mu}_k(t) = \frac{1}{k} \sum_{1 \leq i \leq k} X_i(t)$ and $\tilde{\mu}_k(t) = \frac{1}{n-k} \sum_{k+1 \leq i \leq n} X_i(t)$. If the mean changes, the
169 difference $P_k(t)$ is large for some values of k and t . To deal with the infinite dimension
170 of the observations (curves), we consider the projections of the functions $P_k(\cdot)$ on the
171 principal components of the data. In fact, principal component analysis represents
172 functional data as $X_i(t) = \mu(t) + \sum_{1 \leq l \leq \infty} \eta_{i,l} v_l(t)$, where $\mu(t)$ is the functional mean, $\eta_{i,l}$
173 are the scores and $v_l(t)$ are the eigen-functions of the covariance operator (Hall and
174 Hosseini-Nasab 2006). These projections can be expressed in terms of functional scores,
175 which can be easily computed using the R package “fda”. We consider the estimated
176 scores $\hat{\eta}_{i,l}$ corresponding to the largest L eigenvalues given by:

$$177 \quad \hat{\eta}_{i,l} = \int \{X_i(t) - \bar{X}_n(t)\} \hat{v}_l(t) dt, \quad i = 1, 2, \dots, n \quad j = 1, 2, \dots, L \quad (2)$$

178 with $\bar{X}_n(t)$ is the sample mean function and $\hat{v}_l(t), l = 1, \dots, L$ are the estimated eigen-
179 functions of the covariance operator. It is supposed that $k = [n\alpha]$ where $\alpha \in (0,1)$ and
180 $[\cdot]$ denotes the integer part. Note that $P_k(t)$ does not change if the $X_i(t)$ are replaced by
181 $X_i(t) - \bar{X}_n(t)$. Hence, $P_k(t)$ can be written as:

$$182 \quad P_k(t) = \sum_{1 \leq i \leq k} (X_i(t) - \bar{X}_n(t)) - \frac{k}{n} \sum_{1 \leq i \leq n} (X_i(t) - \bar{X}_n(t)) \quad (3)$$

183 Consequently, the projections are defined by $\int P_k(t) \hat{v}_l(t) dt = \sum_{1 \leq i \leq n\alpha} \hat{\eta}_{i,l} - \frac{[n\alpha]}{n} \sum_{1 \leq i \leq n} \hat{\eta}_{i,l}$ and
184 are used for testing whether the mean function remains constant. For this purpose, the
185 following statistic is considered:

186
$$S_{n,L} = \frac{1}{n^2} \sum_{l=1}^L \lambda_l^{-1} \left(\sum_{1 \leq i \leq n_z} \hat{\eta}_{i,l} - \frac{k}{n} \sum_{1 \leq i \leq n} \hat{\eta}_{i,l} \right)^2 \quad (4)$$

187 where $\hat{\lambda}_1, \hat{\lambda}_2, \dots, \hat{\lambda}_L$ denote the L -estimated eigenvalues. The test rejects the hypothesis
 188 H_0 if $S_{n,L}$ is greater than the corresponding critical value, tabulated in Berkes et al.
 189 (2009).

190 While this test does not take into account the temporal dependence it will be
 191 considered here as a first simple step to introduce the functional change point detection
 192 framework in hydrology. Few other researchers recognize this limitation and propose
 193 some improvements. For instance, a more complex approach has been proposed by
 194 Zhang et al. (2011) in order to take into account the temporal dependence. For sake of
 195 simplicity, this latter will not be considered in the current analysis. In the interim, the
 196 functional approach being used here may, nevertheless, serves as a stepping-stone
 197 towards this more complex approach.

198 **3. Results**

199 Results of application of the proposed method to the above-mentioned data set are
 200 compared with those obtained in Seidou and Ouarda (2007). In the latter, the change
 201 point detection method has been applied separately to the peak, the duration and the
 202 volume of flood events occurring in spring and summer. The first step to apply the
 203 functional method consists on performing a functional principal component analysis
 204 where the first principal components explaining large part of the data variance are,
 205 therefore, to be retained. For the Romaine river station, we retained the first four
 206 principal components as they represent 83% of the explained variance. In the hypothesis

207 testing, we set the first type error at 5%. By applying the functional method for the
 208 Romaine river station, we obtain a change point at the year 1984. This suggests that we
 209 can split the set of curves into the following two segments \overline{TD}_1^a : 1961-1984 and \overline{TD}_2^a :
 210 1985-2000, of size 24 and 16, respectively as shown in Figure 2.a. We can see from
 211 Figure 3 that based on the mean, the median and the modal curves, the two obtained
 212 segments have two different peaks. The peak of the first segment is significantly higher
 213 than that of the second. One can also note that changes affect not only the peak, but also
 214 the duration, the volume and the peak date of the flood event as well. Indeed, in both
 215 classes, flood events began at the same time, but last longer in \overline{TD}_1^a .

216 In a second step, we reiterate the procedure on the obtained two segments. We
 217 therefore only find a change point on the segment \overline{TD}_1^a at the year 1968. Consequently,
 218 we obtain the three following periods \overline{TD}_1^b : 1961-1968, \overline{TD}_2^b : 1969-1984 and \overline{TD}_3^b :
 219 1985-2000 of respective size 8, 16 and 16. According to the Figure 4, we can see that
 220 based on the mean curve, flood events of the segment \overline{TD}_2^b begin before those of the
 221 \overline{TD}_1^b , however floods in both segments end at the same time. Accordingly, the flood
 222 durations for the segment \overline{TD}_2^b are larger than those of the segment \overline{TD}_1^b . While flood
 223 events in both \overline{TD}_2^b and \overline{TD}_1^b have almost the same peak, the functional approach seems
 224 to be able to detect the difference in the duration of the flood events. Flood events of the
 225 segment \overline{TD}_2^b begin at the same time with the flood events of the segment \overline{TD}_3^b , and then
 226 they take end at the same time with the segment \overline{TD}_1^b . Moreover, the two segments \overline{TD}_1^b
 227 and \overline{TD}_2^b have almost the same peak. Consequently, the segment \overline{TD}_2^b can be considered

228 as an intermediate period that enables the transition from the flood regime of the segment
229 \overline{TD}_1^b to the flood regime of the segment \overline{TD}_3^b .

230 In conclusion, for the Romaine river station, functional change point method, has
231 detected two change points, the first at year 1984 and the second at years 1968 as shown
232 in Figure 2.a. This result has divided flood events for the Romaine river station into three
233 periods: the first with very large floods, which begins later, a second intermediate period,
234 and a third period characterized by less important floods which starts early. For the
235 comparison of the functional change point results with a traditional method approach we
236 applied the Bayesian approach of Seidou and Ouarda (2007) to the peak, the volume and
237 the duration of flood events separately. The method of Seidou and Ouarda (2007) based
238 on the duration detects a change point at the year 1987. The same method, however,
239 based on the volume and the peak detects a change point at the year 1985, which is closer
240 to the first change point detected by the proposed functional approach (at year 1984). The
241 Bayesian approach based on the volume and the peak separately was not able to detect
242 the second change point in the segment \overline{TD}_1^a . This is due to the fact that this change does
243 not affect the peak or the volume, but mainly affects the occurrence time of flood events.
244 The functional approach allows detecting this change in the occurring time of flood
245 events because it directly considers a large part of the information contained on the entire
246 discharge series, including information on shape, peak time, duration, etc...

247 For the Moisie river station, using the functional approach, we choose the first
248 four principal components since they represent 85% of the explained variance. In the
249 hypothesis testing, we set the first type error at 5%. We obtain a change point at the year

250 1981 which suggests splitting the set of curves into two segments as follows, \overline{TD}_1^c : 1968-
251 1981 and \overline{TD}_2^c : 1982-1991, of size 14 and 10, respectively. We, then, reiterate the
252 procedure on the obtained two segments, but no change point was detected. Therefore,
253 we can conclude that this method allows detecting just one change point at year 1981 as
254 shown in Figure 2.b. Figure 5 shows the mean curve, the median curve and the modal
255 curve of flood hydrograph corresponding to the two obtained segments. This figure
256 shows that flood events in the two segments \overline{TD}_1^c and \overline{TD}_2^c occur at the same date, but
257 those of the segment \overline{TD}_1^c , last longer and have a larger peak. For the Moisie river station
258 we test the existence of a change point on the peak, the volume and the duration
259 separately using the method of Seidou and Ouarda (2007). Only, the method based on the
260 peaks detects a change point at the year 1978.

261 **4. Discussions**

262 It is worth noting that the purpose of the comparison with the conventional
263 approach is not to show that the functional approach performs better, but rather to check
264 whether this approach gives results consistent with those obtained using a traditional
265 approach. Note that, when the focus is only on one characteristic of the flood event, such
266 as the peak, the volume or the duration, traditional univariate approach preferred.
267 However, the functional approach takes into account all the characteristics of the flood
268 event simultaneously, hence, if no preferences on the flood event characteristic, the
269 functional approach is recommended. Then, the graphical representation of the median
270 curve, the mode curve and the mean curve is helpful to summarize the differences
271 between the different flood periods after the detection of the change point.

272 It should be borne in mind that numerous caveats apply to our findings. First, the
273 proposed functional framework suffers from an edge effects issue and therefore is unable
274 to identify possible changes near the beginning and the end of the data record.
275 Nevertheless, this is a common issue for the traditional change point approaches. Further
276 theoretical studies using generated (known) functional data sets may help to quantify this
277 issue, as well to answer many other questions such as the determination of the minimum
278 record length in order to detect a change. Secondly, the proposed approach does not allow
279 detecting multiple change points simultaneously, and thus need to be iterated for each
280 segment until no further change point is detected. Finally, as problems in hydrology often
281 involve missing data, the proposed functional approach lacks the ability of handling
282 missing data, and thus unable to take full advantage of the whole data record that may be
283 available. For instance, a complete data records are available for Romaine river station
284 from 1957 to 2012 as well for Moisie river station from 1966 to 2012, while, in contrast,
285 our analysis was mainly limited to data records from 1961 to 2000 for Romaine river
286 station and from 1968 to 1991 which are the longest periods for which there is no missing
287 data.

288 In change point analysis, if a significant change is detected in hydrological
289 characteristics, then it is important to try to understand the physical reason behind.
290 Change in hydrological characteristics may be caused by climatic factor such as climate
291 variability or climate change, but there may be many other possible explanations, such as
292 anthropogenic change (urbanization, water abstraction etc.), natural catchment changes,
293 and problem linked to data. The best way to improve understanding of change is rather to
294 gather as much information as possible, using, e.g., information about change in the

295 catchment. In addition, related variables, like temperature and precipitation can help to
296 determine whether changes in flow can be explained by climatic factors. Indeed,
297 streamflow depends strongly on the spatial distribution of precipitation in a watershed,
298 and on the interactions between temperature and precipitation which determines whether
299 precipitation falls as rain or snow (Ben Alaya et al. 2014).

300 In a warming climate it is expected that the atmosphere's water holding capacity
301 will increase with warming according to the Clausius-Clapeyron (C-C) equation (Collins
302 et al. 2013), which may lead to more intense precipitation events that may directly affect
303 streamflow and flood events behaviours. In addition, climate variability through oceanic
304 and atmospheric oscillations on a large scale known as teleconnections, such as the North
305 Atlantic Oscillation (NAO), El Nino-Southern Oscillation (ENSO) and Pacific Decadal
306 Oscillation (PDO), influences the variability and trends in the climate system (Hurrell
307 and Van Loon 1997, Rogers 1997) and thus may in turn affect characteristics of flood
308 events.

309 Based on the obtained results, the frequency of flood events which occur later has
310 decreased at both Romaine river and Moisie river stations while earlier floods
311 characterized by low peaks and volumes became more frequent. Given the short record
312 length of the data series used, attributing this change to corresponding underlying
313 processes is challenging. Another challenge is that signals such as trends and shifts are
314 superposed on variability arising from the memory within the hydrological system.

315 While the proposed approach is not able to distinguish between shifts and trends
316 that may be present in the data, the results for the Romaine river station reflect hints

317 about the presence of a trend in functional data. Note, however, that the “trend”
318 terminology in case of the sequence of functional curves is not the same used as in case
319 of random variables where sample elements are points. The definition of a trend in case
320 of the sequence of functional curves requires, first, to define an extended notion of order
321 that tell us in which case a curve can be considered to be higher than another. Such a
322 definition may not be, however, uniquely determined. To the best of our knowledge, a
323 first attempt for functional trend analysis has been proposed by Fraiman et al. (2014).
324 Nevertheless, we think that a more comprehensive way to handle this concern is to
325 account for the notion of autocorrelation in the sequence of functional curves that has
326 been proposed by Zhang et al. (2011).

327 Note that change point analyses are only descriptive. Hence, they cannot answer
328 questions about how the hydrological system works in a non-stationary climate, and,
329 therefore, cannot be used to predict future conditions. Indeed, seeking answers to those
330 questions requires a hydrological modelling. Nevertheless, the proposed functional
331 change point framework, as a mathematical descriptive tool, can play a very important
332 role for scientific investigations. It can help to get first quantitative clues about what
333 happened in the past. Unlike traditional change point approaches, the conclusions reached
334 from the proposed framework are enhanced by providing reach mathematical pictures
335 summarizing the mean, mode and median curves describing flood regimes. Those
336 pictures are obtained within a mathematical framework that rigorously explains how they
337 were obtained and how the conclusions were reached. Those steps can be easily applied
338 to outputs of hydrological models, whether deterministic or stochastic, to rigorously
339 check and test whether they reproduce similar pictures and same conclusions that have

340 been drawn from original data. As suggested in previous works, the interpretation of
341 hydrological models, particularly in a non-stationary climate is always challenging
342 (Montanari and Koutsoyiannis 2014, Serinaldi and Kilsby 2015, Serinaldi and Kilsby
343 2018). On the other hand, by following the first clues given by a reach descriptive
344 analysis, we may achieve a deep understanding about the complexity of the real
345 mechanism. Our finding can serve as a starting point toward an effective calibration of
346 hydrological models, or can merely be used for model testing.

347 As recommended by Koutsoyiannis and Montanari (2015), including additional
348 information from prior physical knowledge about the physical process involved is
349 essential to build a successful hydrological model that can be used to predict the future.
350 In this respect, our approach opens the door to think on how to take full advantage of the
351 functional framework from a modelling perspective. This can be achieved by casting the
352 hydrological modelling in a functional regression framework by including major factors
353 that influence flood events as covariates. The implementation of such approach, however,
354 is not straightforward and is outside the scope of the current paper. Nevertheless, we
355 think that, slowly, over the course of several steps, starting from functional descriptive
356 tools, a pathway can be paved for development of sound functional regression models
357 and perhaps eventual inclusion of additional knowledge from key factors involved in
358 generating flood events.

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362 **5. Conclusions**

363 The purpose of the present paper is to propose a new context of the change point
364 detection of flood hydrographs using functional data framework. A functional change
365 point approach is presented and adapted to flood events. An application is performed for
366 two hydrological stations in the province of Quebec, Canada. The presented functional
367 approach is compared to a classical Bayesian univariate approach applied to the peak, the
368 volume and the duration of flood events separately. Based on this comparison, it has been
369 shown that the functional approach gives results that are consistent with the traditional
370 univariate approach. The functional approach has the benefit that it provides a
371 comprehensive way to handle the flood event as a curve within a defined statistical
372 framework and thus an opportunity for a better exploitation of the information contained
373 in a discharge time series.

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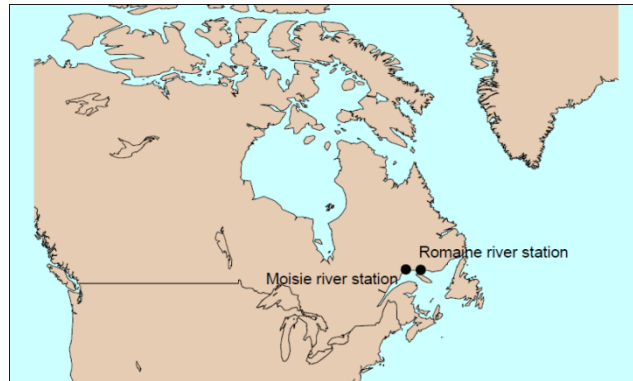
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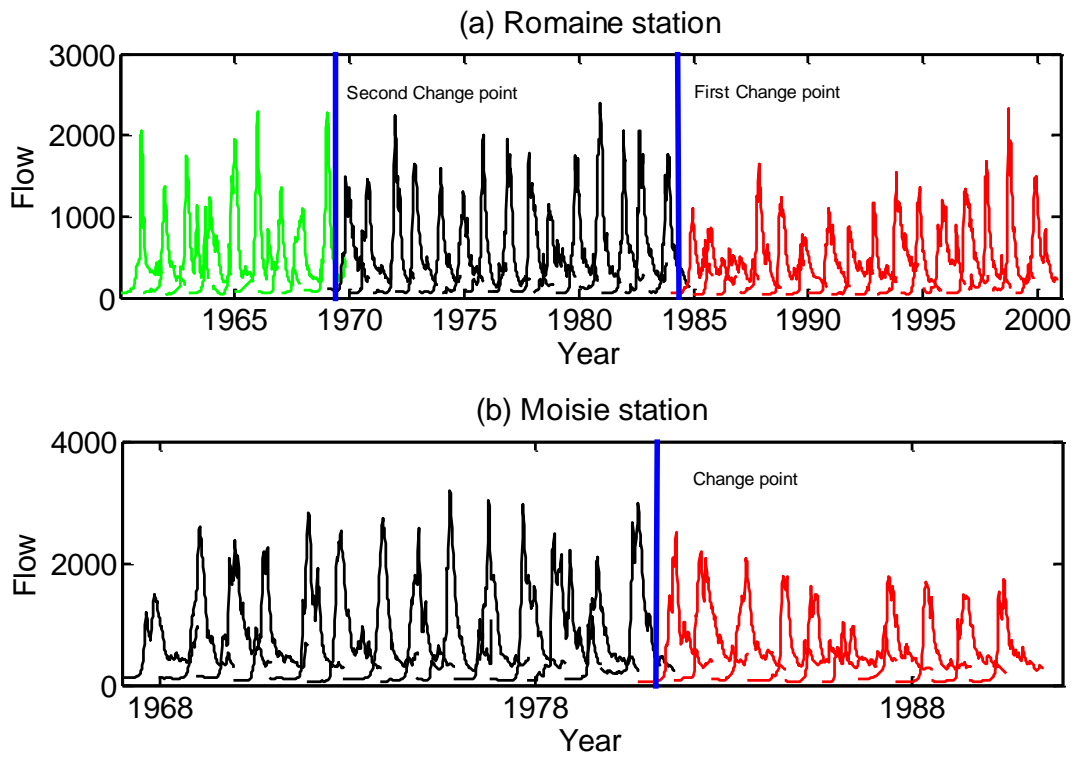


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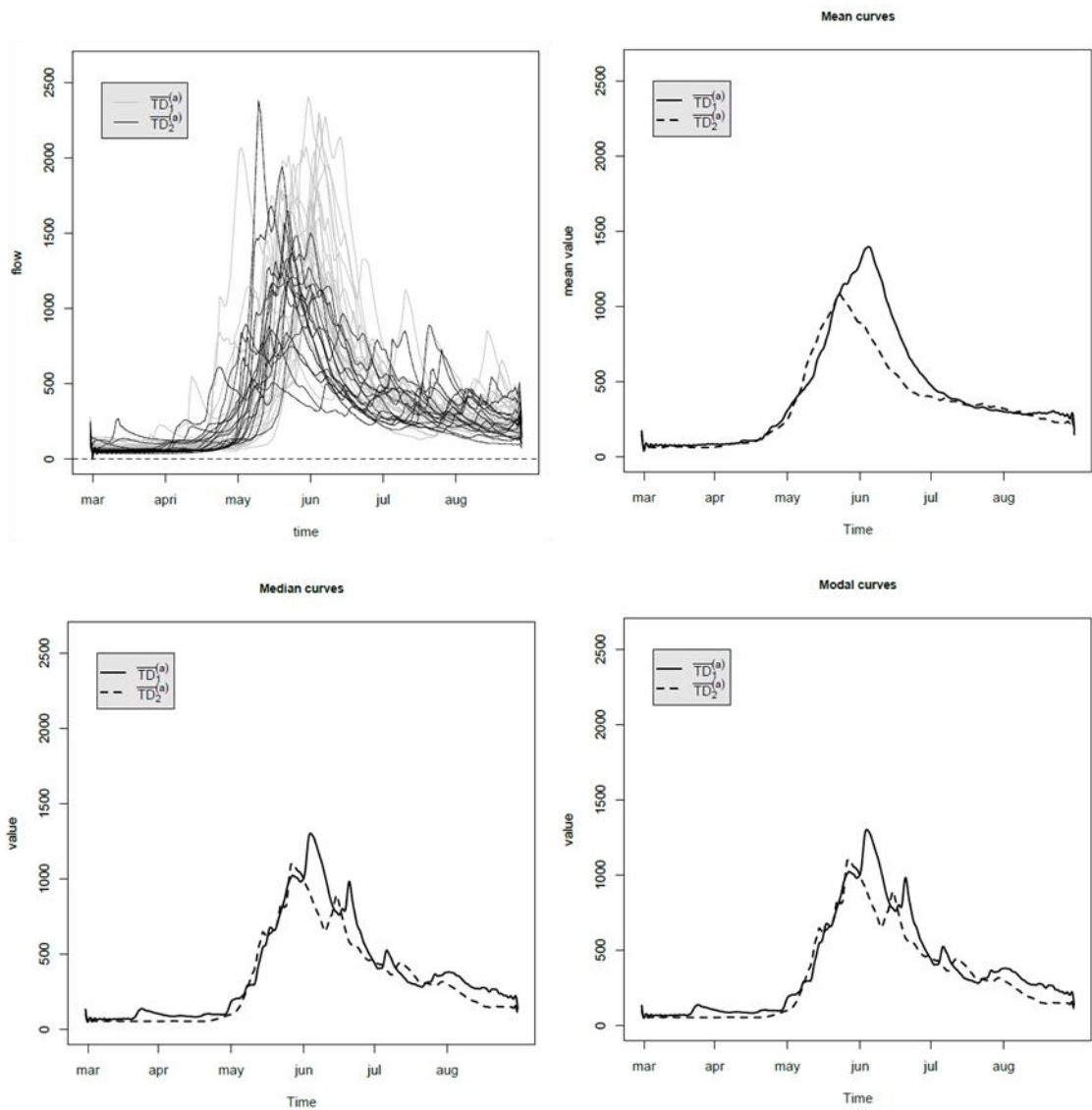


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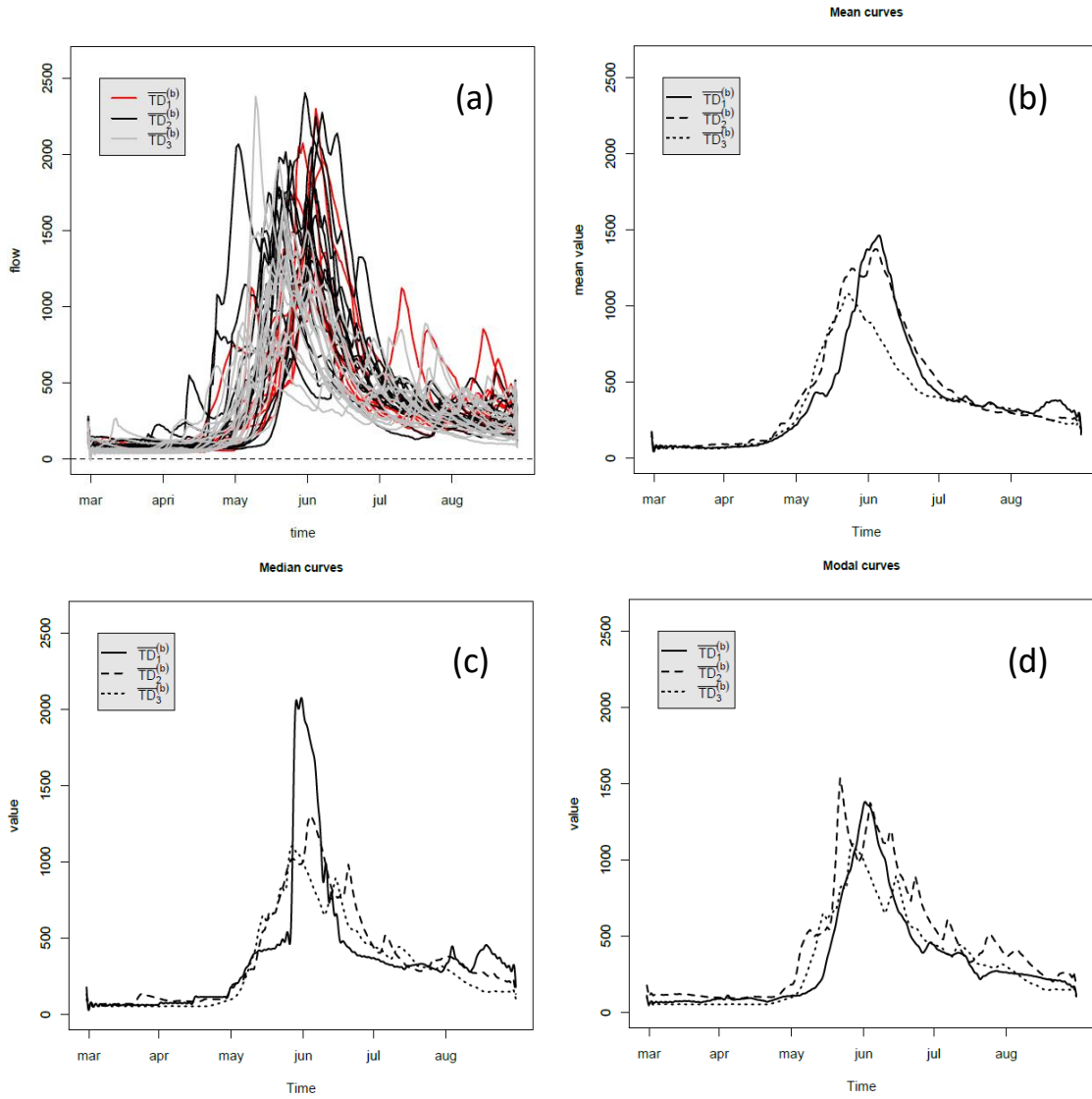
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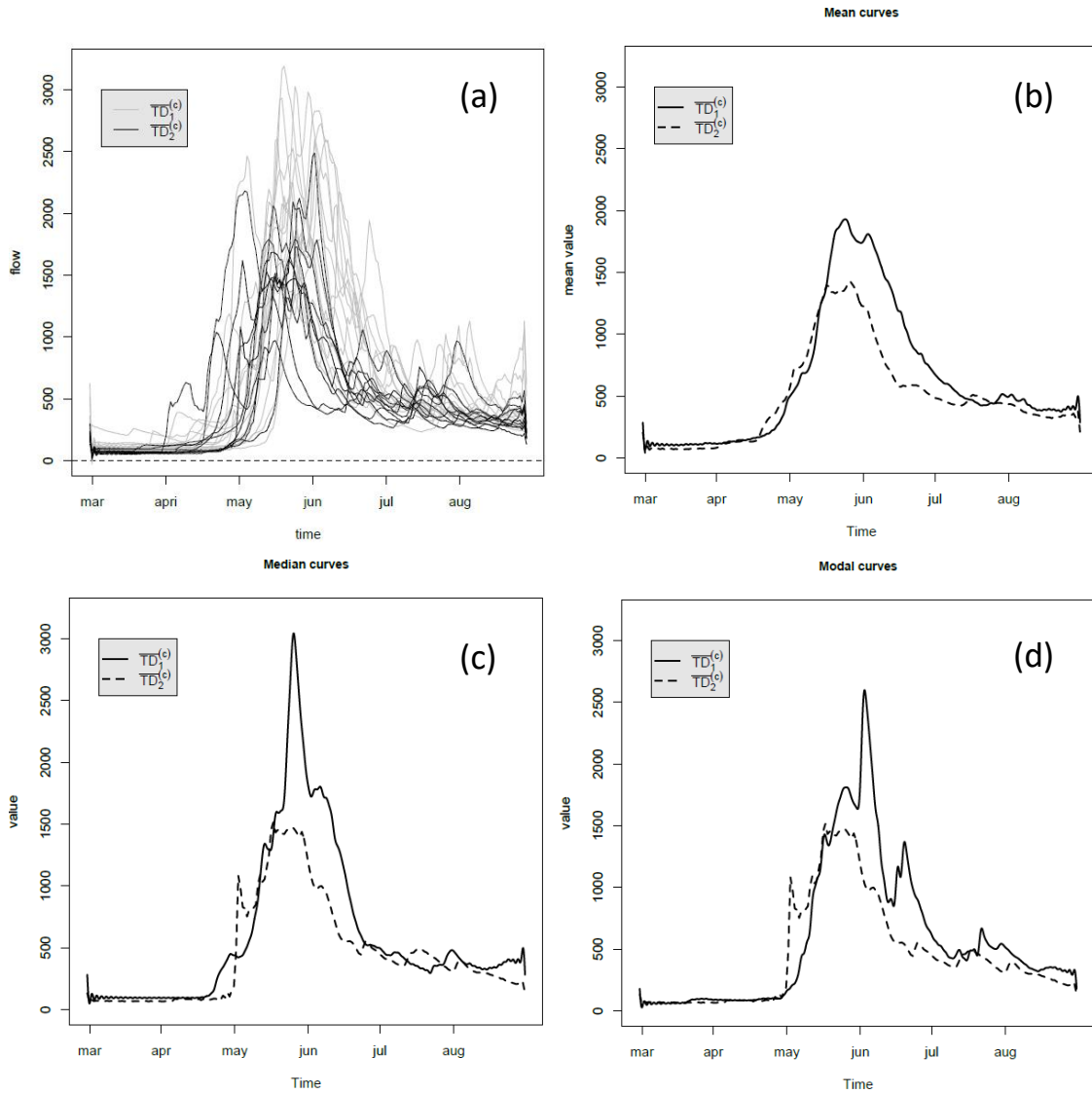
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